

Modelling the Milky Way in the Gaia era

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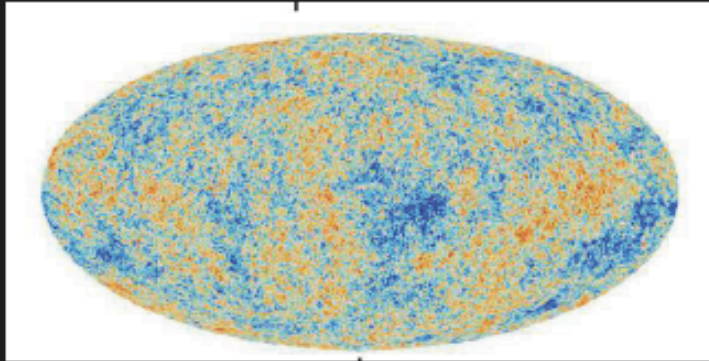


MW dynamics main questions

- Decipher the structure of the Galaxy, and of each of its components (stellar pops, gas), including its **dark matter** distribution, **in configuration space and phase-space** (total mass? core vs. cusp? , ...)
- Understand the various steps in the Galaxy formation process, understand **internal** secular processes (e.g., effect of spiral arms, bar) and **external** environmental ones (e.g., interactions with satellites)
- What are the exact roles of spirals (+ today's number of arms, pitch angle, pattern speed?) and the bar (length, pattern speed?) in the secular evolution (radial migration), how did they evolve? ...

MW dynamical models

TOP - DOWN



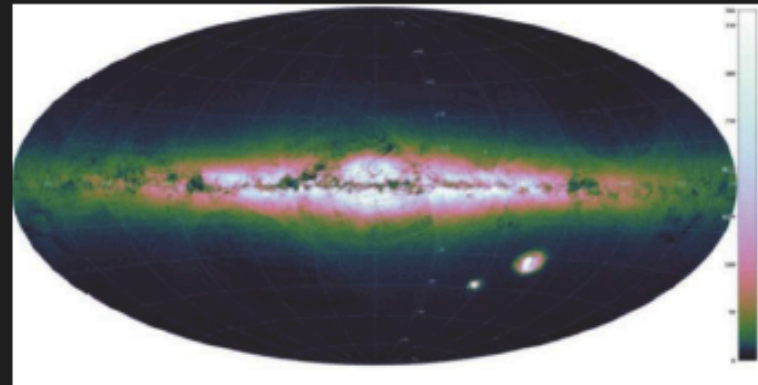
↓
N-body + hydro
simulations



BOTTOM - UP

$$f(\mathbf{x}, \mathbf{v}, t)$$
$$\Phi(\mathbf{x}, t)$$

↑
Collisionless Boltzmann
equation





Jeans theorem

- If integrable system: $df_0/dt = 0 \Leftrightarrow f_0(I_1, I_2, I_3)$
- Natural phase-space coordinates for regular orbits in (quasi)-integrable systems: **actions \mathbf{J} and angles θ**
= phase-space canonical coordinates such that $H=H(\mathbf{J})$
 $\Rightarrow f_0(\mathbf{J})$ with \mathbf{J} adiabatic invariants
- **A triplet of actions defines a regular orbit, angles tell us where the star is along that orbit**
- Phase-space is filled by orbital tori
 \Rightarrow use AGAMA ([Vasiliev 2019](#))



ACTIONFINDER

- Deep learning algorithm ([Ibata et al. 2021](#)) designed to transform a **sample of phase-space measurements along orbits** in an (**unknown**) static potential into action and angle coordinates, using the fact that stars along a same orbit have the same actions
- Start from "toy" potential (isochrone) with known actions and search for canonical transform:

$$G = G(\boldsymbol{\theta}, \boldsymbol{J}') \quad \longrightarrow \quad \begin{aligned} \boldsymbol{J} &= \boldsymbol{J}' + \frac{\partial G}{\partial \boldsymbol{\theta}} \\ \boldsymbol{\theta}' &= \boldsymbol{\theta} + \frac{\partial G}{\partial \boldsymbol{J}'} . \end{aligned}$$

The neural network then **searches for G**, minimizing a loss function (basically the spread in actions along each orbit)



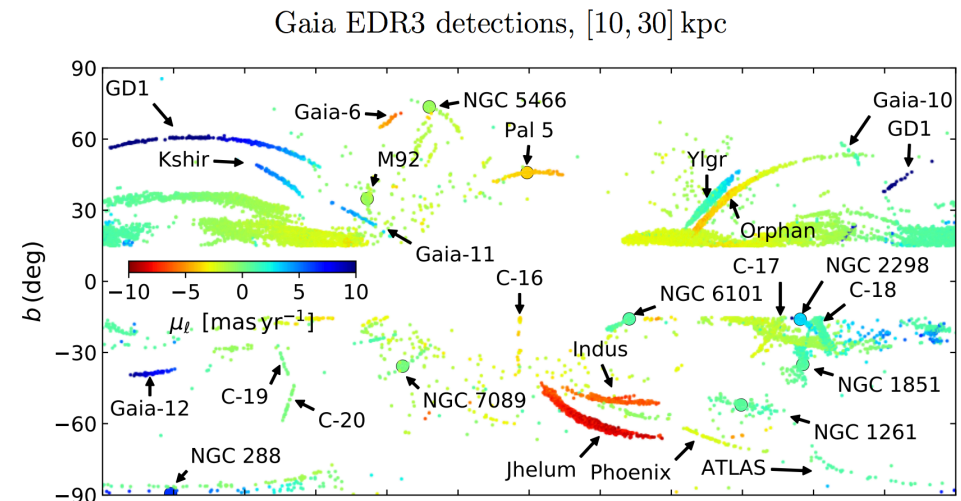
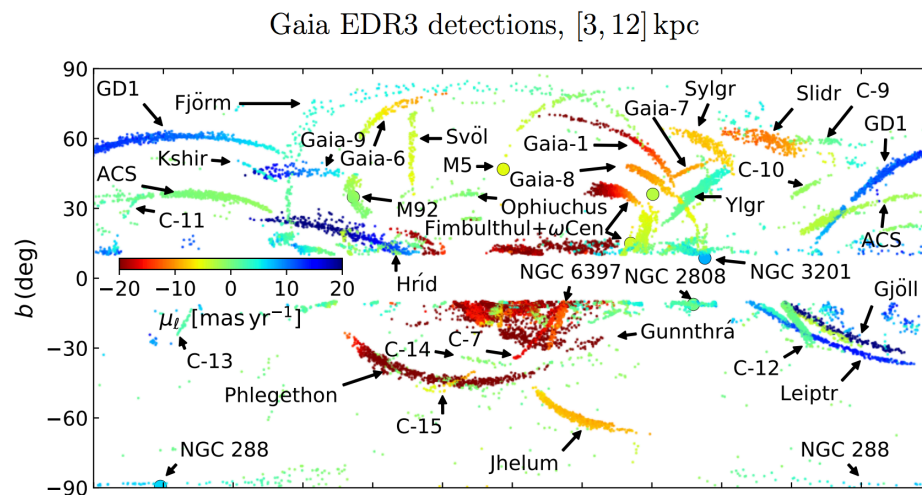
ACTIONFINDER

- With 8 points per orbit and 128 orbits (hence 1024 phase-space points), recovers the actions and angles from the Torus machinery of Binney & McMillan with 0.6% precision
- But most importantly: **recovers the (unknown) Hamiltonian and therefore Galactic potential**

Stellar streams *nearly* trace orbits

Streams (Ibata et al., Gaia EDR3):
32 streams in Gaia DR2, 7 new ones without
an obvious progenitor in EDR3

Find single stellar pops. and
integrate streams orbits in a tube
by exploring all distances and
radial vel. until stream candidate
found (STREAMFINDER)



15 with a globular cluster progenitor
(good distance, SSP template, and GC on the actual orbit)

Modelling the MW disc

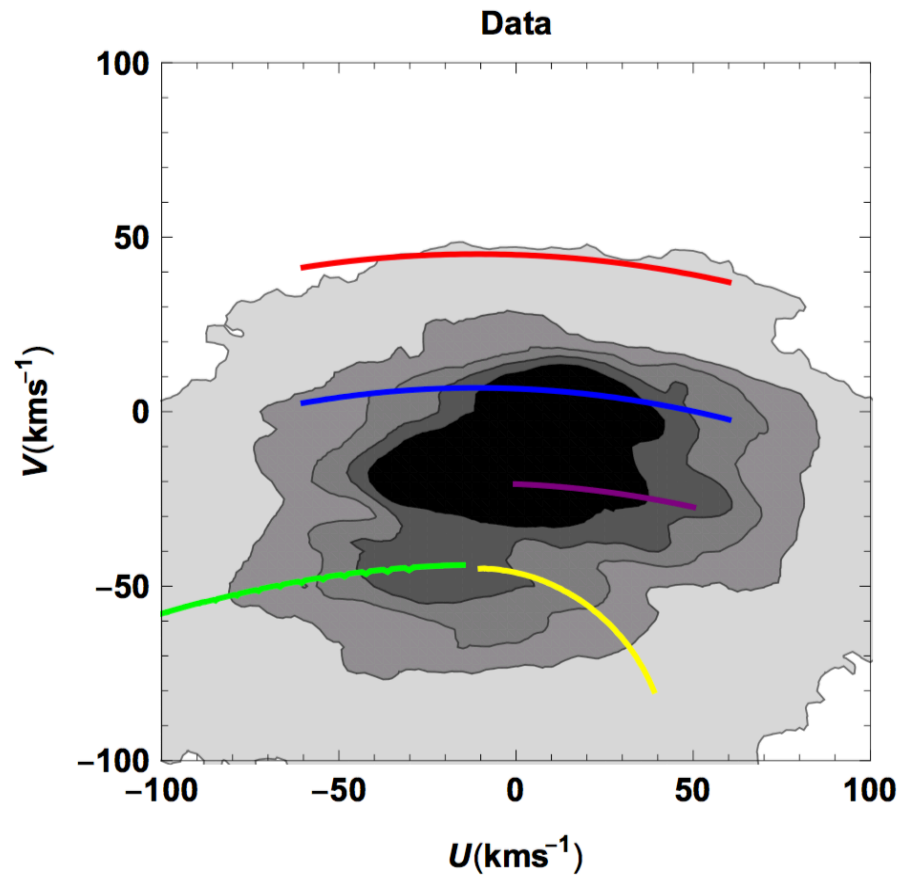
Adjust combination of parametric DFs:

$$f_0(J_R, J_\phi, J_z) = \frac{\Omega(R_g(J_\phi))}{(2\pi)^{3/2} 2\kappa(R_g(J_\phi))} \underbrace{\frac{\tilde{\Sigma}(R_g(J_\phi))}{\tilde{\sigma}_r^2(R_g(J_\phi)) \tilde{\sigma}_z^2(R_g(J_\phi)) z_0}}_{\text{radial distribution in } R_g(J_\phi)} \times \underbrace{e^{-\frac{J_R^2}{\tilde{\sigma}_r^2} - \frac{J_z^2}{\tilde{\sigma}_z^2}}}_{\text{velocity ellipsoid together with the velocity disp. dependence in previous factor}}$$

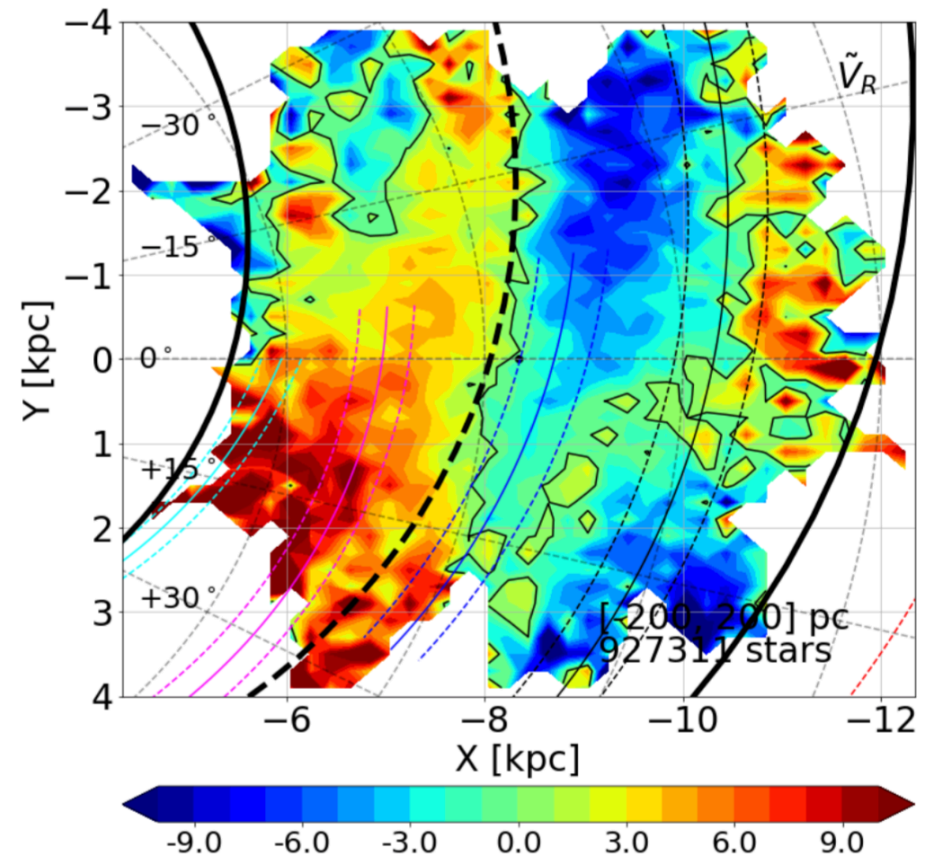
Even better: **non-parametric DF**: adjust with neural nets

But not so « simple »: the disc is perturbed by both internal non-axisymmetries and external perturbations!

Modelling the MW disc

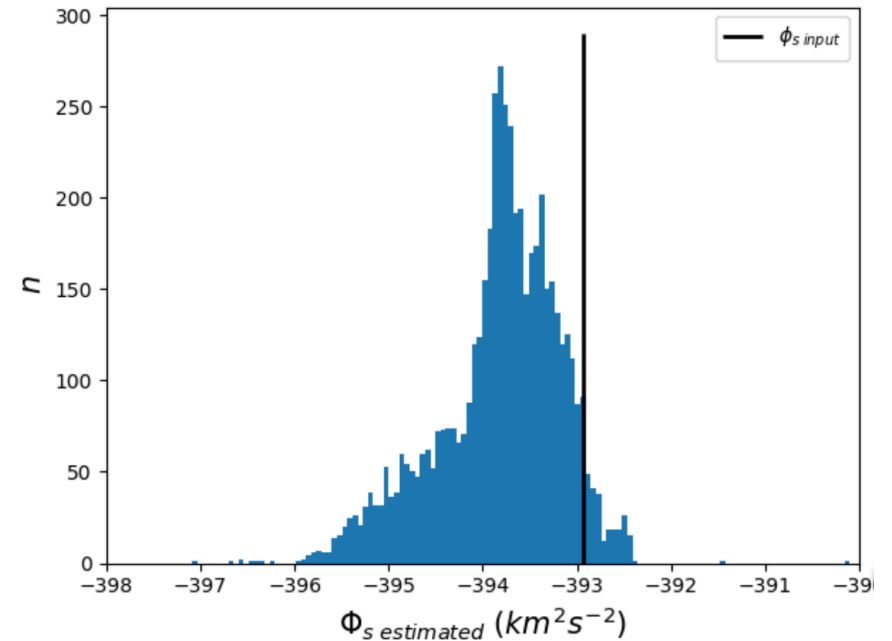
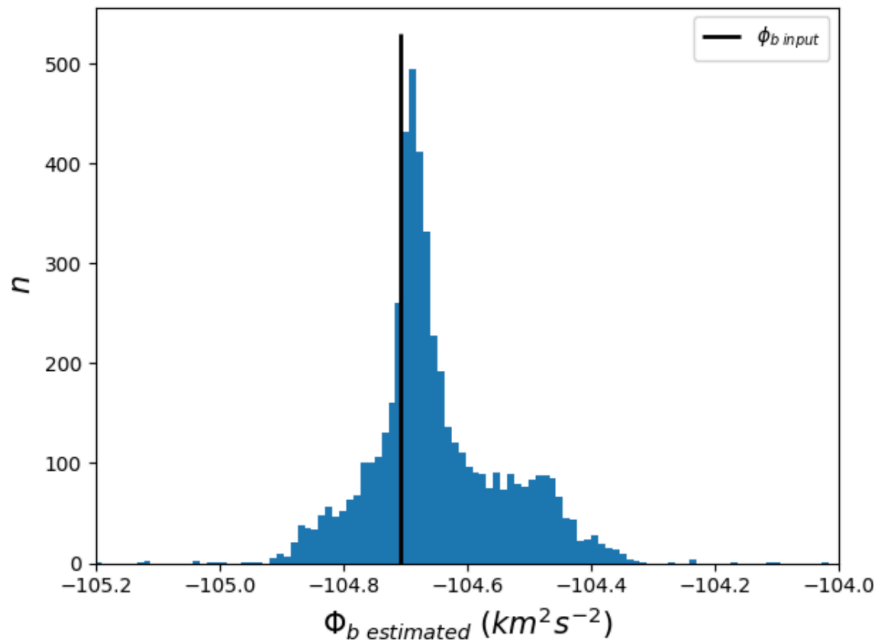


Local velocity space



Galactocentric radial velocity map

Expressing the bar potential in actions and angles



$$\Phi_1(R, \varphi, z) = \text{Re} \left\{ \sum_{j,l} \phi_{jml}(J_R, J_z, J_\varphi) e^{i(j\theta_R + m\theta_\varphi + l\theta_z)} \right\}$$

Al Kazwini et al. (2021)



Linearized CBE

$$\frac{df_1}{dt} = \frac{\partial f_0}{\partial \mathbf{J}} \cdot \frac{\partial \Phi_1}{\partial \boldsymbol{\theta}}$$

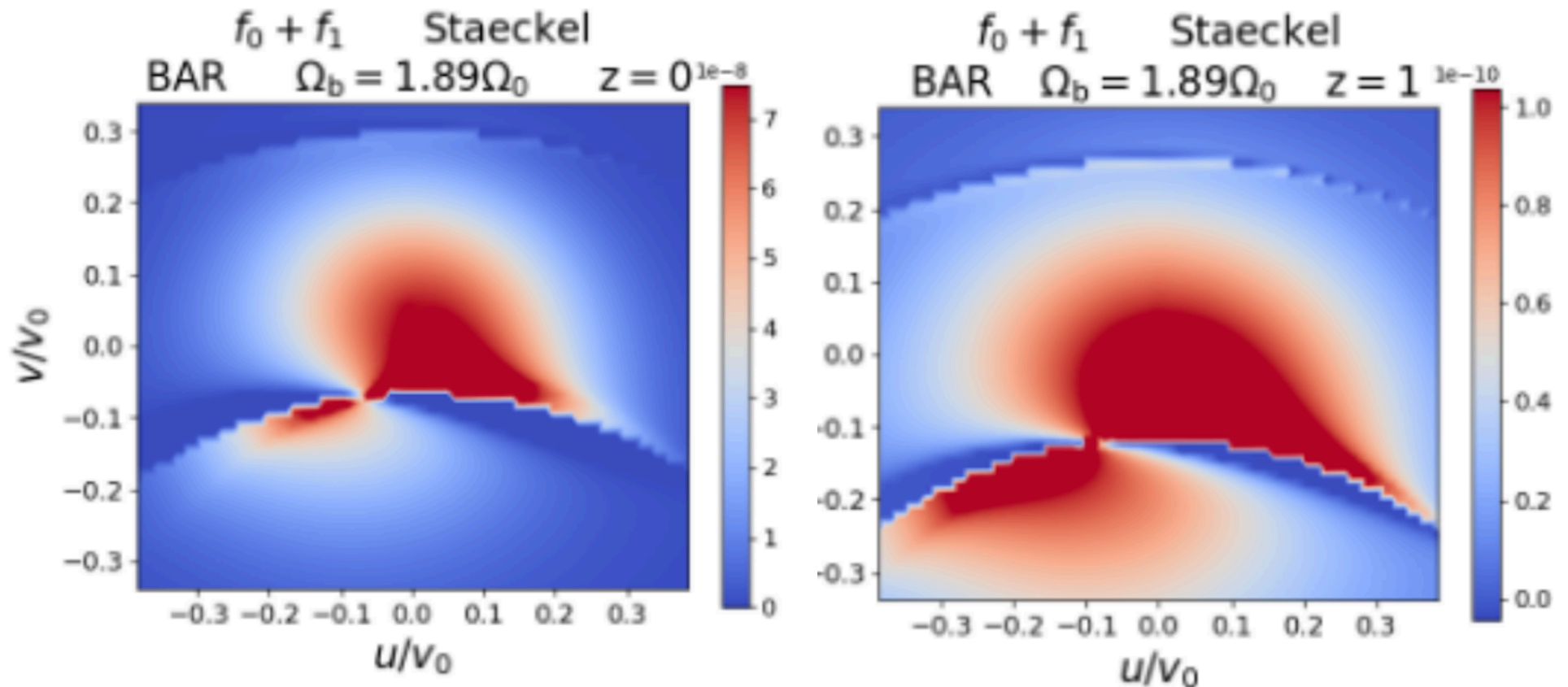
Integrate from zero amplitude bar to plateau of constant amplitude:

$$f_1(\mathbf{J}, \boldsymbol{\theta}, t) = \text{Re} \left\{ \sum_{j,l=-n}^n f_{jml} e^{i[j\theta_R + m(\theta_\varphi - \Omega_p t) + l\theta_z]} \right\},$$

$$f_{jml} = \phi_{jml} \times \frac{j \frac{\partial f_0}{\partial J_R} + m \frac{\partial f_0}{\partial J_\varphi} + l \frac{\partial f_0}{\partial J_z}}{j\omega_R + m(\omega_\varphi - \Omega_p) + l\omega_z}$$

Monari et al. (2016)

E.g., imposing $f_1 < f_0$ for resonance (1,2,0) of a fast bar:



Displacement of the resonance with z (corotation moves faster)
 + depends on the potential \Rightarrow new constraints with Gaia DR3

Treating resonances

Consider in-plane resonances (l,m): use **Arnold averaging principle**

=> change to slow angles that almost don't evolve at resonance and **average over fast angles** :

$$\begin{aligned}\theta_s &= l\theta_R + m(\theta_\phi - \Omega_b t), & J_\phi &= mJ_s, \\ \theta_f &= \theta_R, & J_R &= lJ_s + J_f.\end{aligned}$$

$$\overline{H} = H_0(J_f, J_s) - m\Omega_b J_s + \text{Re} \left\{ \phi_{lm}(J_f, J_s) e^{i\theta_s} \right\}$$

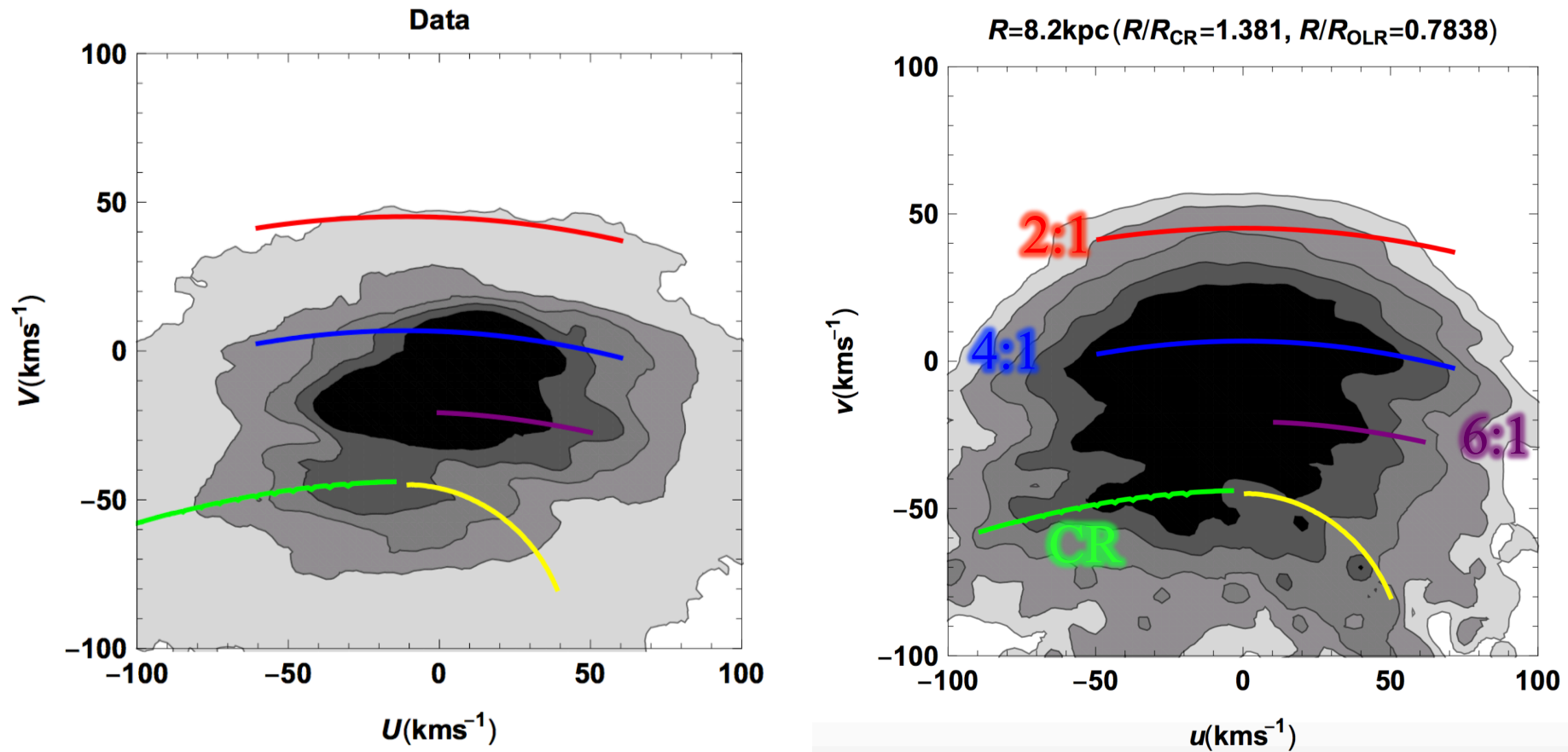
For each J_f , define J_{sres} such that $\omega_s=0$ and expand around J_{sres}

=> Hamiltonian of a pendulum of angle θ_s

=> New canonical transform to pendulum actions and angles (J_p, θ_p)

=> Phase-mix the original DF over θ_p

Treating resonances



Monari et al. (2019)

$V_{\odot} = 0 \text{ km/s}$, declining RC allows to get a more realistic $V_{\odot} = 8 \text{ km/s}$



Ridges as a function of azimuth

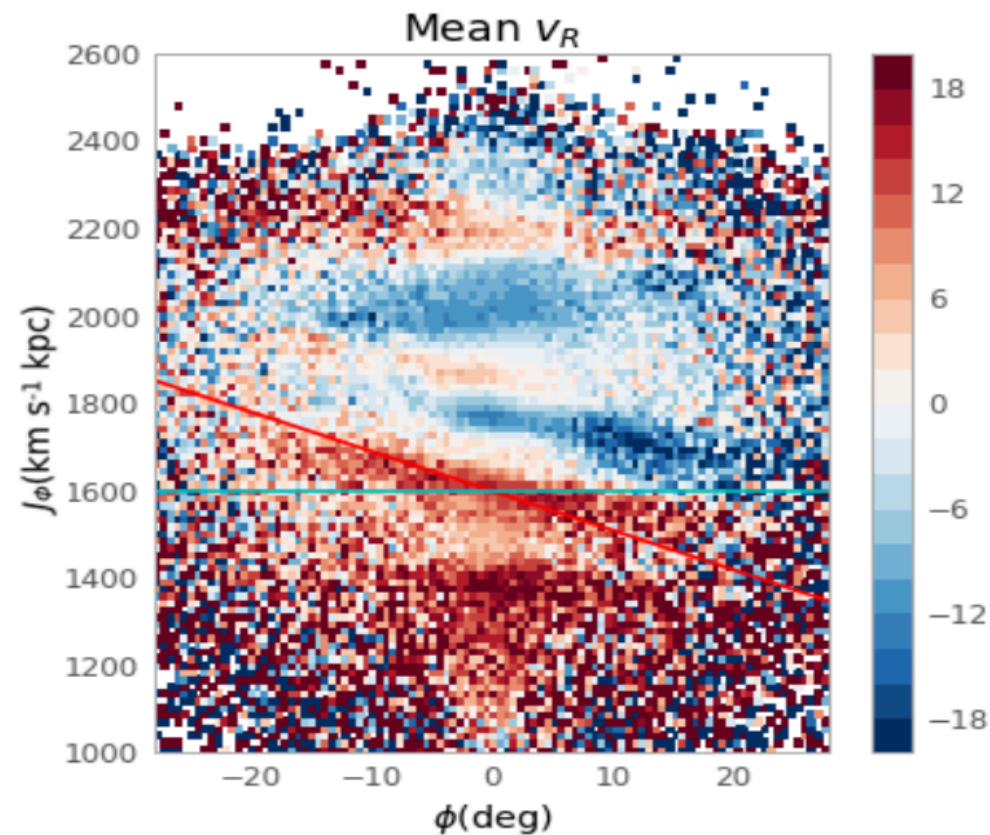
$$\begin{aligned}\theta_s &= l\theta_R + m(\theta_\phi - \Omega_b t), & J_\phi &= mJ_s, \\ \theta_f &= \theta_R, & J_R &= lJ_s + J_f.\end{aligned}$$

At the $(l,m) = (1,2)$ OLR, the azimuthal angles of trapped orbits can vary fast while the angular momentum varies slowly.

But NOT at the $(l,m)=(0,2)$ CR, where any large change in azimuthal angle is accompanied by a large change of angular momentum

=> The J_ϕ location of the CR varies faster in azimuth than the OLR

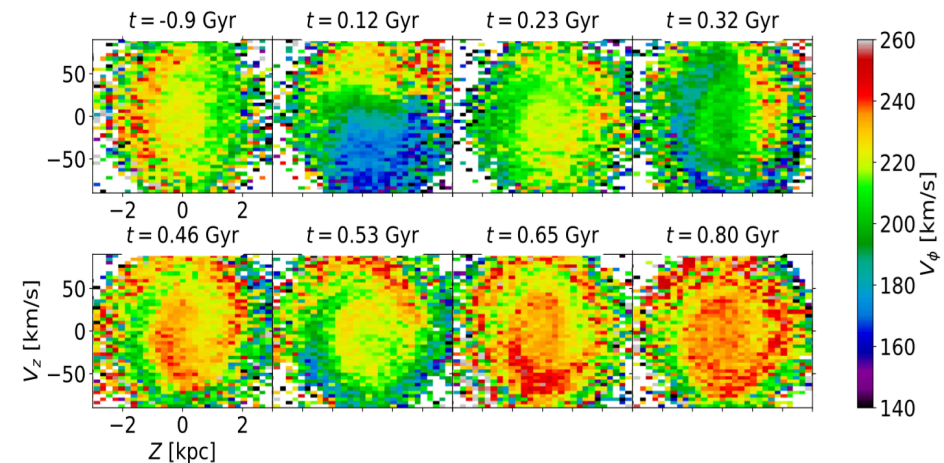
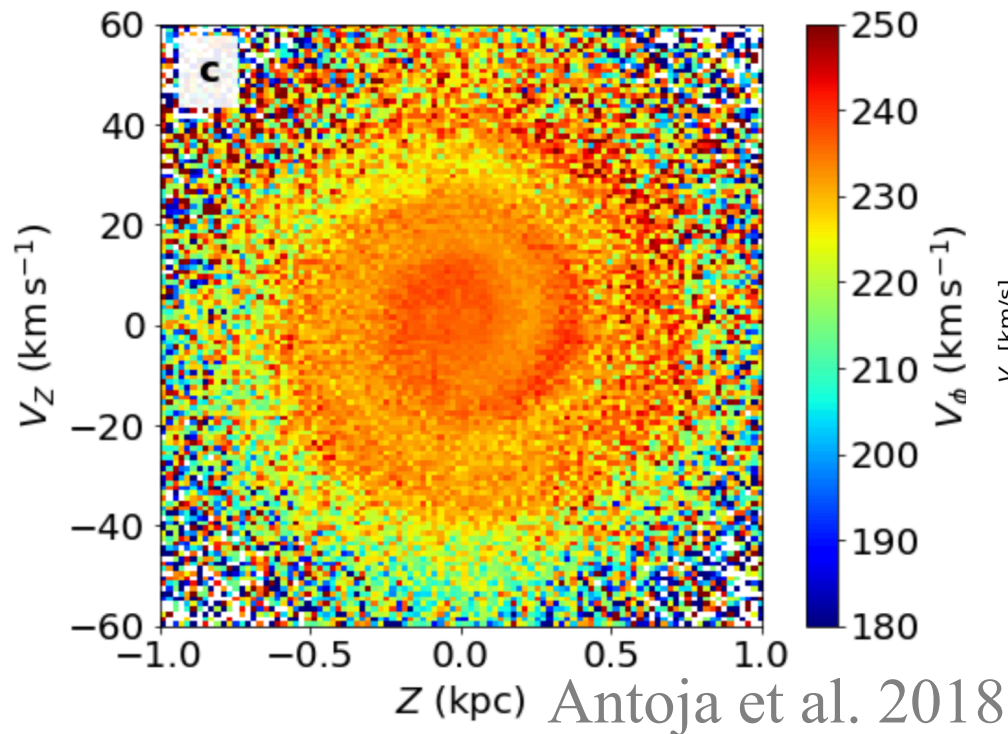
Ridges as a function of azimuth



400 pc annulus around the Sun
(StarHorse bayesian distance estimates)

[Monari et al. 2019](#)

The disk is vertically perturbed too



Laporte et al. 2018
(last pericentric passage of
Sgr dwarf at $t=0$)

- ⇒ Can traditional Jeans modelling be applied? **NO** (Haines et al. 2019)
- ⇒ Can we neglect self-gravity of the disk? **NO** (Khoperskov et al. 2019)

Similar (but less intense) phase spirals survive > 1 Gyr after bar buckling

The disk is vertically perturbed too

Taking self-gravity into account needs simultaneously solving
CBE and Poisson

=> Use bi-orthogonal basis functions that solve Poisson
(basis functions appropriate for thickened disks)

$$\psi^s(\mathbf{x}, t) = \sum_p a_p(t) \psi^{(p)}(\mathbf{x}); \quad \psi^e(\mathbf{x}, t) = \sum_p b_p(t) \psi^{(p)}(\mathbf{x}) \leftarrow \text{The Sgr dwarf potential}$$

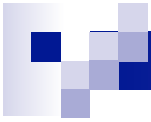
$$a_p(t) = - \int d\mathbf{x} \int d\mathbf{v} f_1(\mathbf{x}, \mathbf{v}, t) \psi^{(p)*}(\mathbf{x}) \quad [\text{equivalent to integrating over } J \text{ and } \theta]$$

insert solution of linearized CBE and develop
the perturbing potential ($\Psi_s + \Psi_e$) on the basis
functions (as a sum over q)

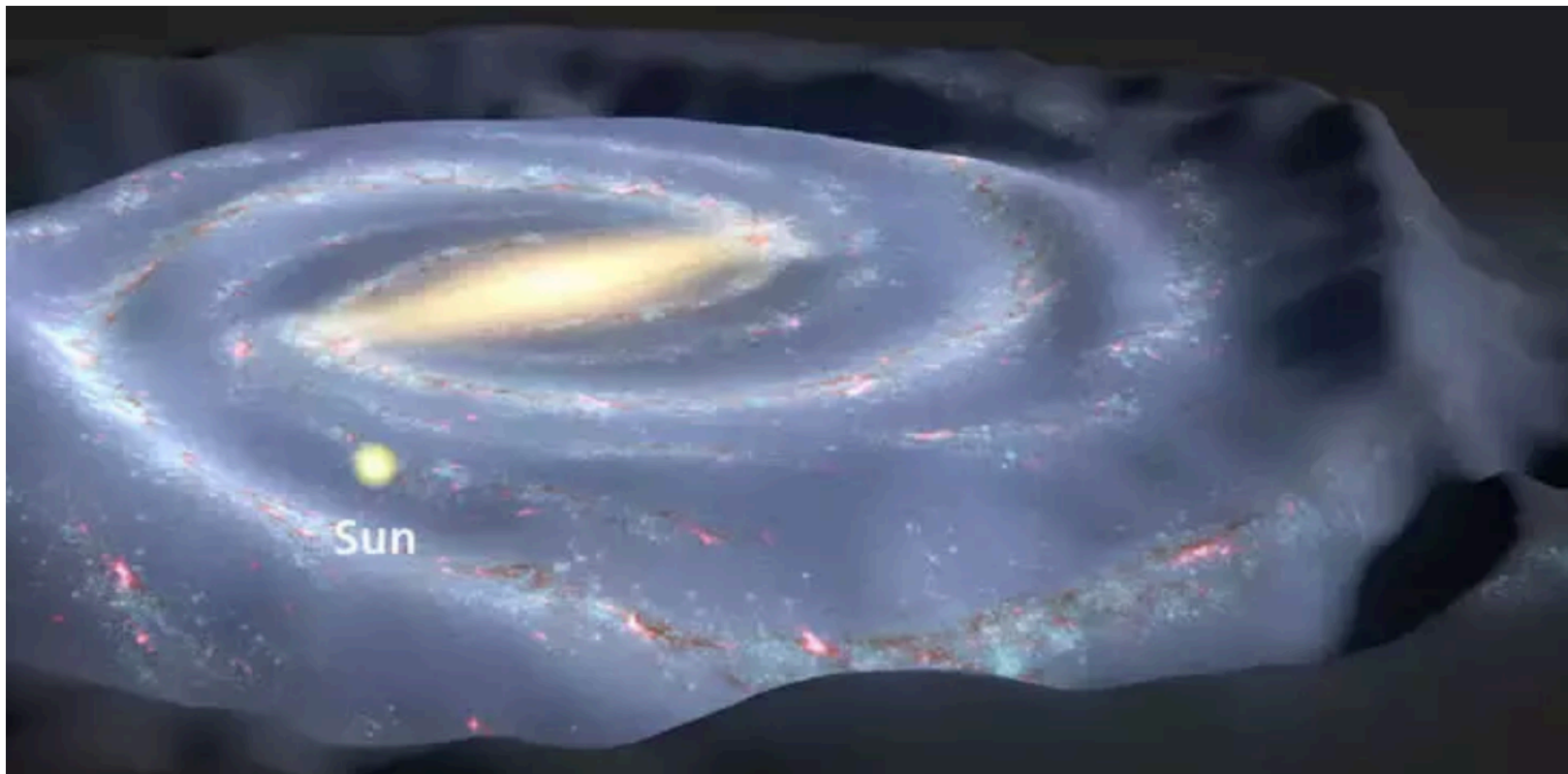


$$\mathbf{a}(t) = \int_0^t d\tau \mathbf{M}(t - \tau) [\mathbf{a}(\tau) + \mathbf{b}(\tau)]$$

Work led by
S. Rozier



$$\mathbf{M}_{pq}(t) = -i (2\pi)^3 \sum_{\mathbf{n}} \int d\mathbf{J} \, \mathbf{n} \cdot \frac{\partial F_0}{\partial \mathbf{J}} \psi_{\mathbf{n}}^{(p)*}(\mathbf{J}) \psi_{\mathbf{n}}^{(q)}(\mathbf{J}) e^{-i \mathbf{n} \cdot \boldsymbol{\Omega} t}$$





Conclusion and next steps

- Detected dozens of streams => probes of the Galactic potential making use of ACTIONFINDER (+ self-consistent phase-space modelling of DM and testing alternatives)
- Disc: 2D analytic formalism for resonances of bar and spirals
=> **MW bar with CR at 6 kpc** qualitatively reproduces a surprisingly large amount of features in local action-space and velocity-space
- Next step: combine the treatment at resonances with the linear response to combine the bar and spiral arms (when no resonance overlap), **fit** to data on larger scales (velocity field, ridges,...)
- Vertically perturbed disk => Jeans modelling inappropriate
=> **We need to work on the appropriate analytic formalism (Matrix method, SEGAL ANR)**



Data: what's next?

- Next year: Gaia DR3 will improve even more the observational situation (e.g., RVS data for 3.5×10^7 stars down to $G \sim 15$)
- Next year: **WEAVE** as spectroscopic counterpart to Gaia. High-res survey ($R \sim 20000$) will allow chemical labelling to $G \sim 16$ for $\sim 1.2 \times 10^6$ stars
 - + Low-res surveys (disk and HighLat) for $\sim 2.75 \times 10^6$ stars ($R \sim 5000$) deep in the disk and halo down to $G \sim 20$