SORBONNE UNIVERSITÉ

cnrs

Stellar Dynamics in Galactic Nuclei

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Habilitation à diriger les recherches

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Nicholas STONE	— Rapporteur	Martin Weinberg	— Examinateur
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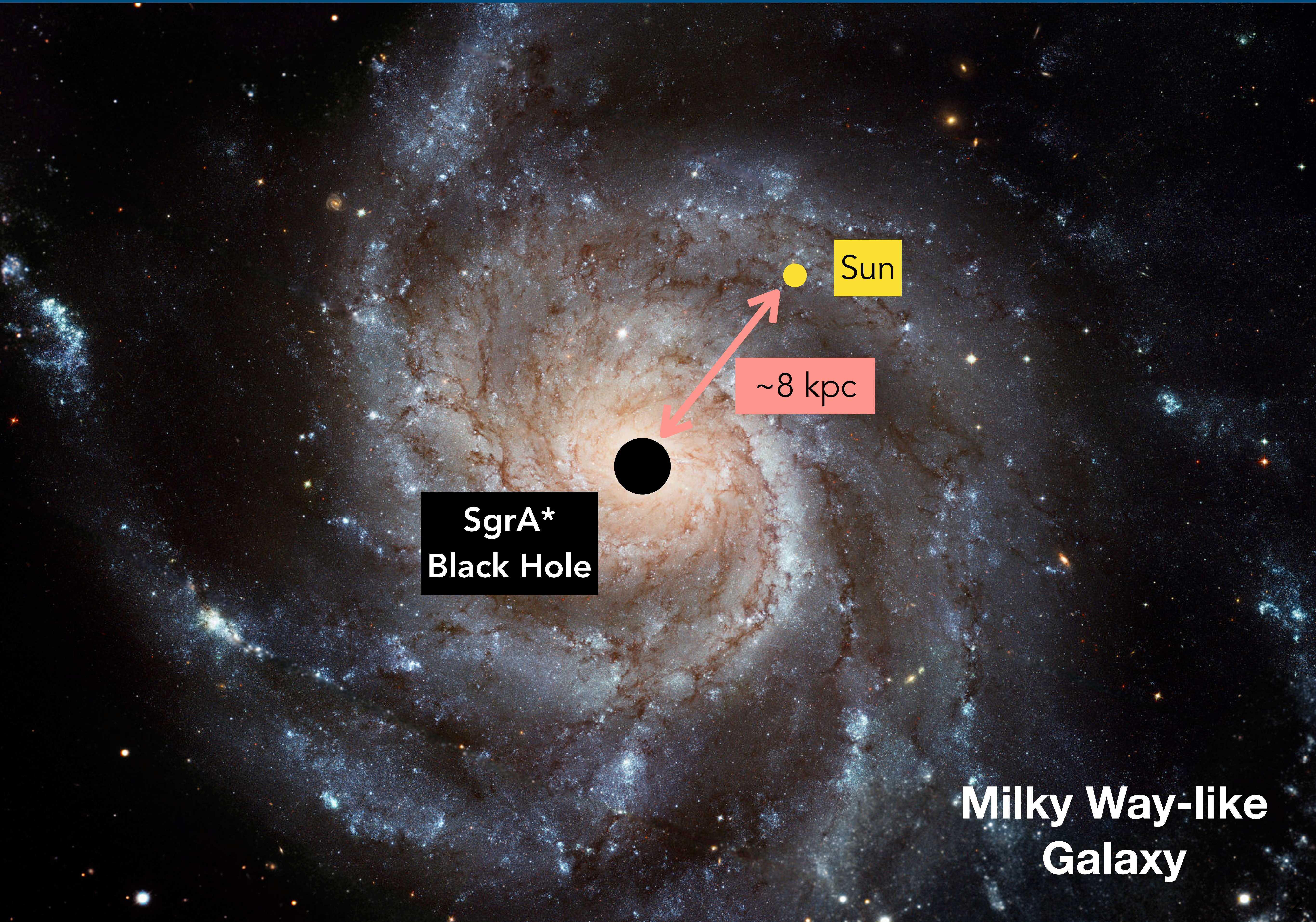
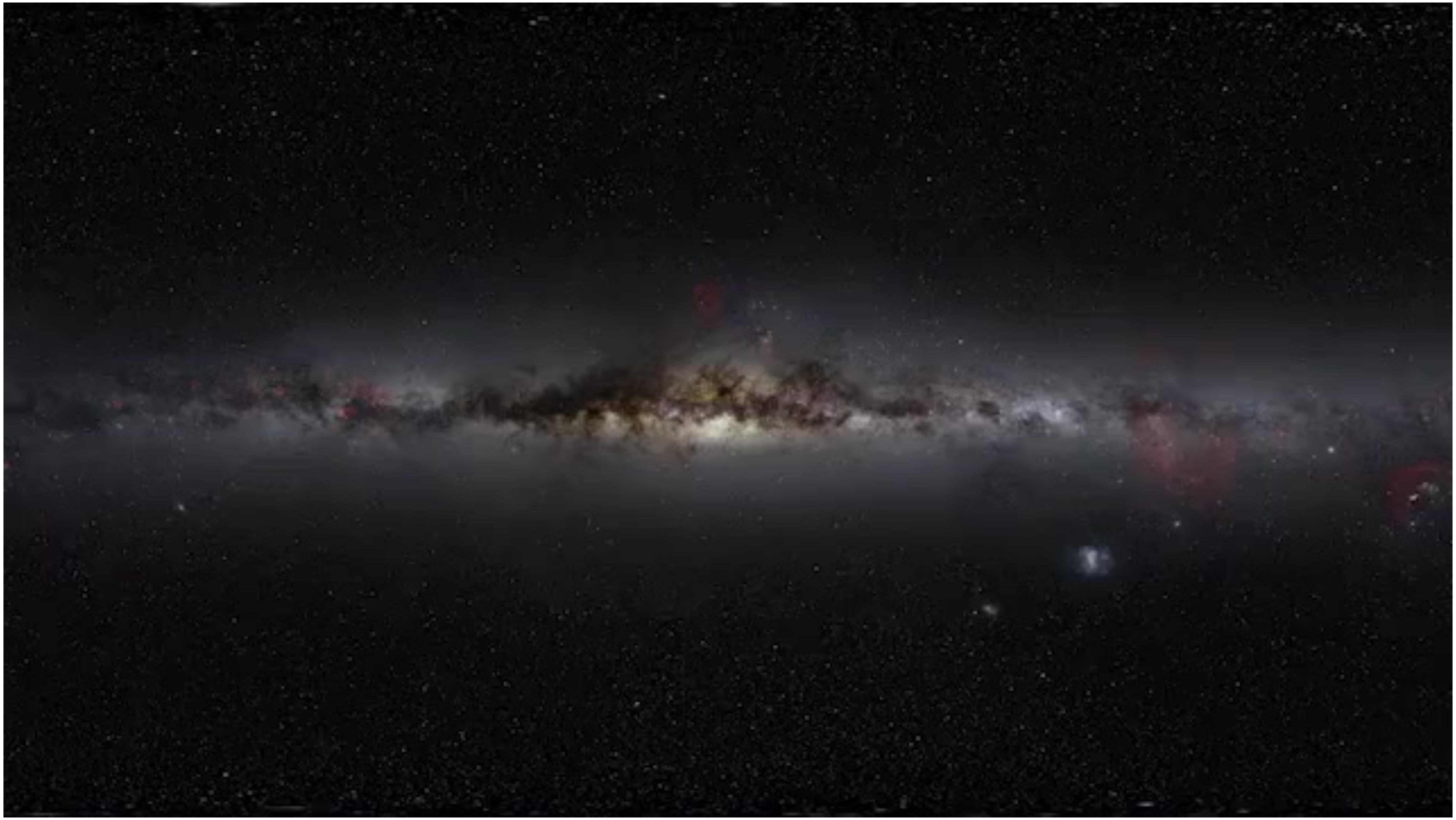

Sun
~8 kpc
SgrA*
Black Hole
Milky Way-like
Galaxy

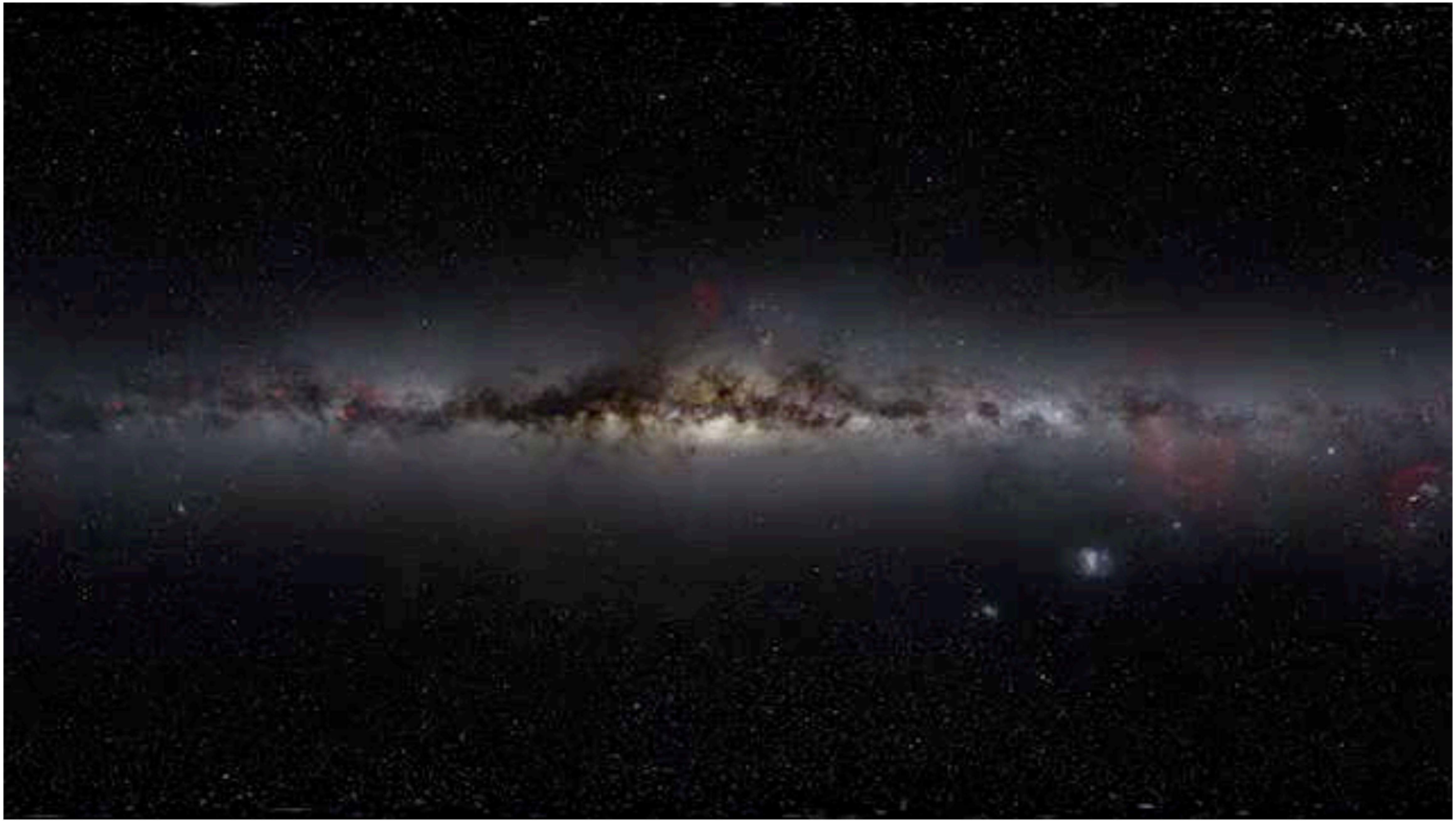
SgrA*, our Galactic Centre



ESO

Milky Way (10^{17} km)

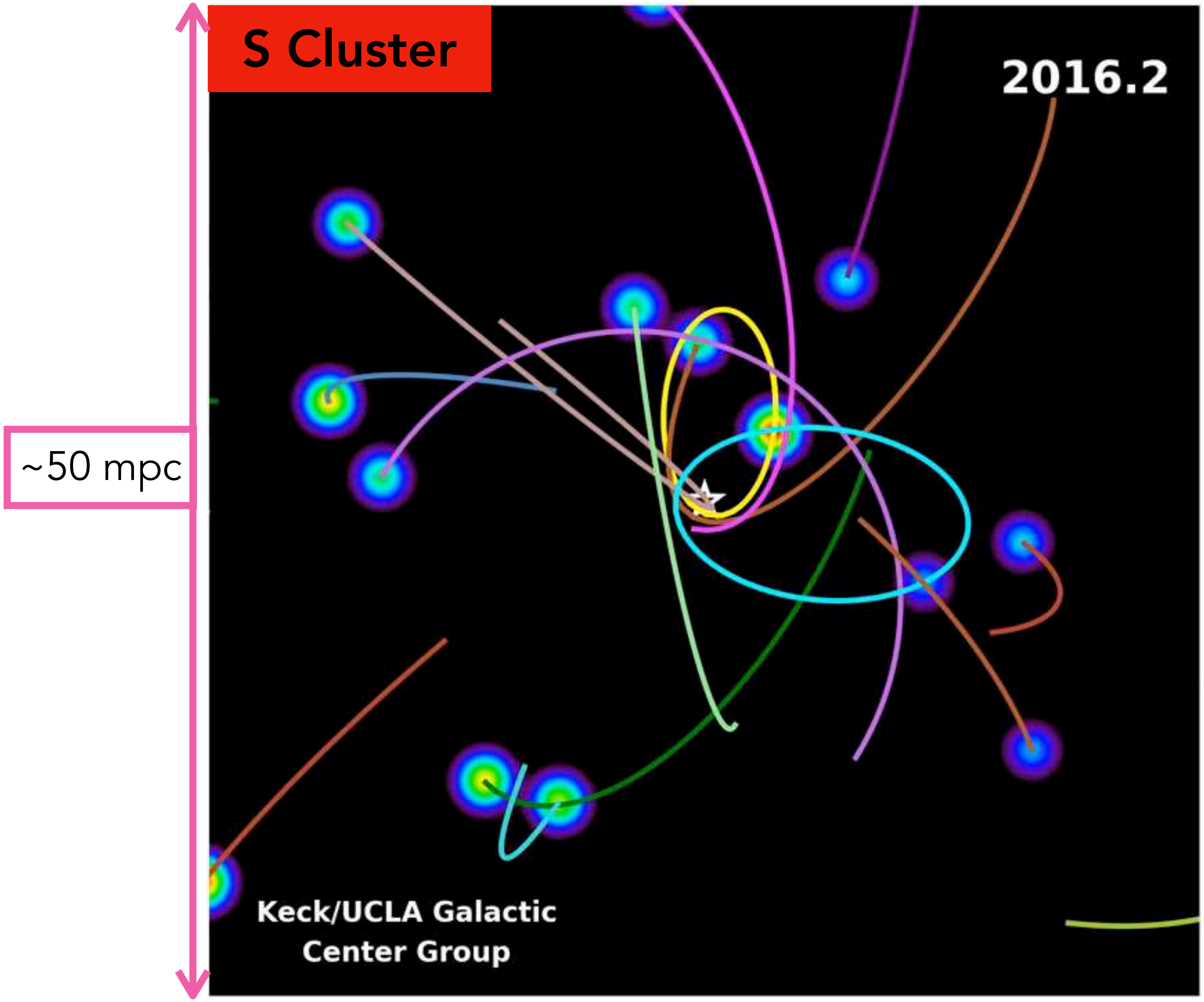
SgrA*, our Galactic Centre



ESO

Zoom (x10,000,000)

SgrA*, at the heart of the Milky Way

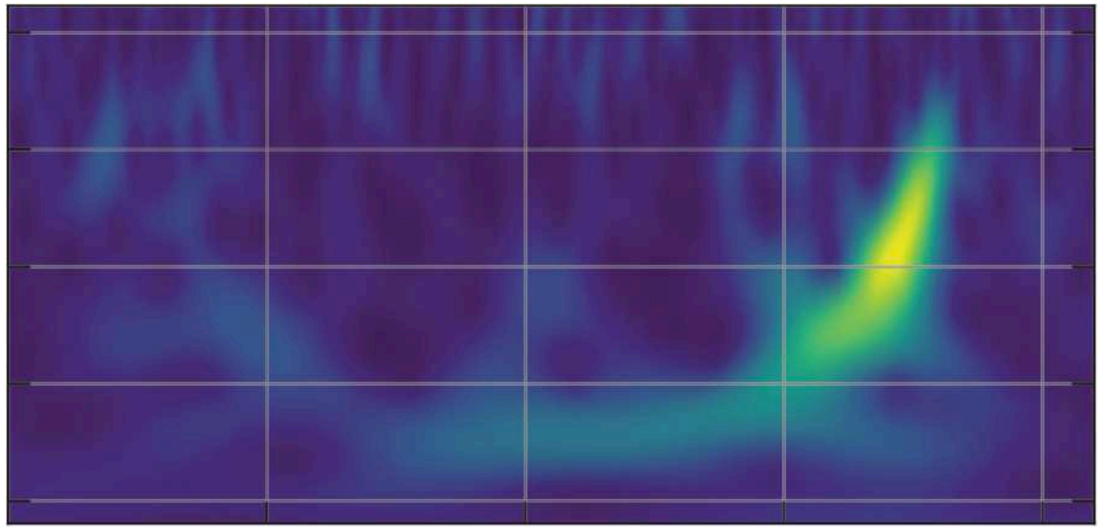


What is the diet of **supermassive black holes**?

Galactic nuclei are exciting

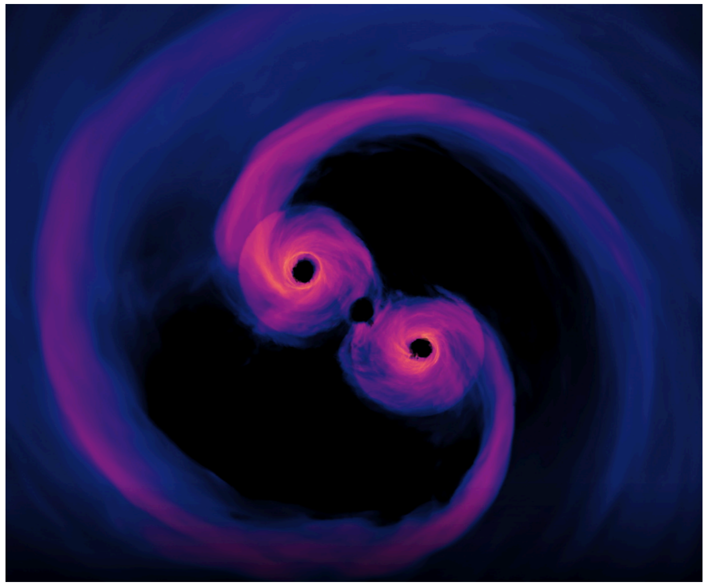
Gravitational waves

LIGO+(2015)



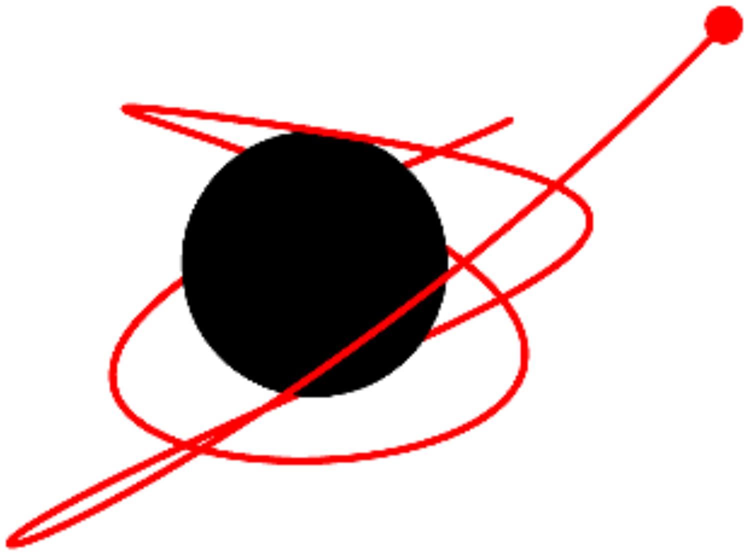
SMBH merger

Pulsar Timing Array



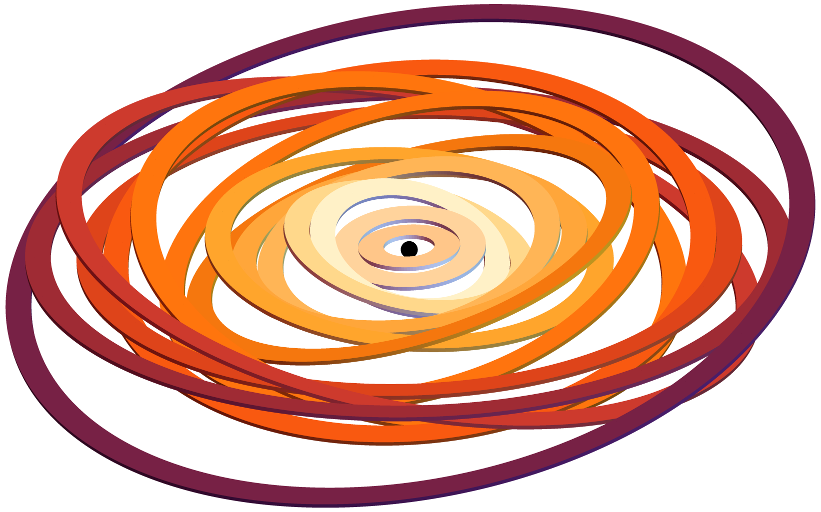
EMRIs

LISA



Discs of IMBHs

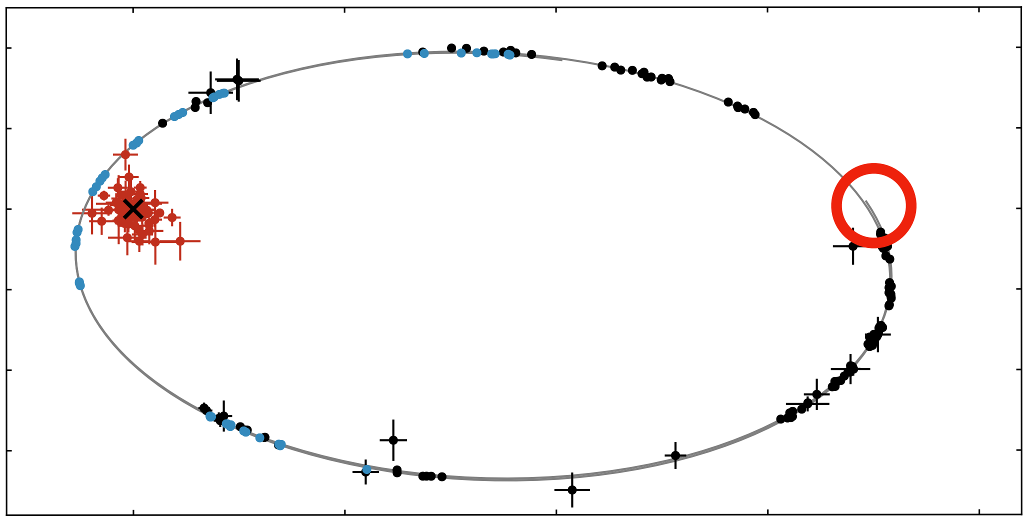
Szölgyen+(2018)



SgrA* is exciting

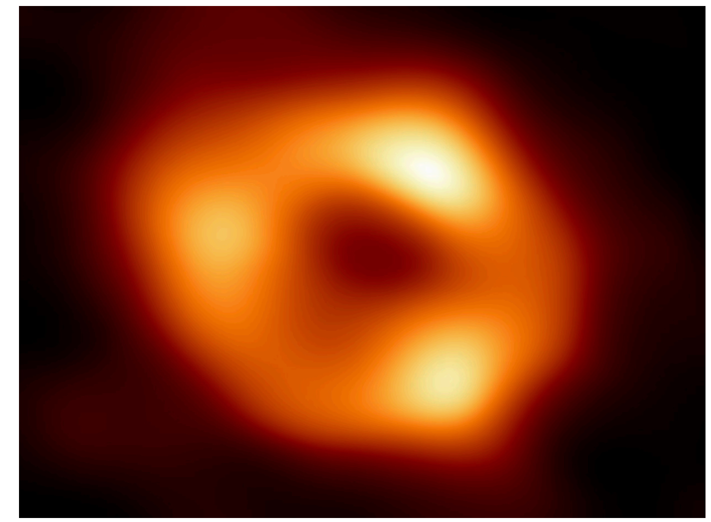
S2's **relativistic precession**

Gravity+(2020)



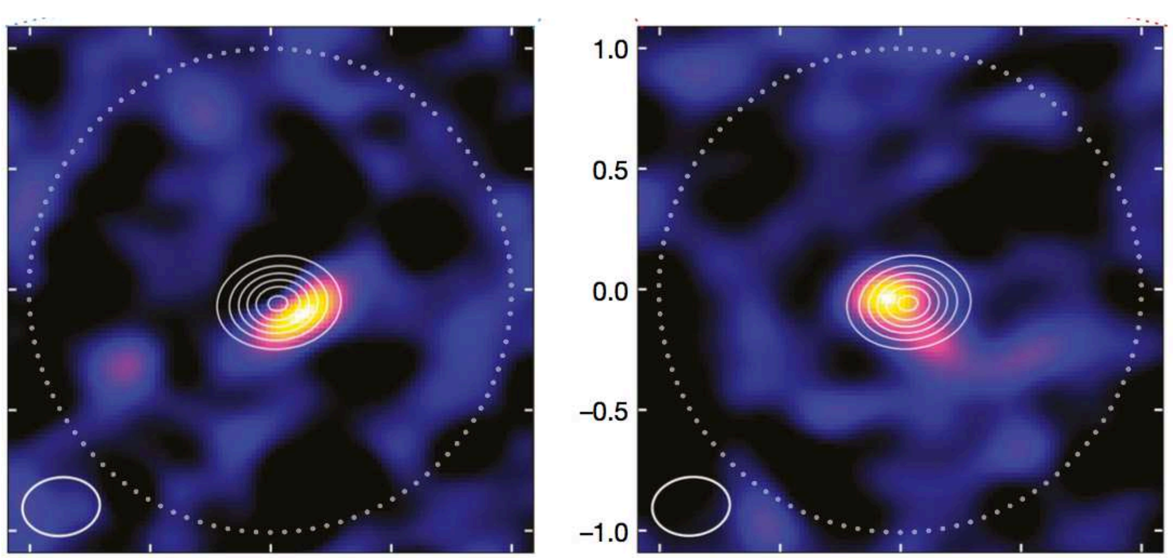
Event Horizon

EHT+(2022)



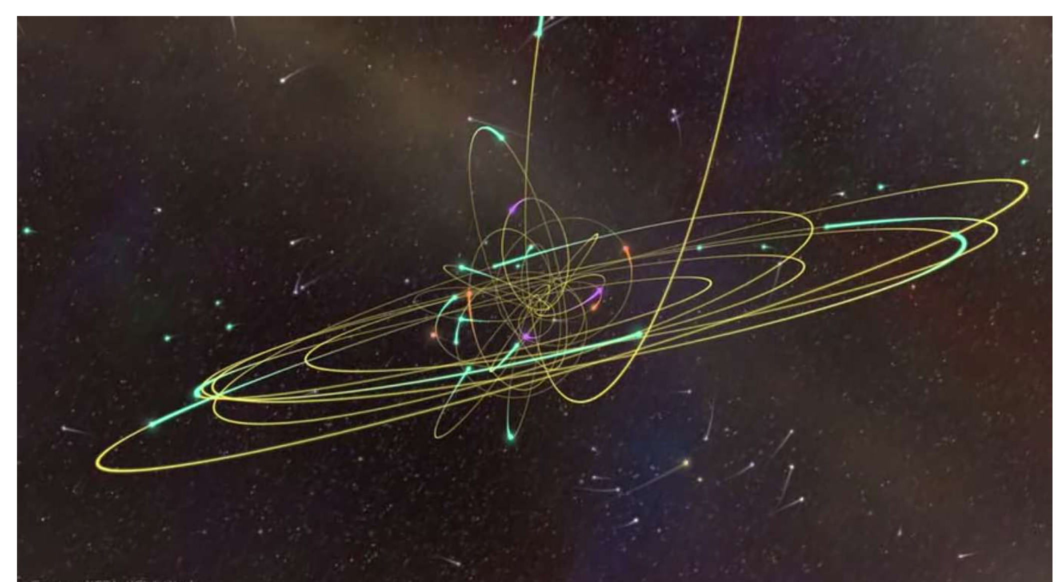
Cold **accretion** disc

Murchikova+(2019)

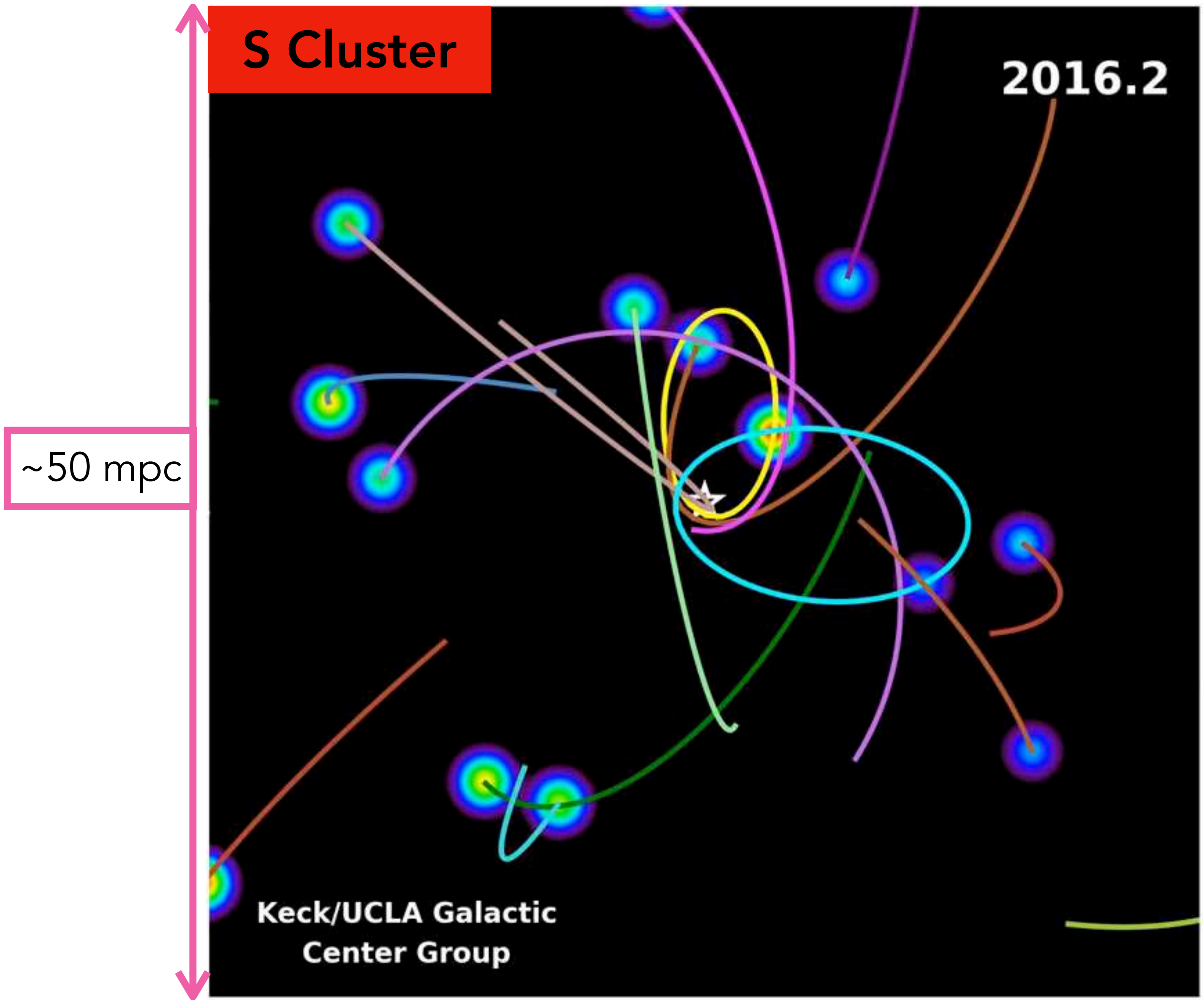


Clockwise **stellar disc**

Keck



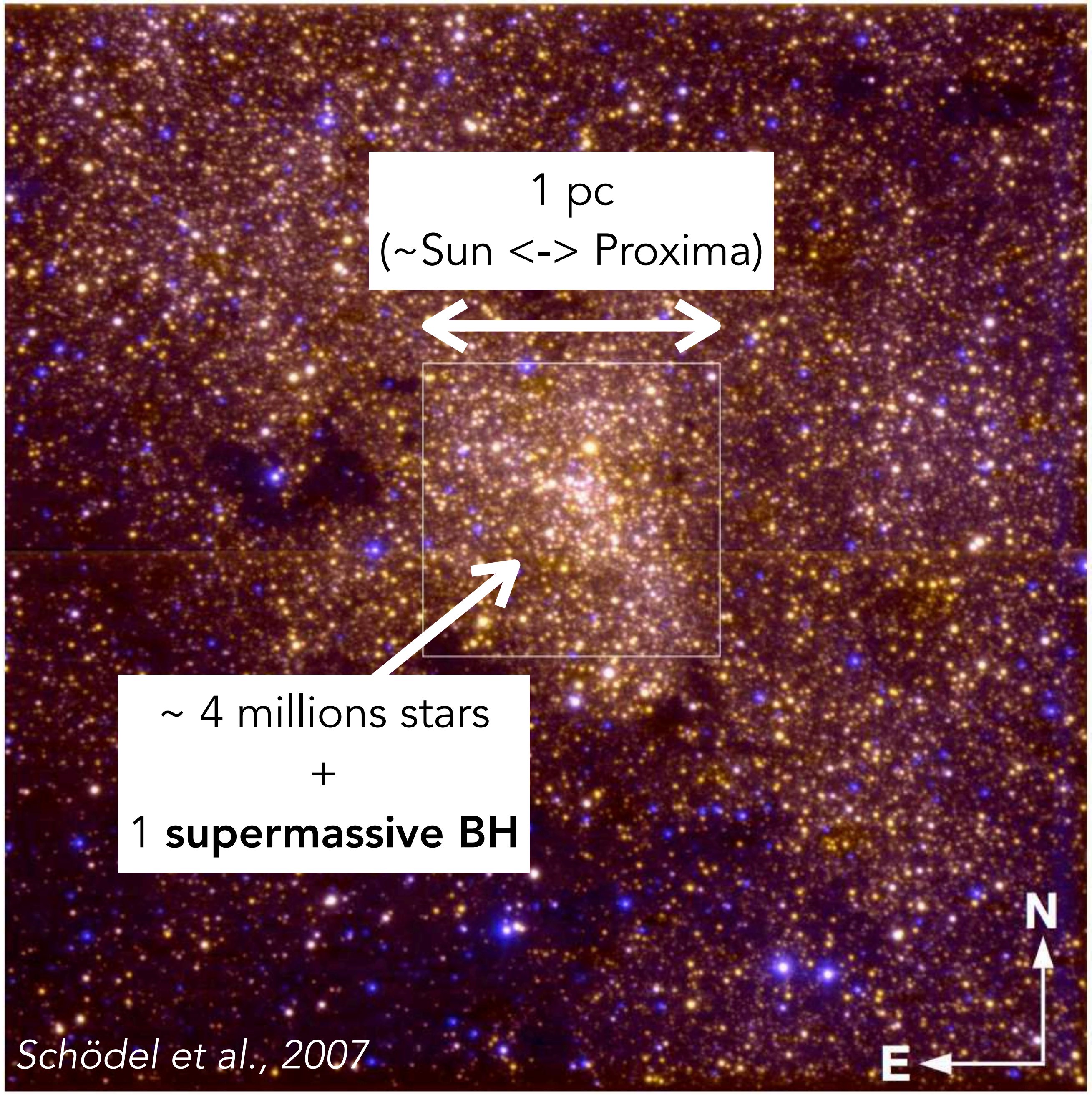
SgrA*, at the heart of the Milky Way



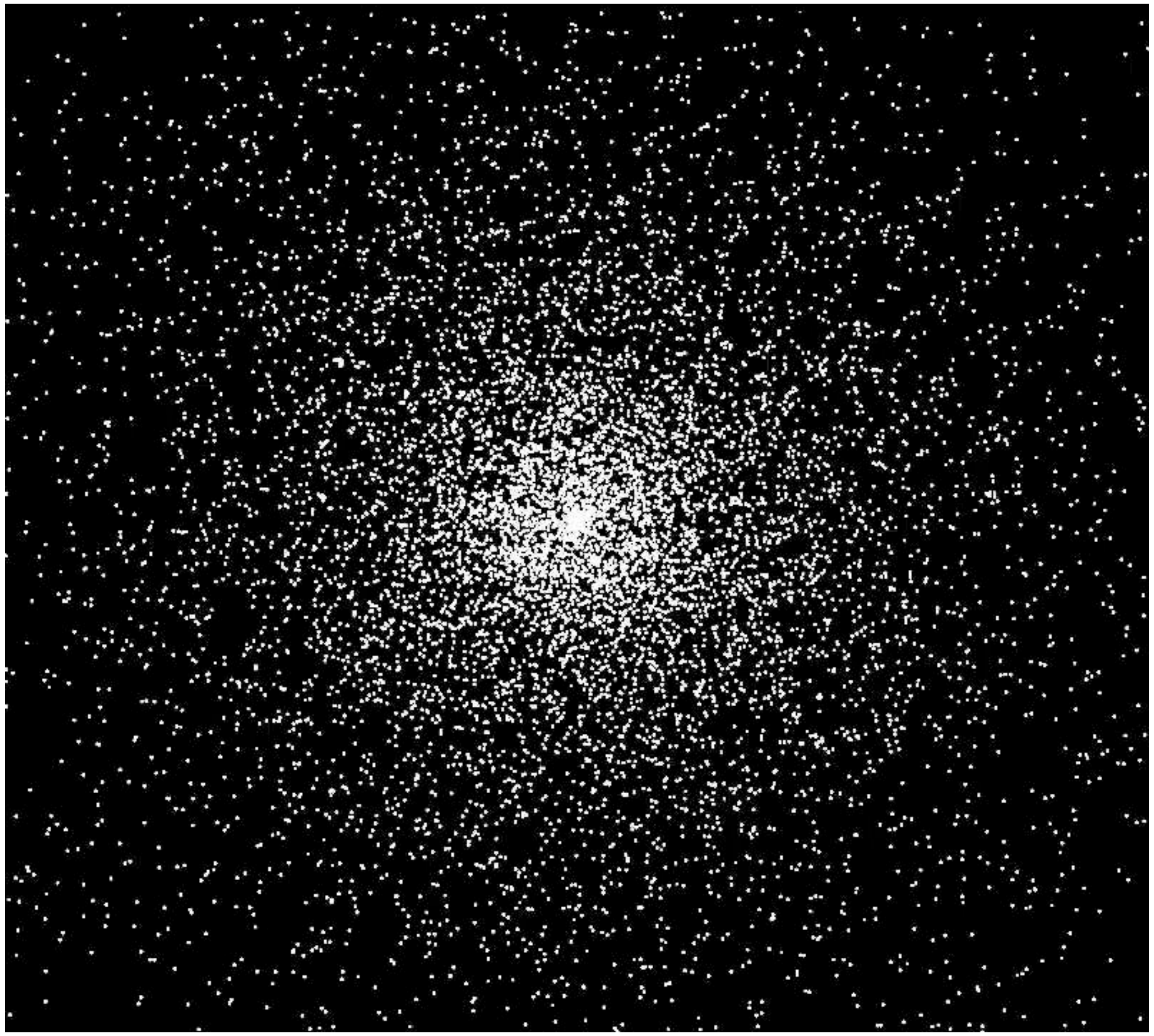
What is the diet of **supermassive black holes**?

Extremely dense environment

Behaves like a **gas of stars**



VLT observations

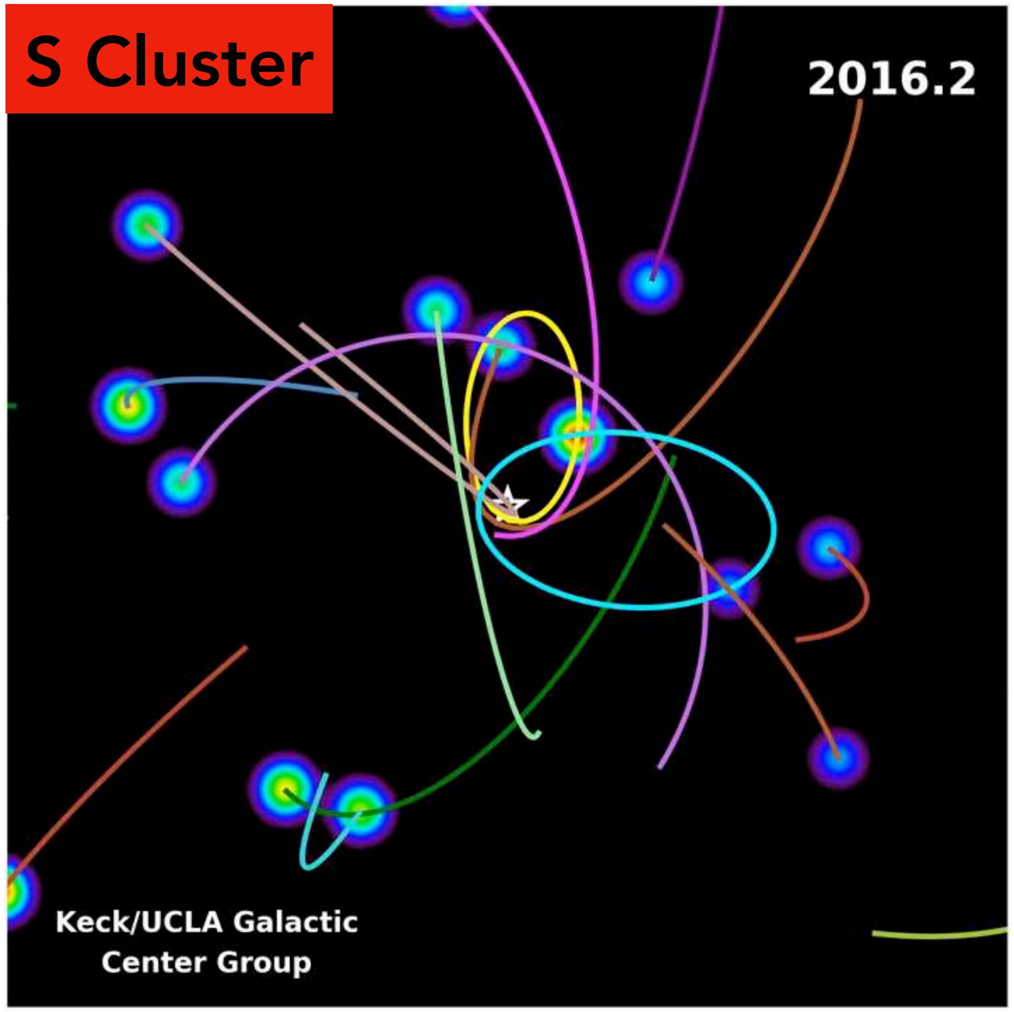


Numerical simulations

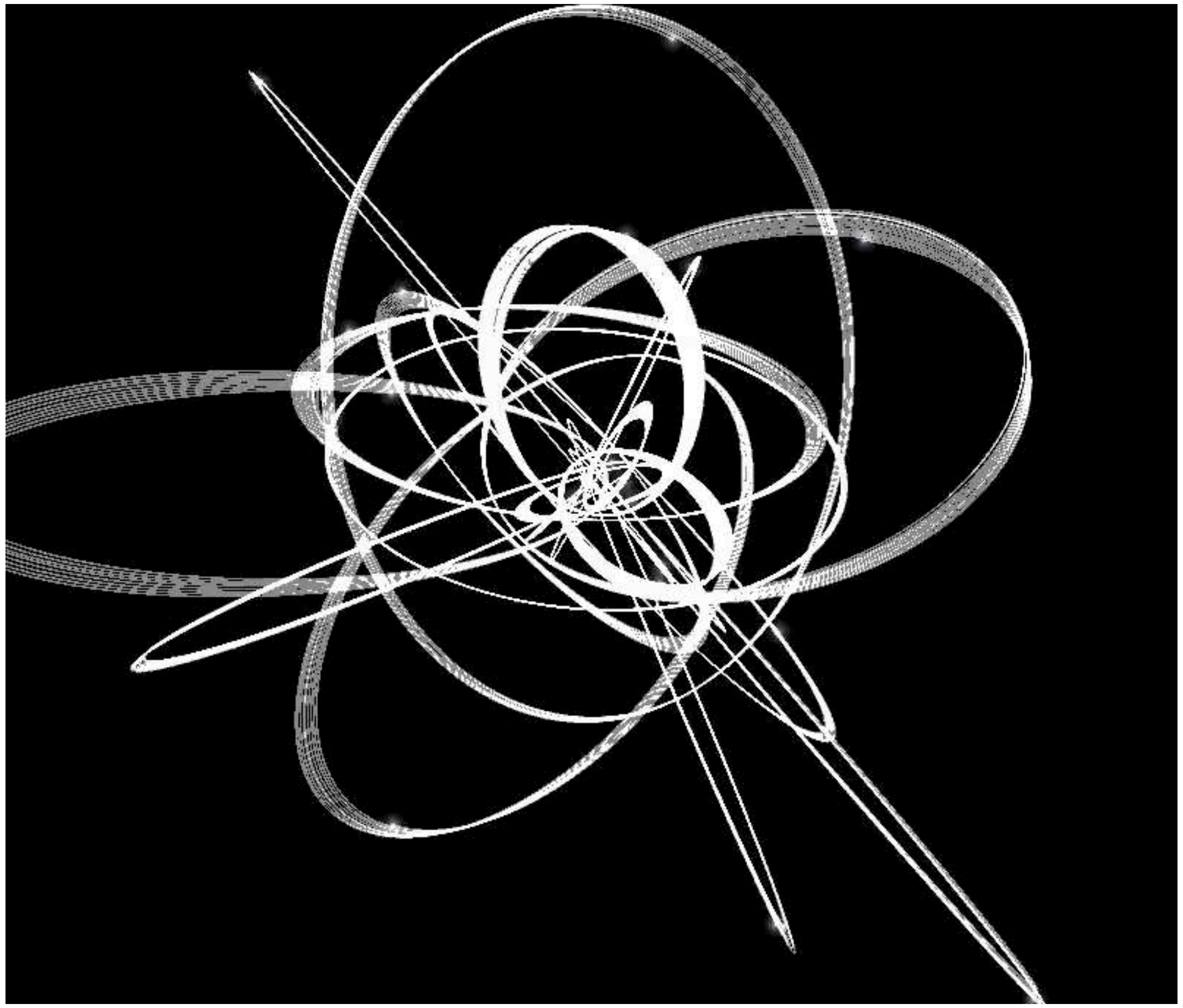
A simple dynamics?

The central BH is **supermassive**

$M_{\mathrm{SgrA}} \simeq 4{,}200{,}000 \times M_{\mathrm{Sun}}$ vs. $M_{\mathrm{Sun}} \simeq 330{,}000 \times M_{\mathrm{Earth}}$



Keck observations

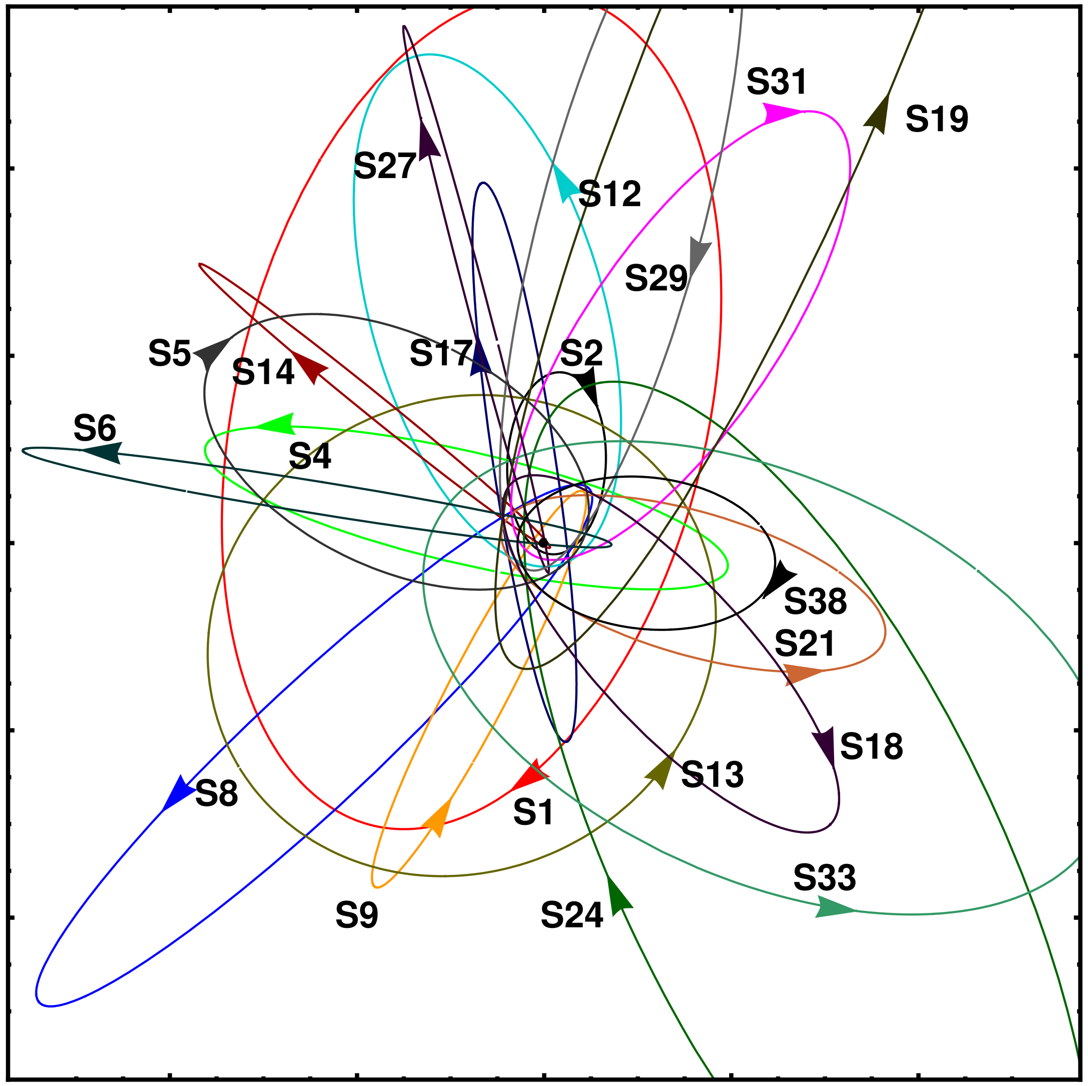


Numerical simulations

Like the Earth around the Sun, stars follow **Keplerian orbits**

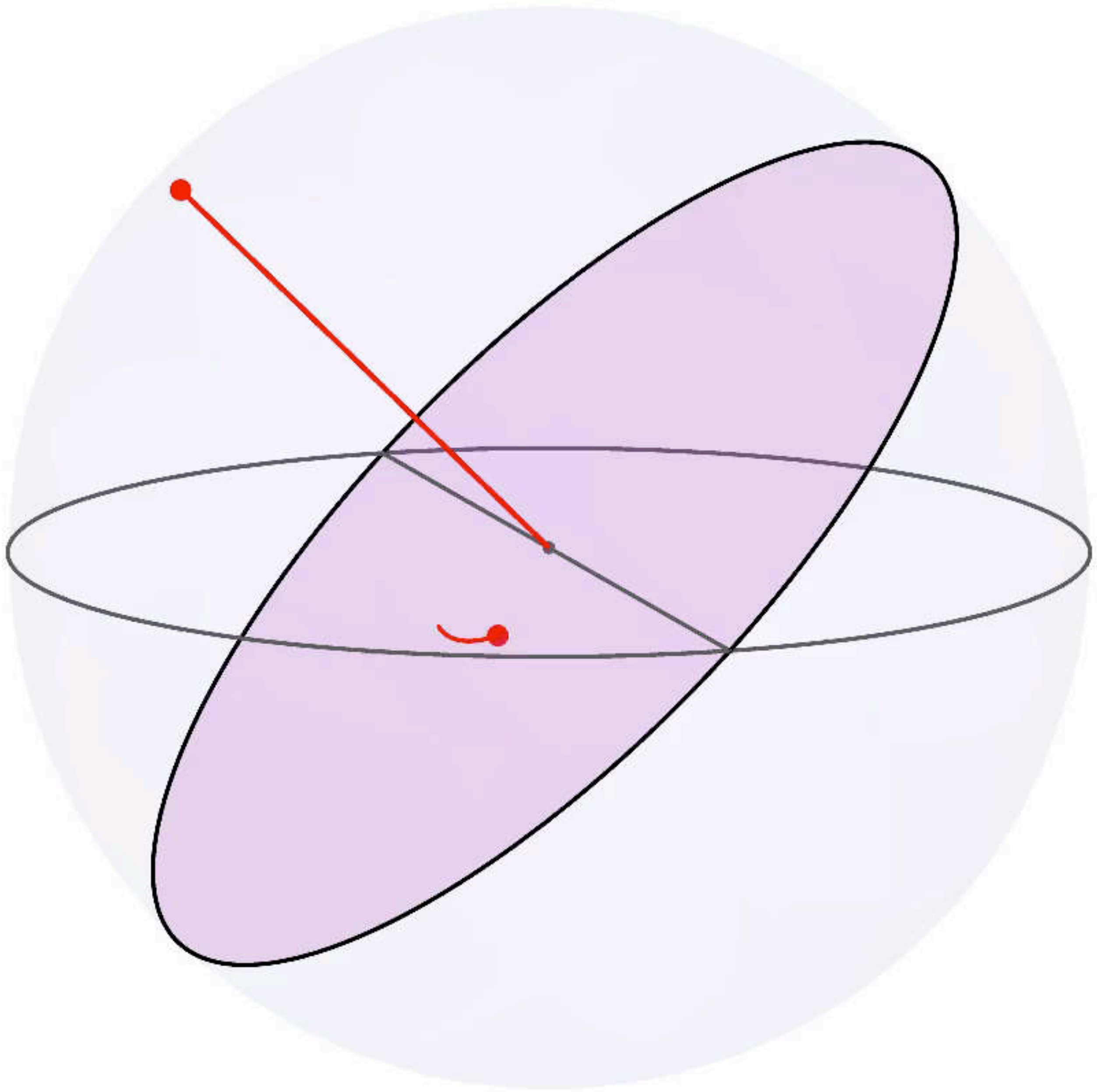
Keplerian orbits

The BH dominates the stars' dynamics



Gillessen et al., 2009

VLT observations

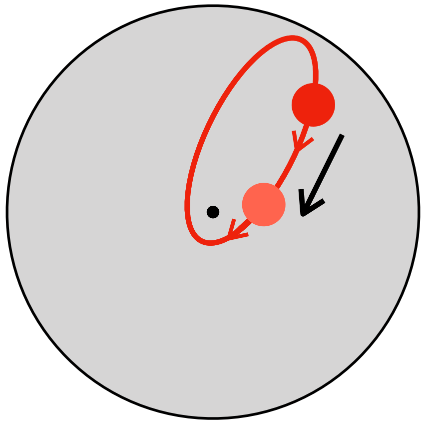


Typical orbit

What is an orbit?

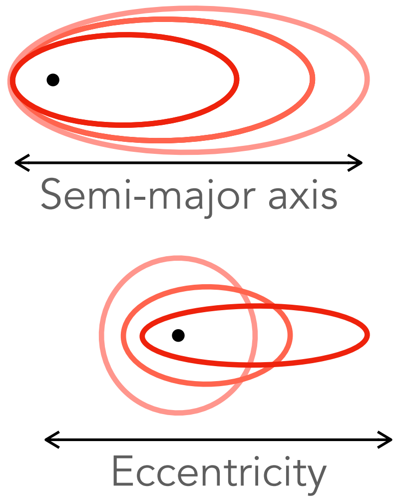
Describing an **orbit**

Position of the star

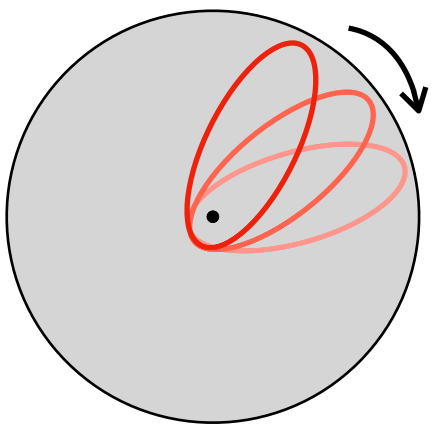


Dynamical motion

Shape of the orbit

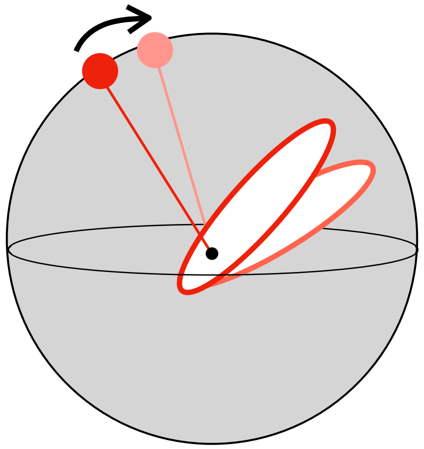


Phase of the orbit

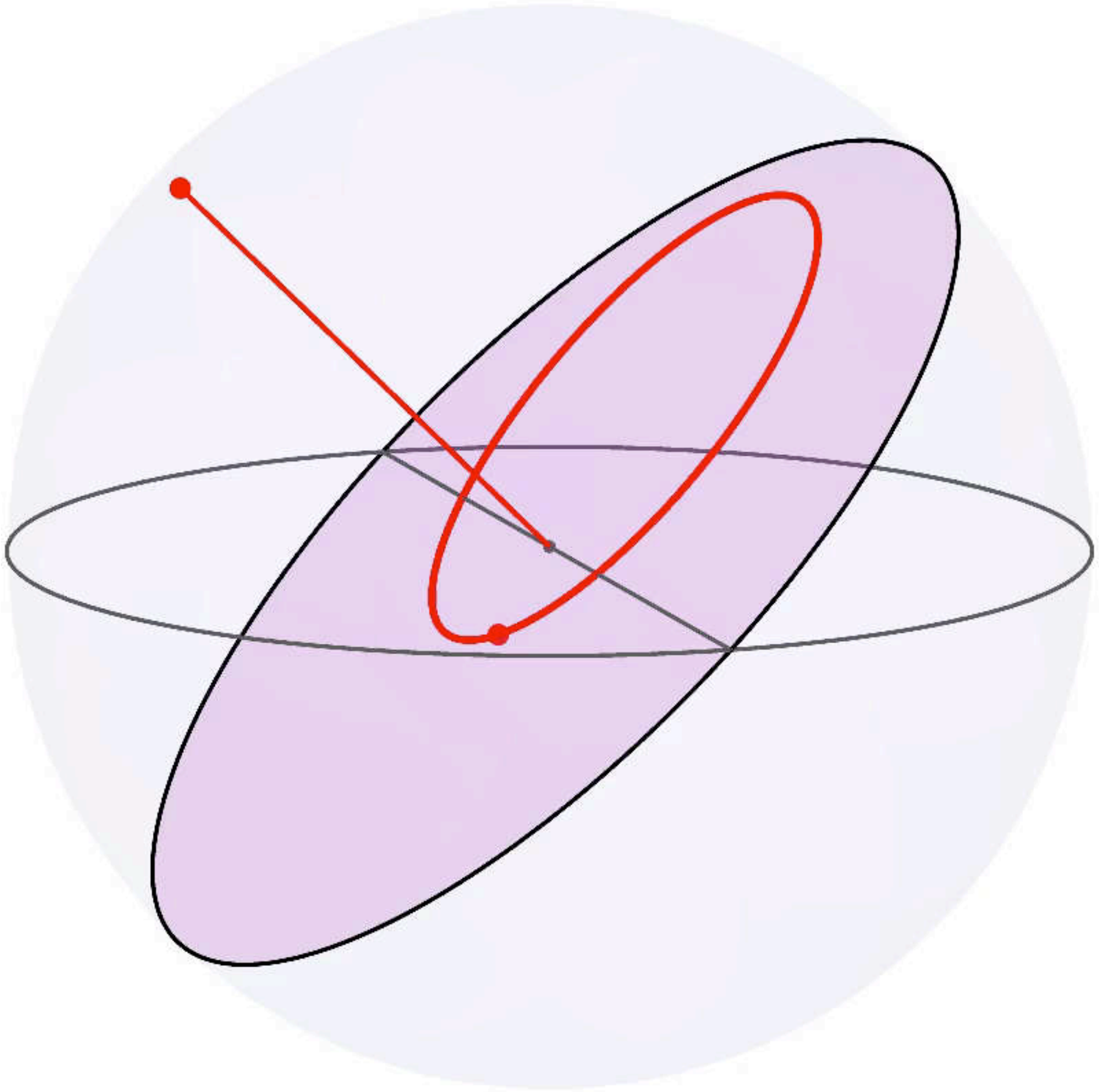


Phase of the pericentre

Orientation of the orbit



Spatial orientation

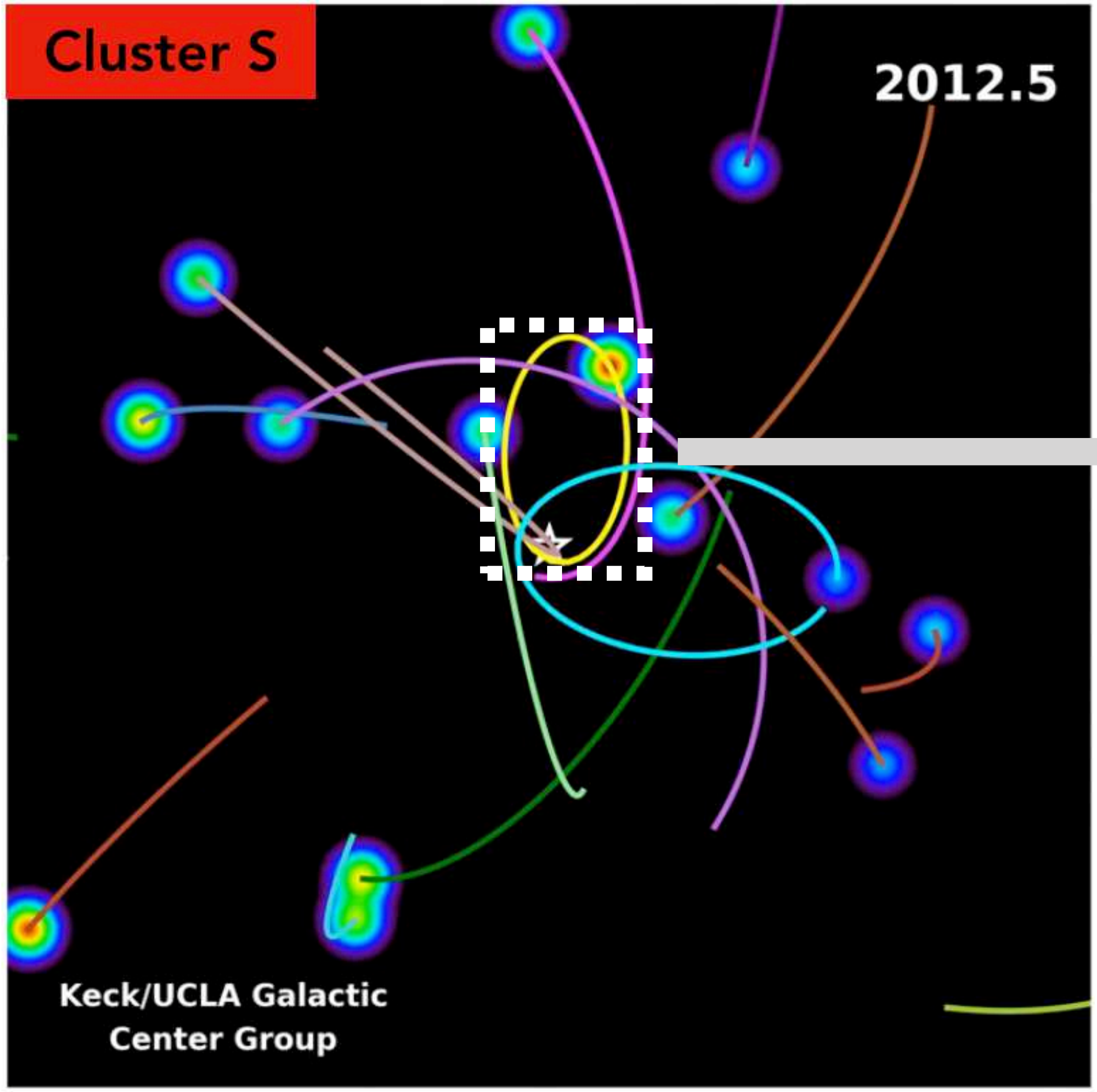


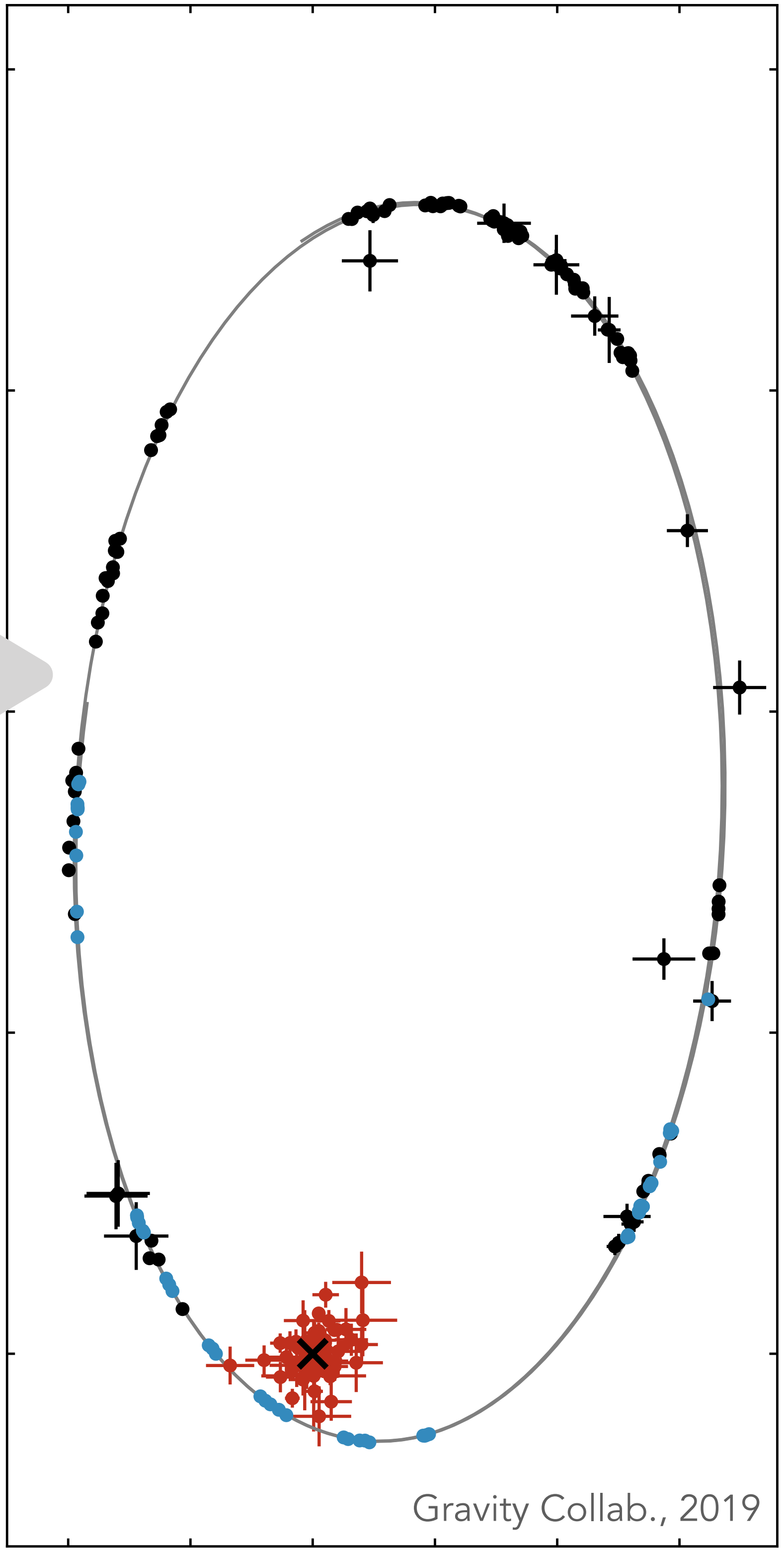
Keplerian orbit

What is the dynamics of **Keplerian orbits**?

S2’s observation

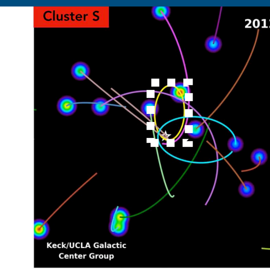
S2’s observations (Gravity)





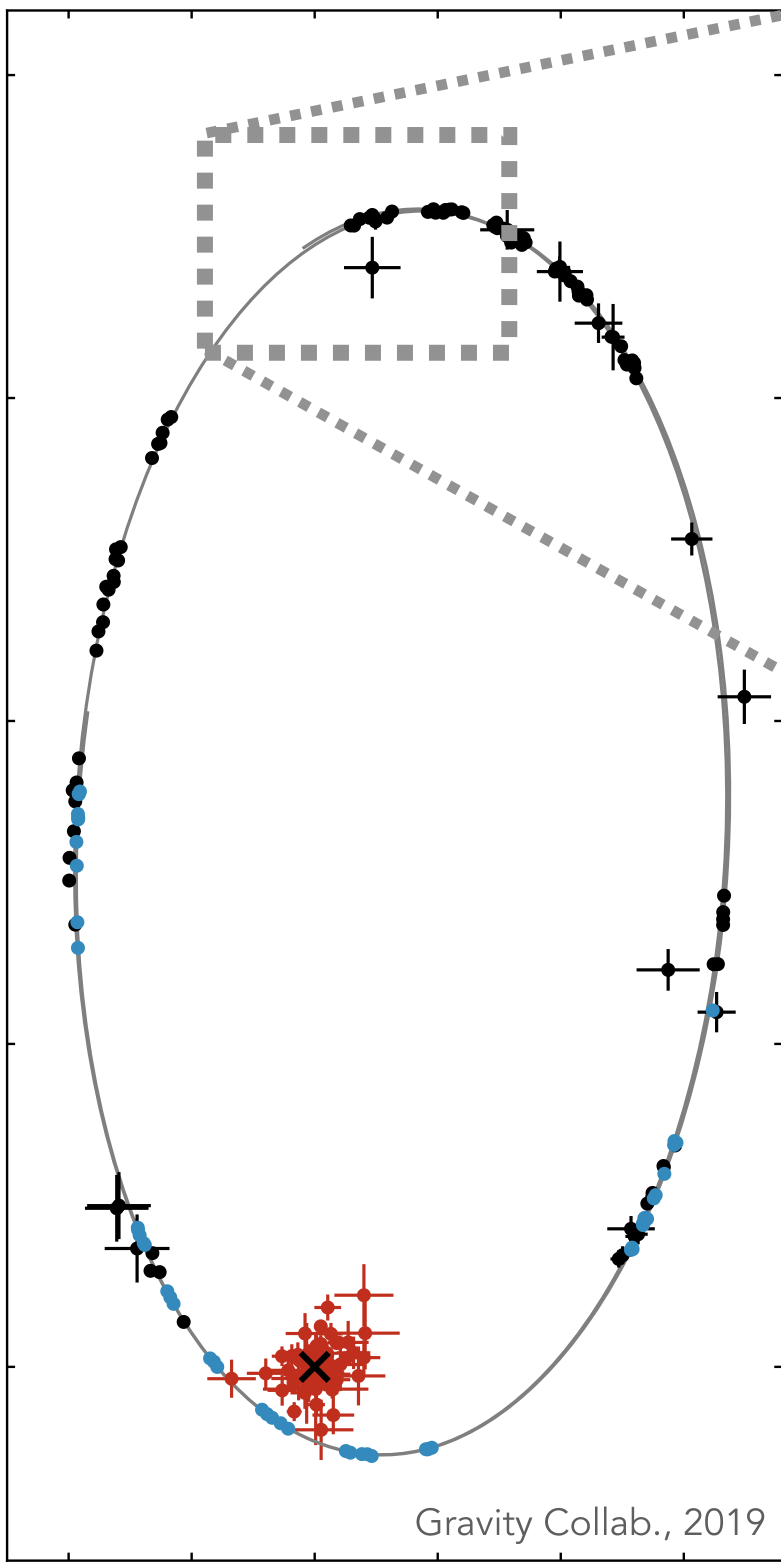
Classical (adaptative) telescope

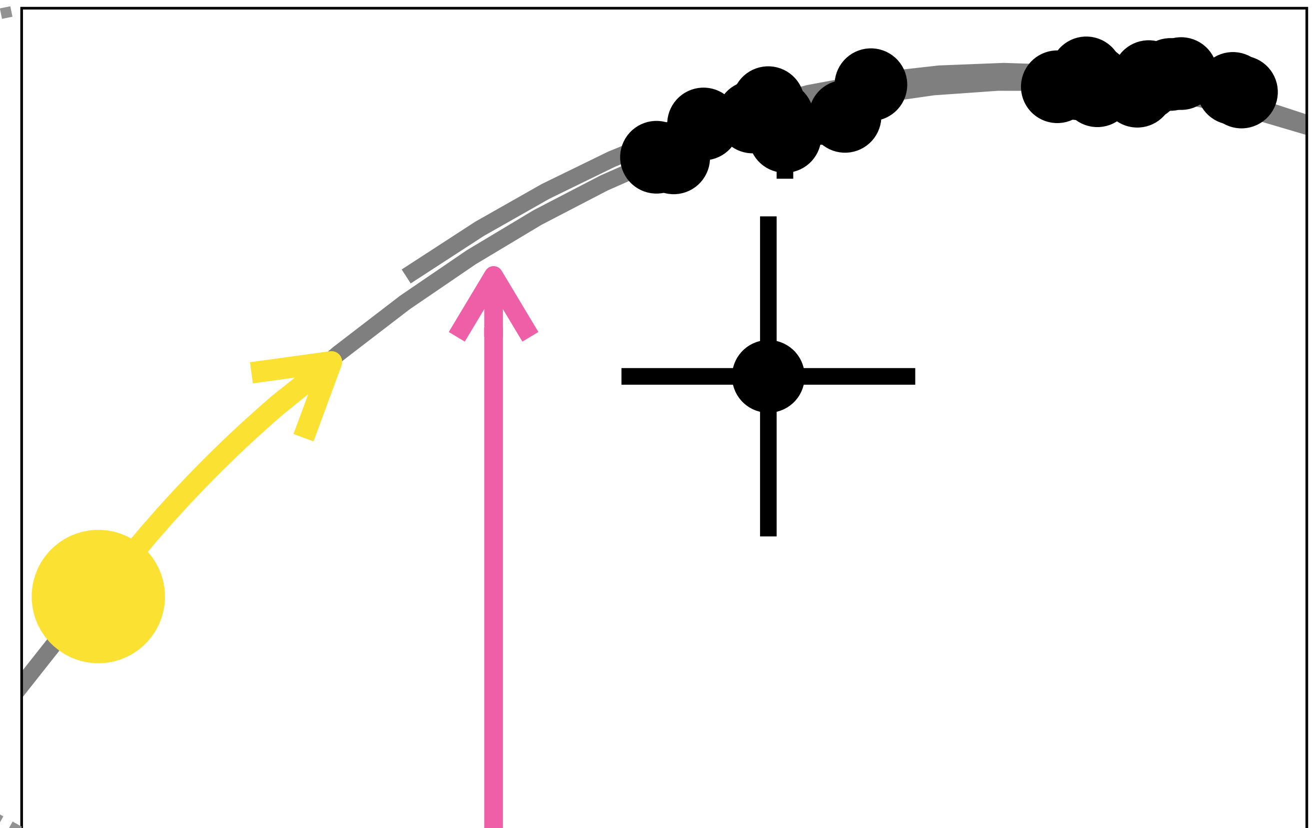
Interferometric telescope



Relativistic precession

Observation of S2 (Gravity)





Precession of the orbit
12′ (=0.2°) per orbit

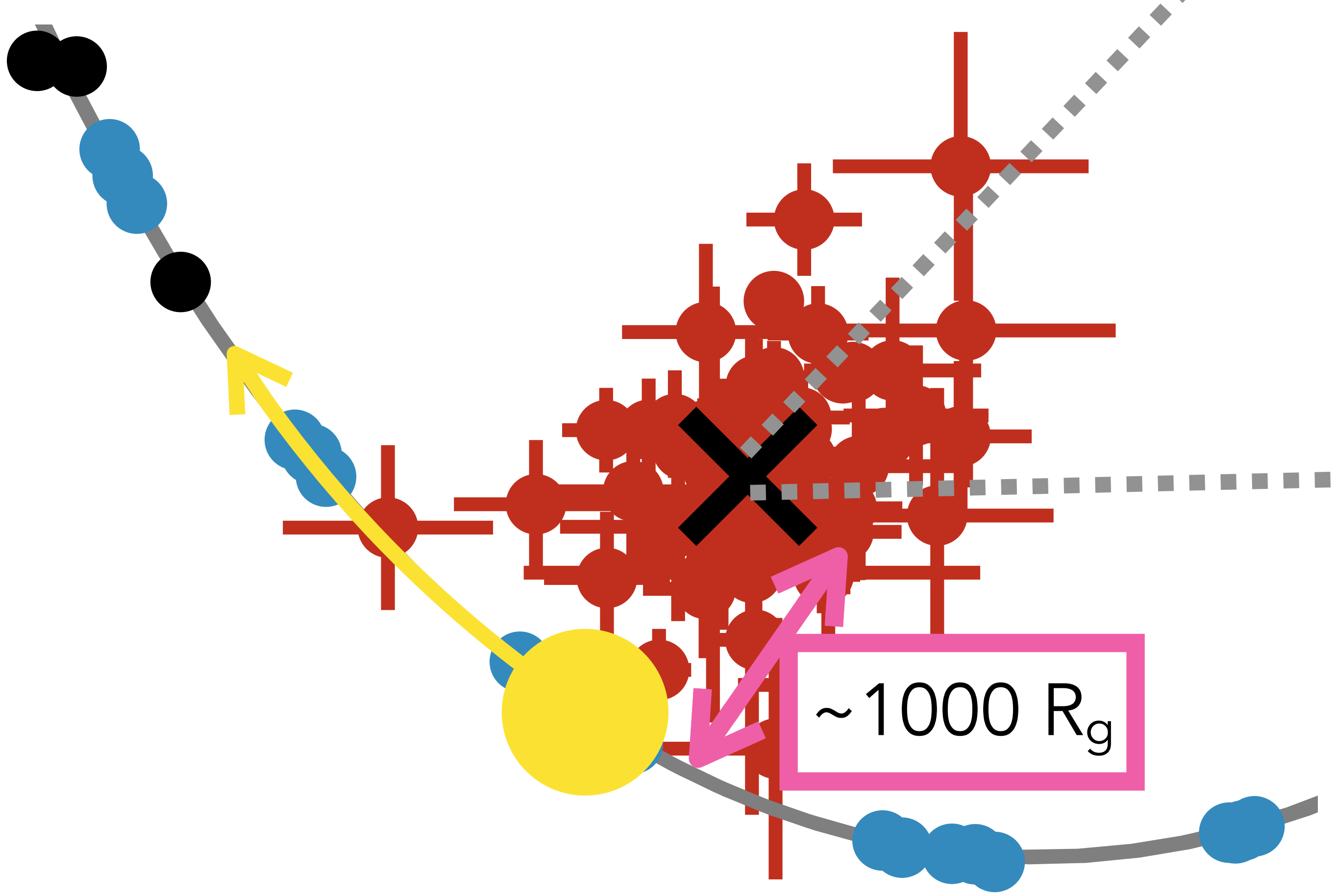
S2 arrived early:

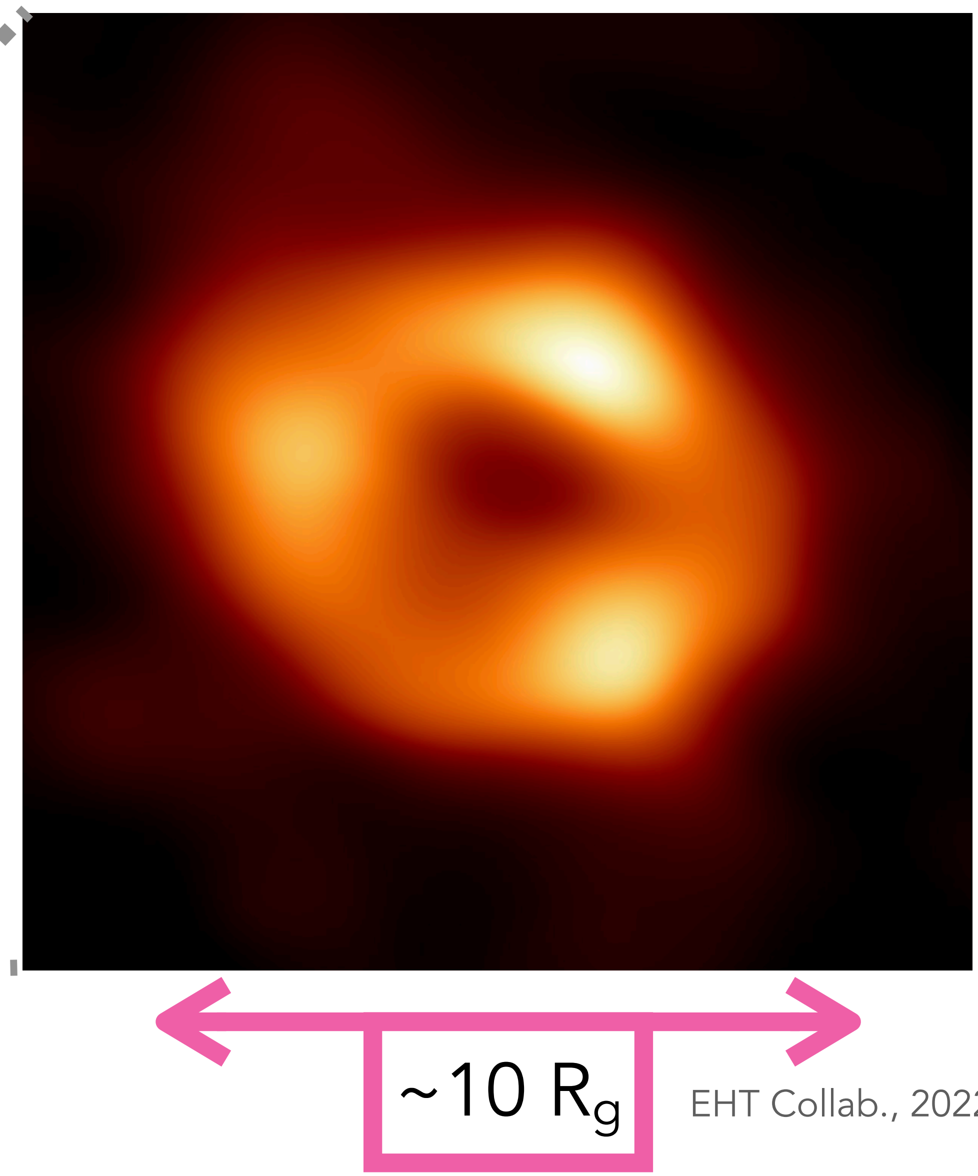
+ Orbit is **non-closed**
+ **Prograde** precession

→ **Relativistic effects**

Black hole's shadow

Event Horizon observation





EHT Collab., 2022

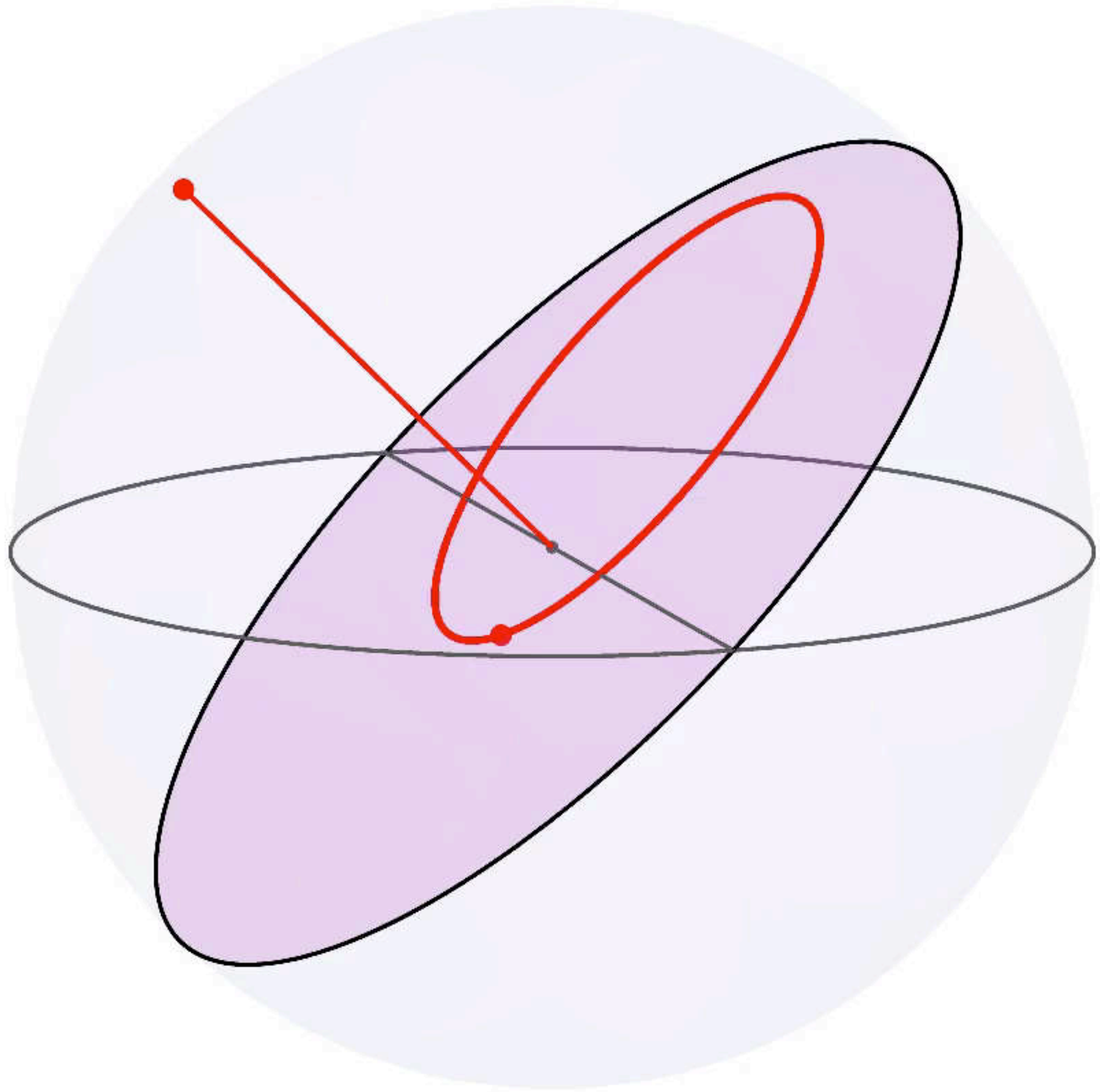
Gravitational radius

$$R_g = \frac{GM_\bullet}{c^2}$$

Pericentre precession

Origins of the **precession**:

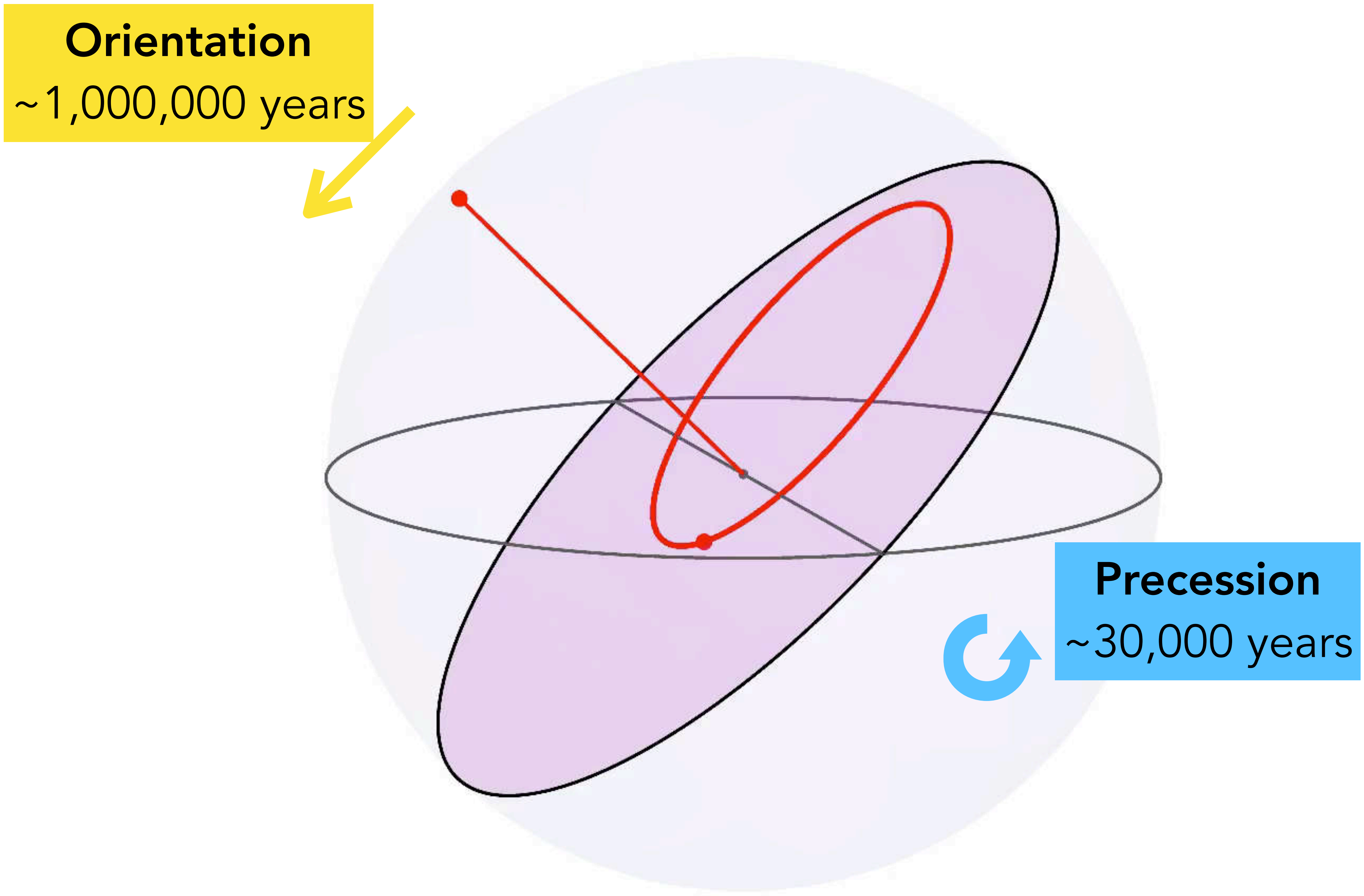
+ **Relativistic** effects from the BH
+ **Perturbations** from other stars



~30,000 years for S2

Orbits **precess** in their planes

Orbits also change in orientations

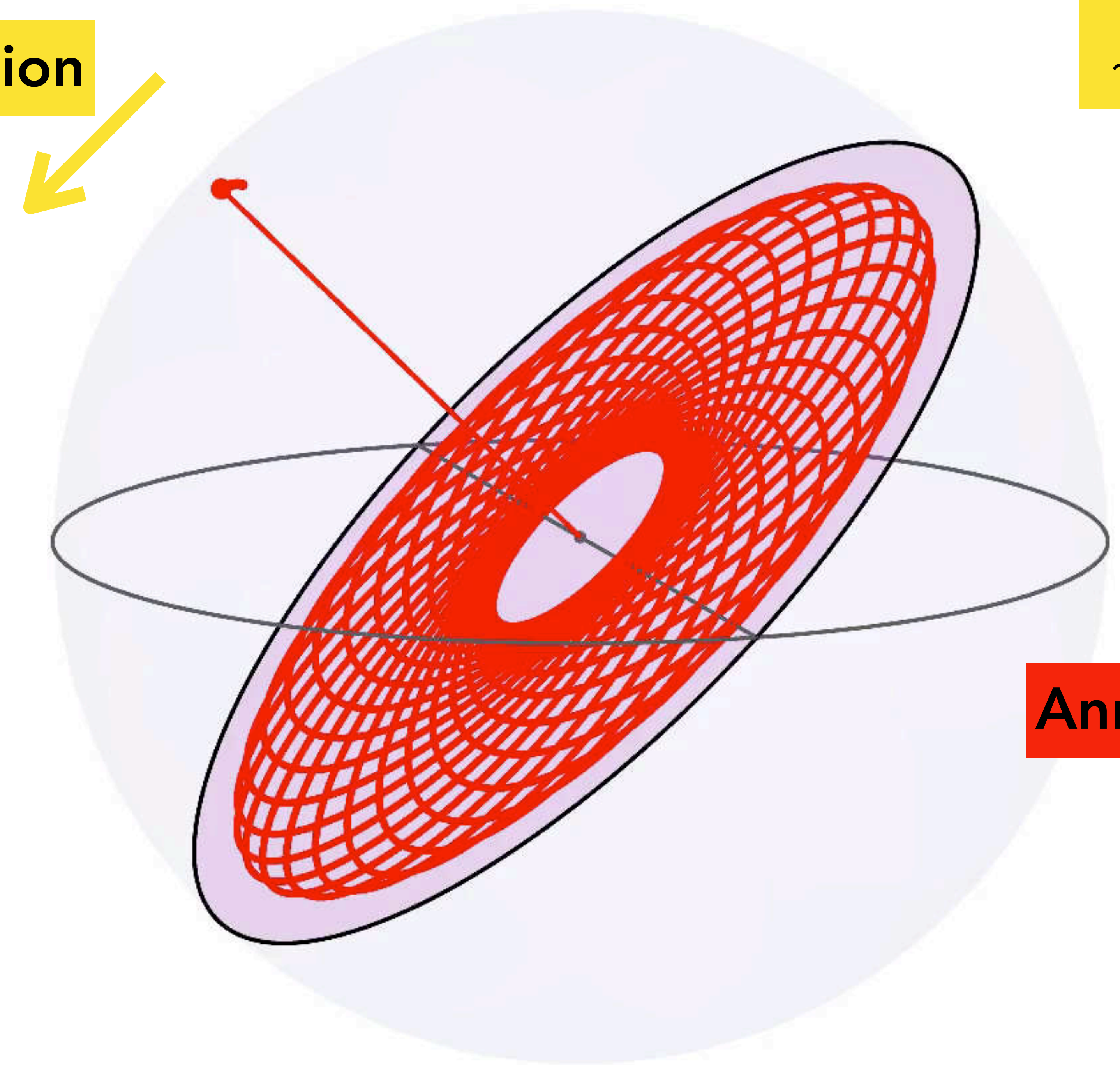


Two timescales: **Precession** $\ll$ **Orientation**

Stellar orientations

Orientation

Typical timescale
~1,000,000 years

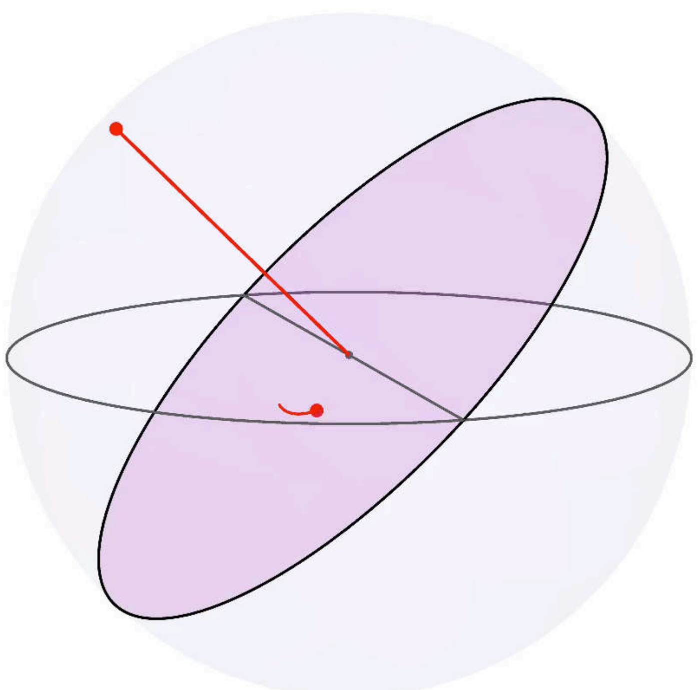


Annuli

After a full precession, **ellipses** become **annuli**

Stellar dynamics

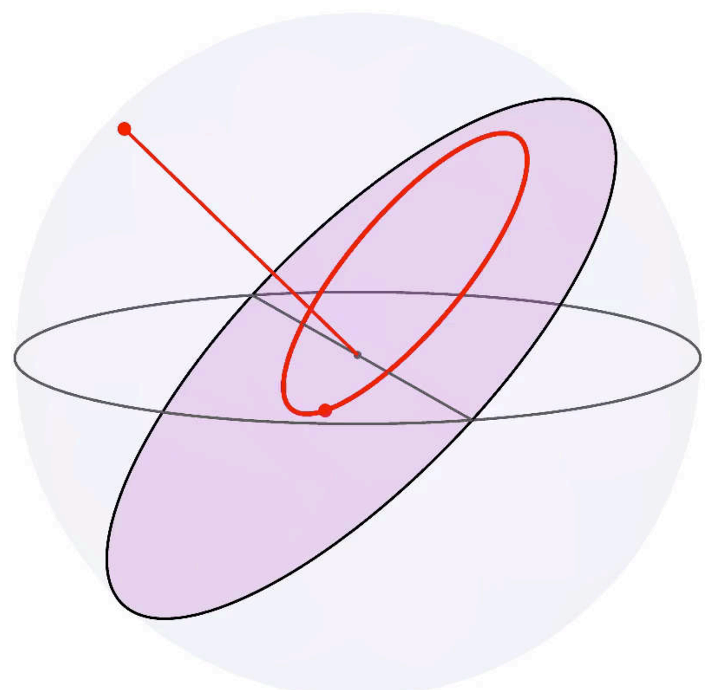
Stars



~10 years

Orbital motion

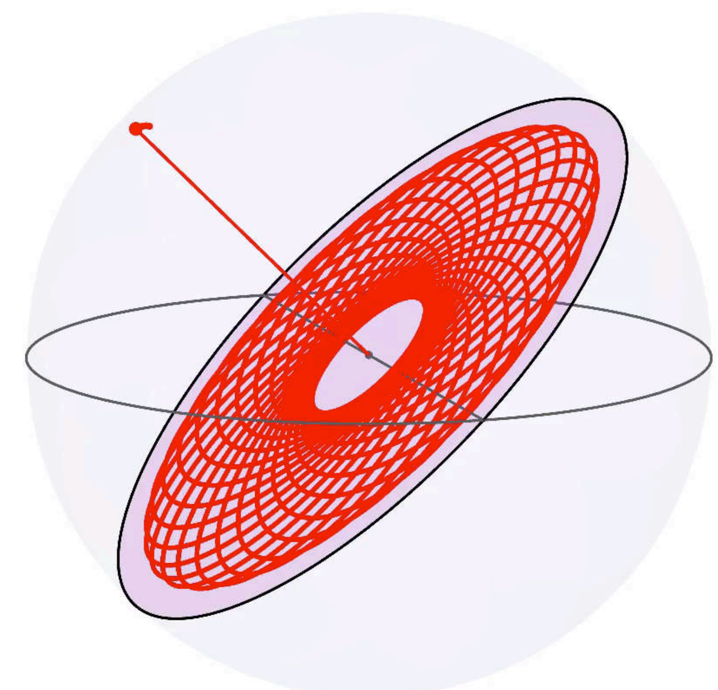
Ellipses



30,000 years

Pericentre precession

Annuli



~1,000,000 years

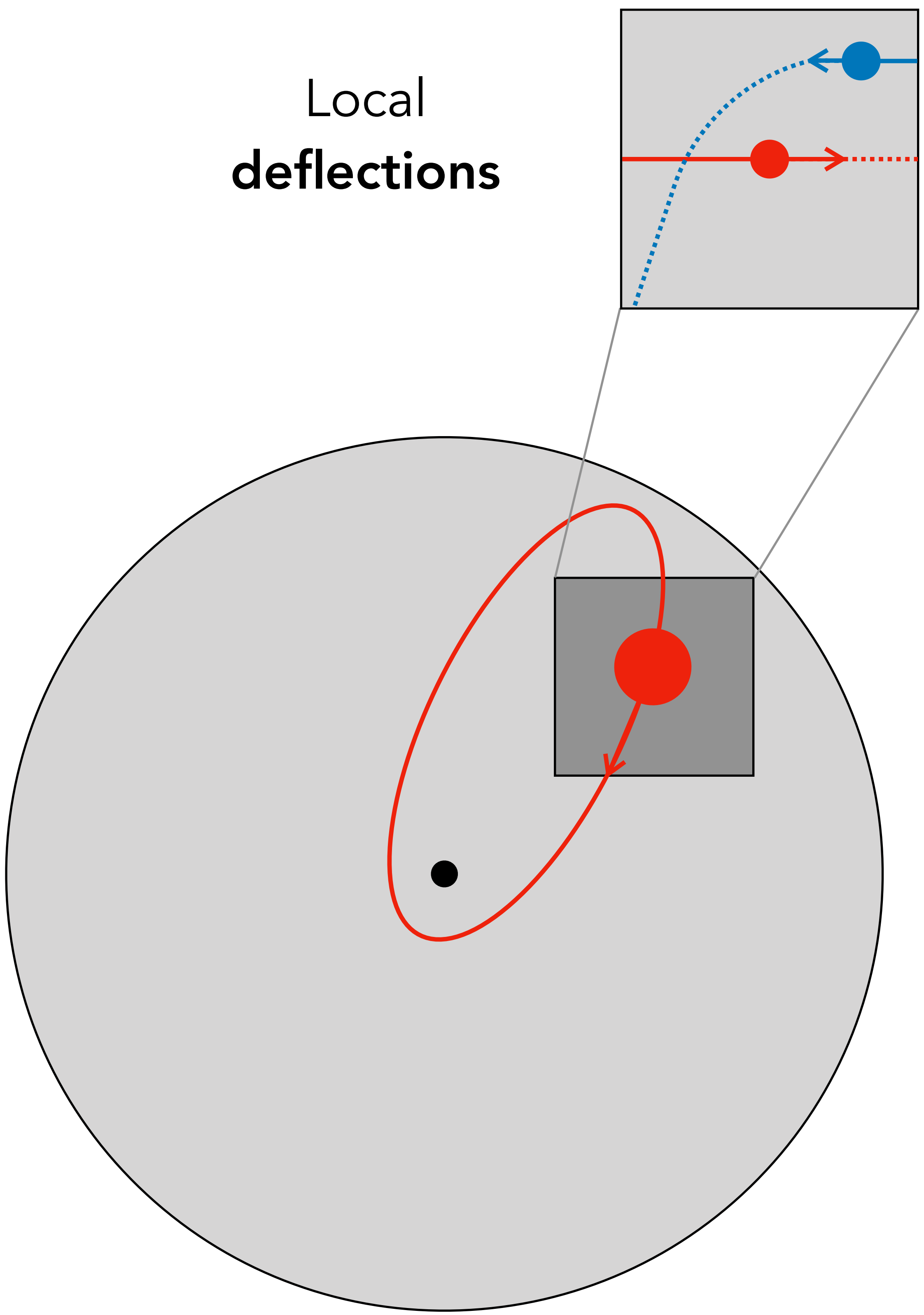
Orientation precession

SgrA* is 10 Gyr orld. We can wait longer.

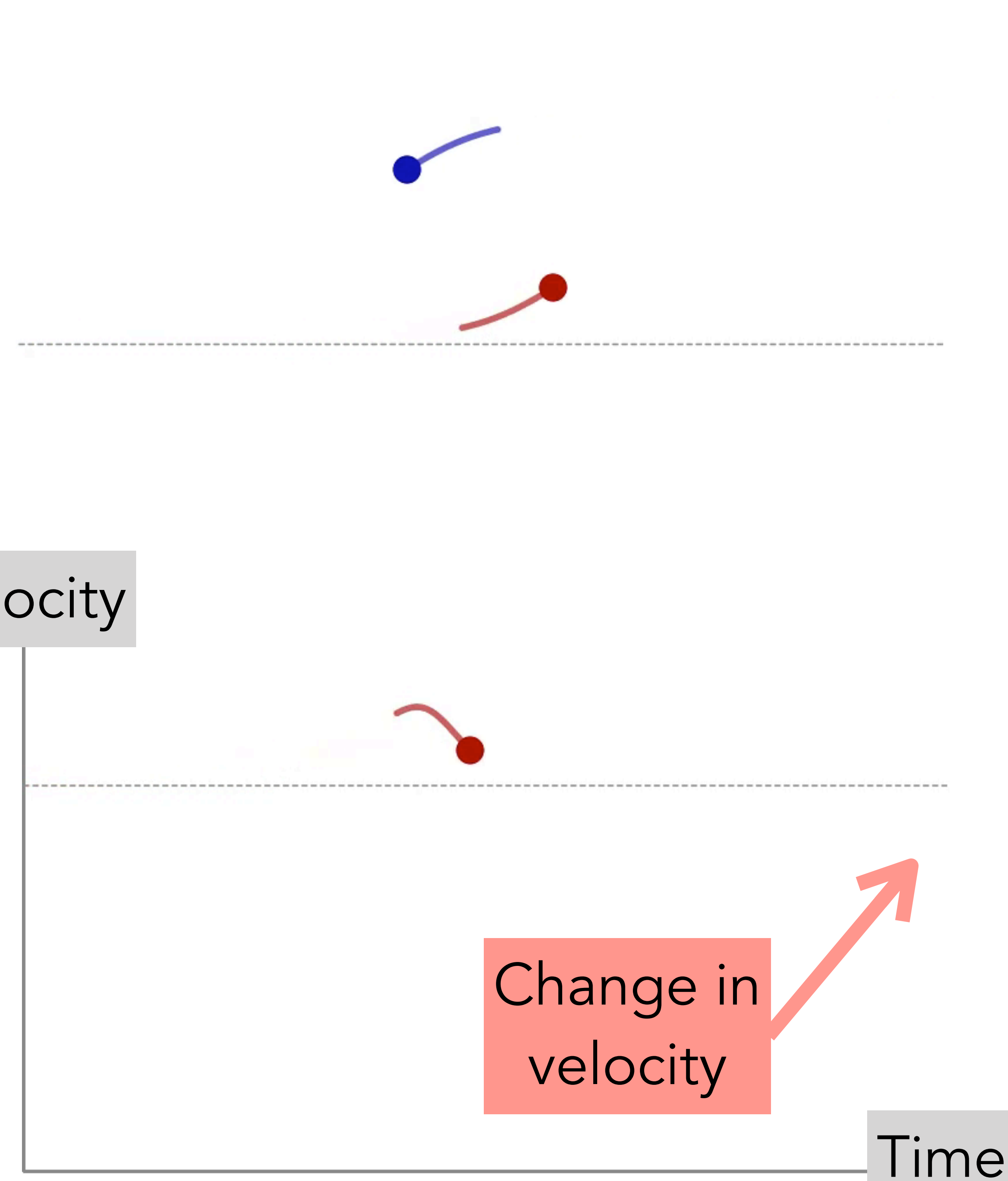
Stellar energy

Orbital distortions sourced by instantaneous **kicks and deflections**

Local
deflections

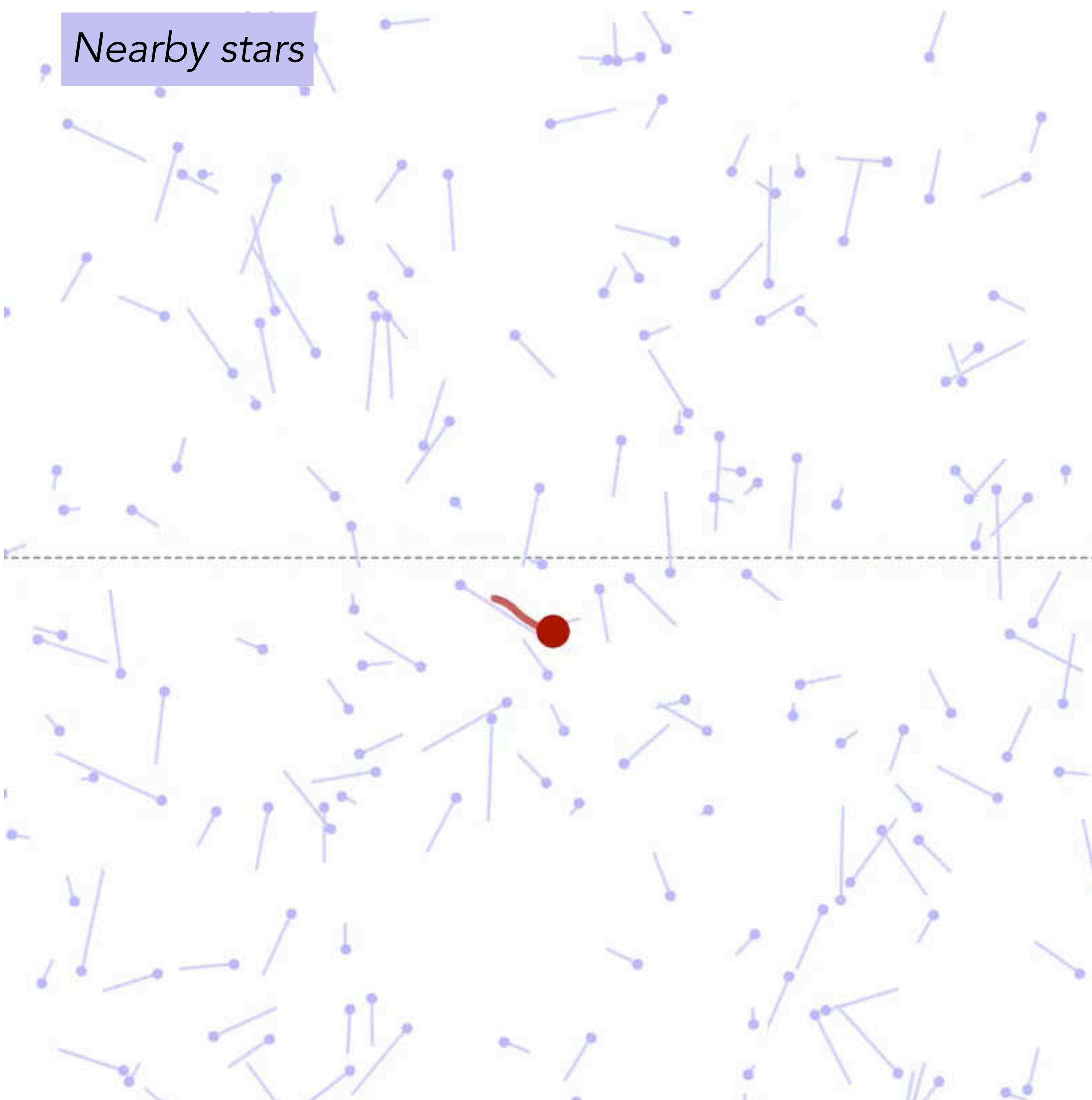


Zoom on the orbit

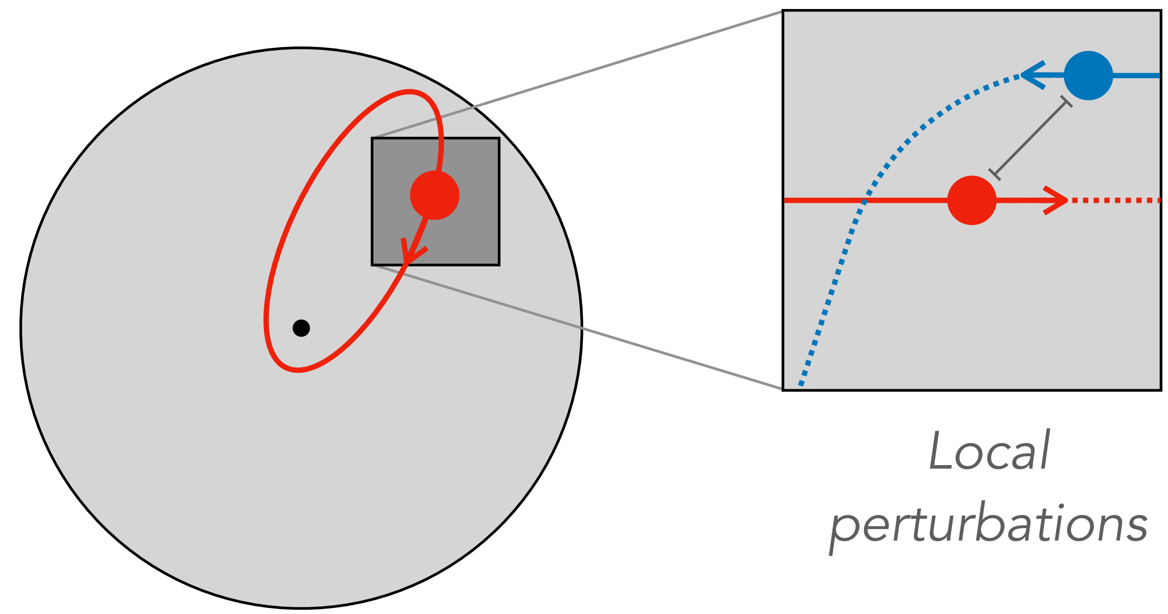


Deflections

The star has a lot of **close neighbours**

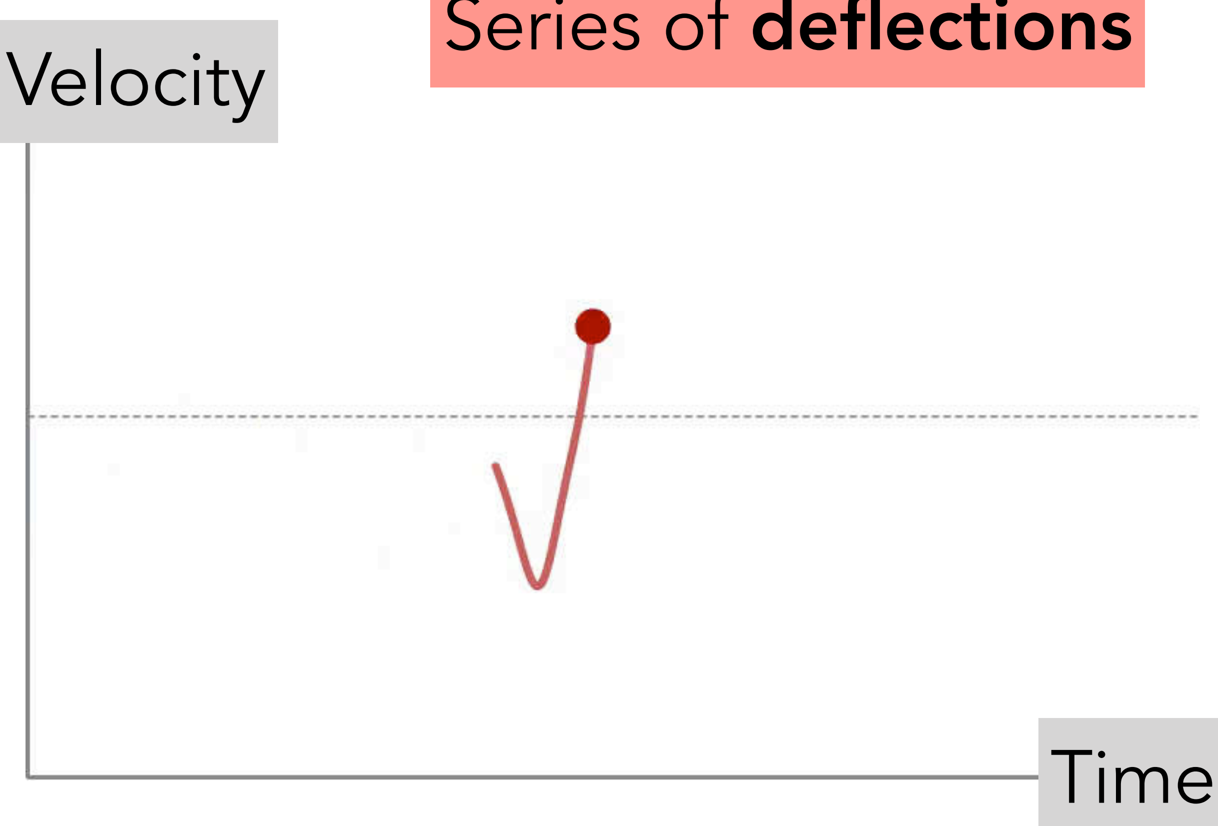


Random walk



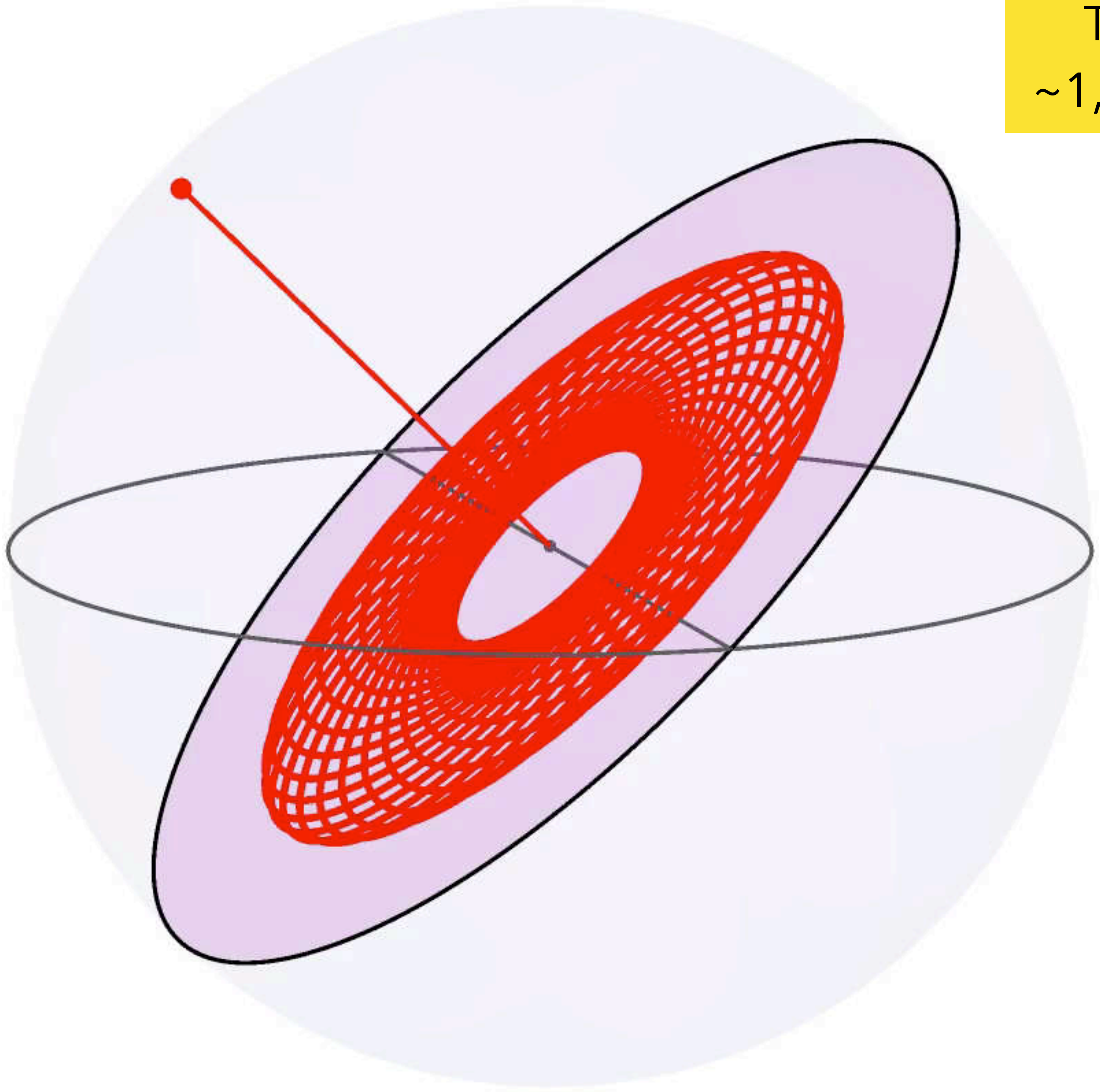
Local perturbations

Series of **deflections**



Stellar energy

Typical timescale
~1,000,000,000 years



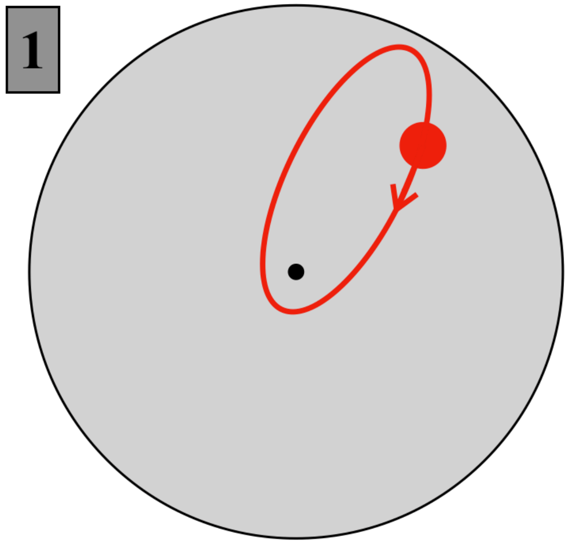
Deflections drive a slow change in the Keplerian energy

Timescales are highly hierarchical

1. Dynamical time

Fast orbital motion induced by the BH

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \Omega_{\mathrm{Kep}}$$



Timescales are highly hierarchical

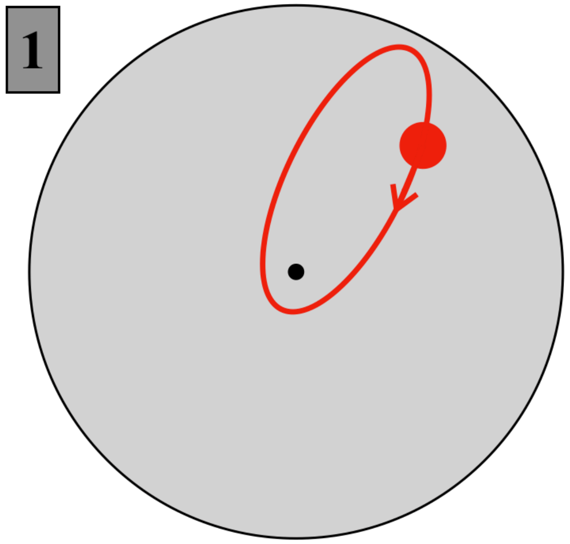
1. Dynamical time
Fast orbital motion induced by the BH

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \Omega_{\mathrm{Kep}}$$

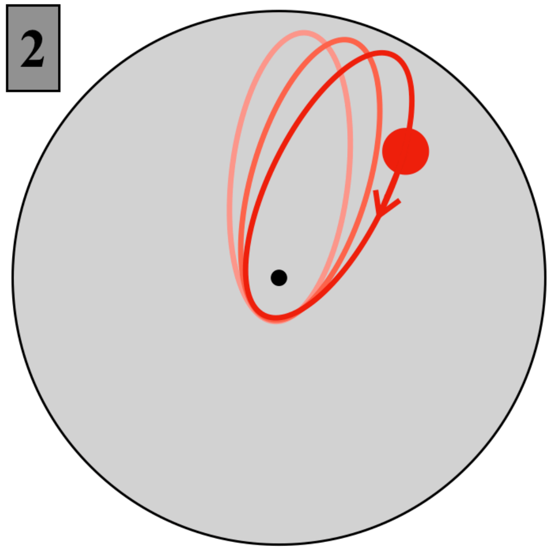
2. Precession time
In-plane precession (mass + relativity)

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \Omega_{\mathrm{p}}$$

1



2



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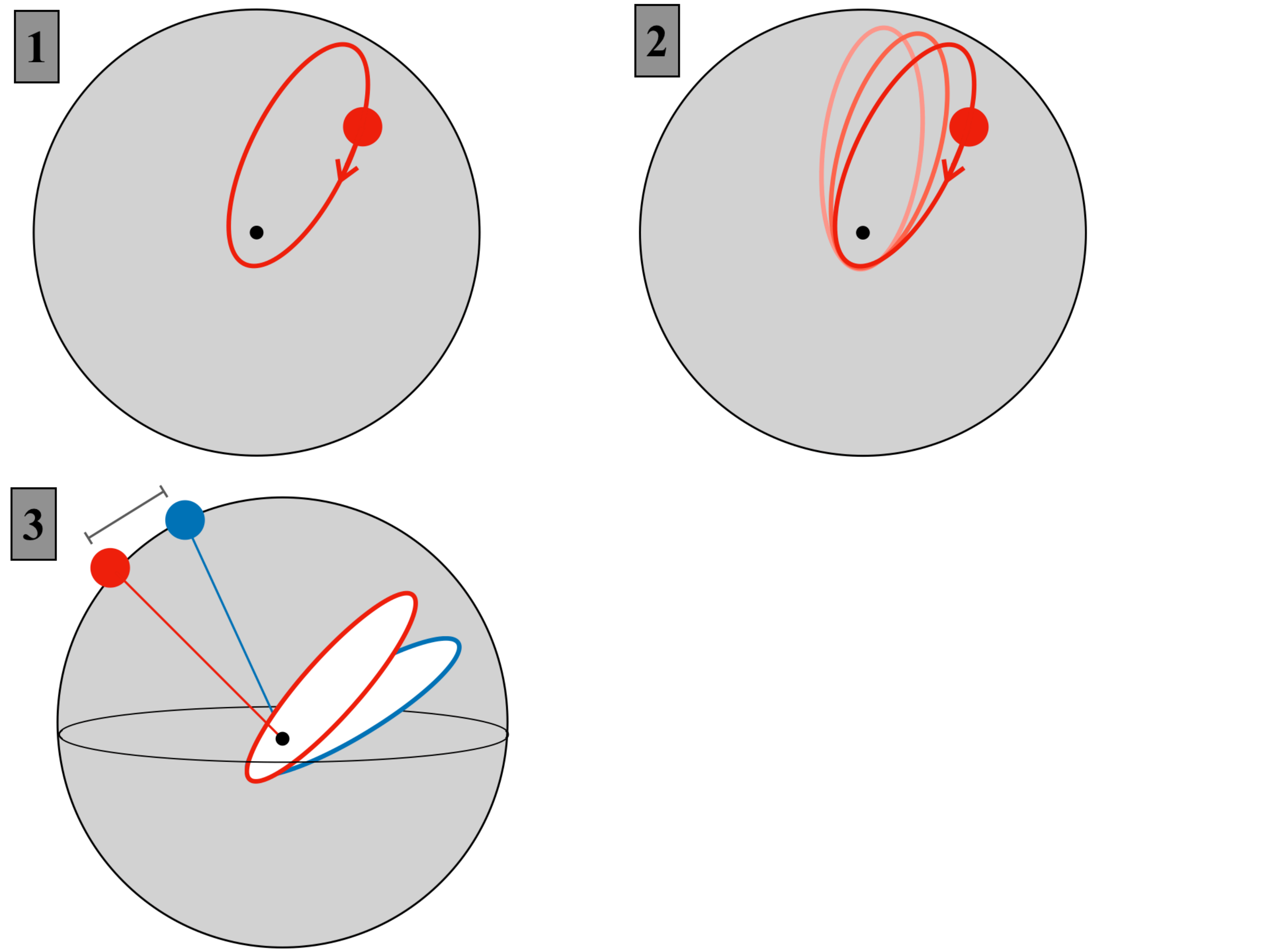
$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \Omega_{\mathrm{p}}$$

3. Vector Resonant Relaxation
Non-spherical torque coupling

$$\frac{\mathrm{d}\hat{\mathbf{L}}}{\mathrm{d}t} = \eta(\hat{\mathbf{L}}, t)$$



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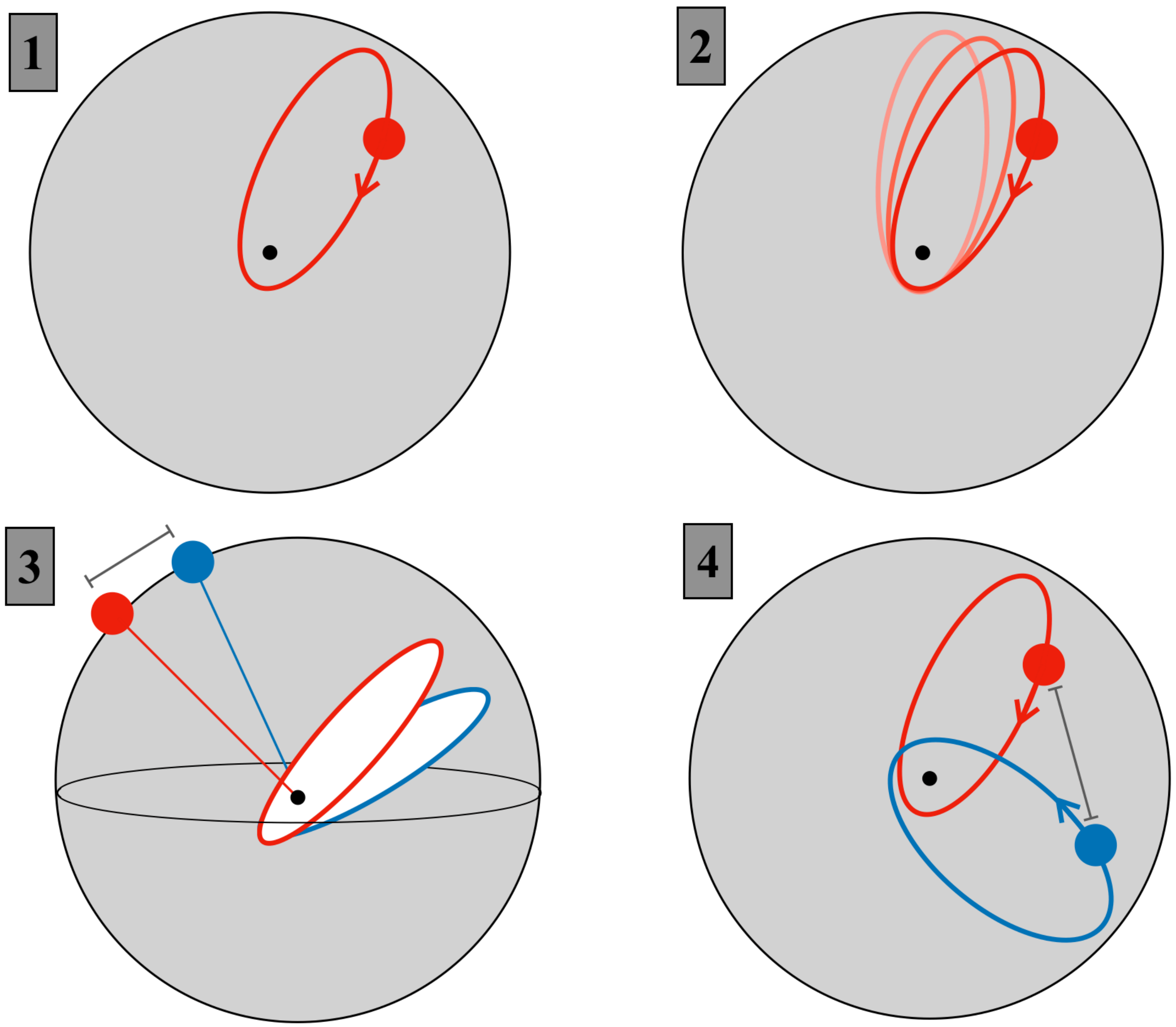
Non-spherical torque coupling

$$\frac{\mathrm{d}\hat{\mathbf{L}}}{\mathrm{d}t} = \eta(\hat{\mathbf{L}}, t)$$

4. Scalar Resonant Relaxation
Resonant coupling on precessions

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \eta(e, t)$$



Timescales are highly hierarchical

1. Dynamical time
Fast orbital motion induced by the BH

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \Omega_{\mathrm{Kep}}$$

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In-plane precession (mass + relativity)

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Non-spherical torque coupling

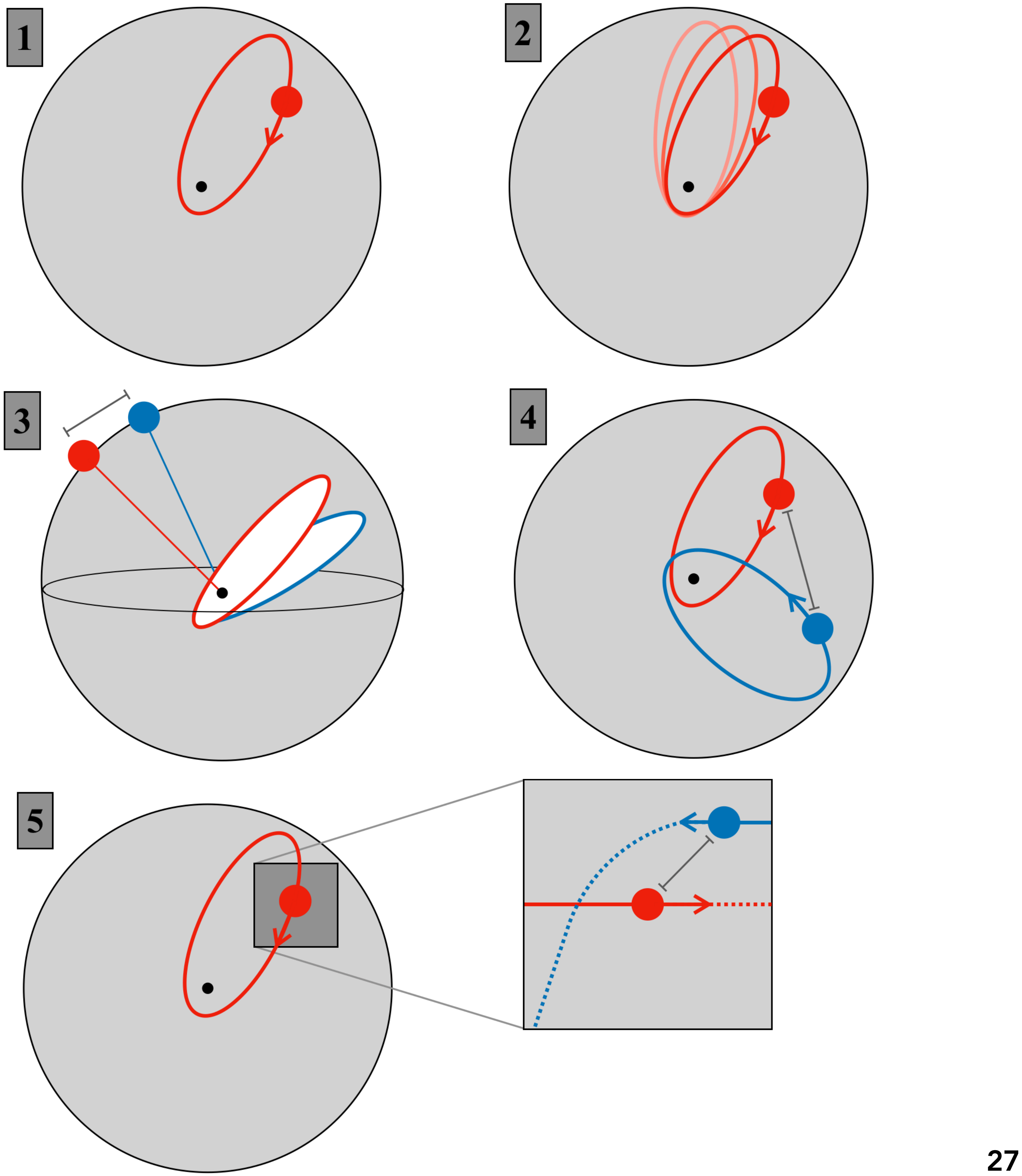
$$\frac{\mathrm{d}\hat{\mathbf{L}}}{\mathrm{d}t} = \eta(\hat{\mathbf{L}}, t)$$

4. Scalar Resonant Relaxation
Resonant coupling on precessions

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \eta(e, t)$$

5. Non-Resonant Relaxation
Local two-body encounters

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \eta(a, t)$$



Long-term evolution

Typical **evolution equation**

Phase-space dynamics

$$\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}t} = f[\mathbf{w}(t)] + \eta[t, \mathbf{w}(t)]$$

Deterministic, constructive motion — *Stochastic perturbation*

Dynamical process

Vector Resonant Relaxation **VRR**	Scalar Resonant Relaxation **SRR**	Non-Resonant Relaxation **NR**

State vector

$\mathbf{w} \to \hat{\mathbf{L}}$	$\mathbf{w} \to e$	$\mathbf{w} \to a$

Constructive motion

$\mathrm{d}\hat{\mathbf{L}}/\mathrm{d}t = \mathrm{Lense-Thirring}$	$\mathrm{d}\omega/\mathrm{d}t = \mathrm{Schwarzschild}$	$\mathrm{d}\mathbf{x}/\mathrm{d}t = \mathbf{v}$

Stochastic dynamics

Random perturbations

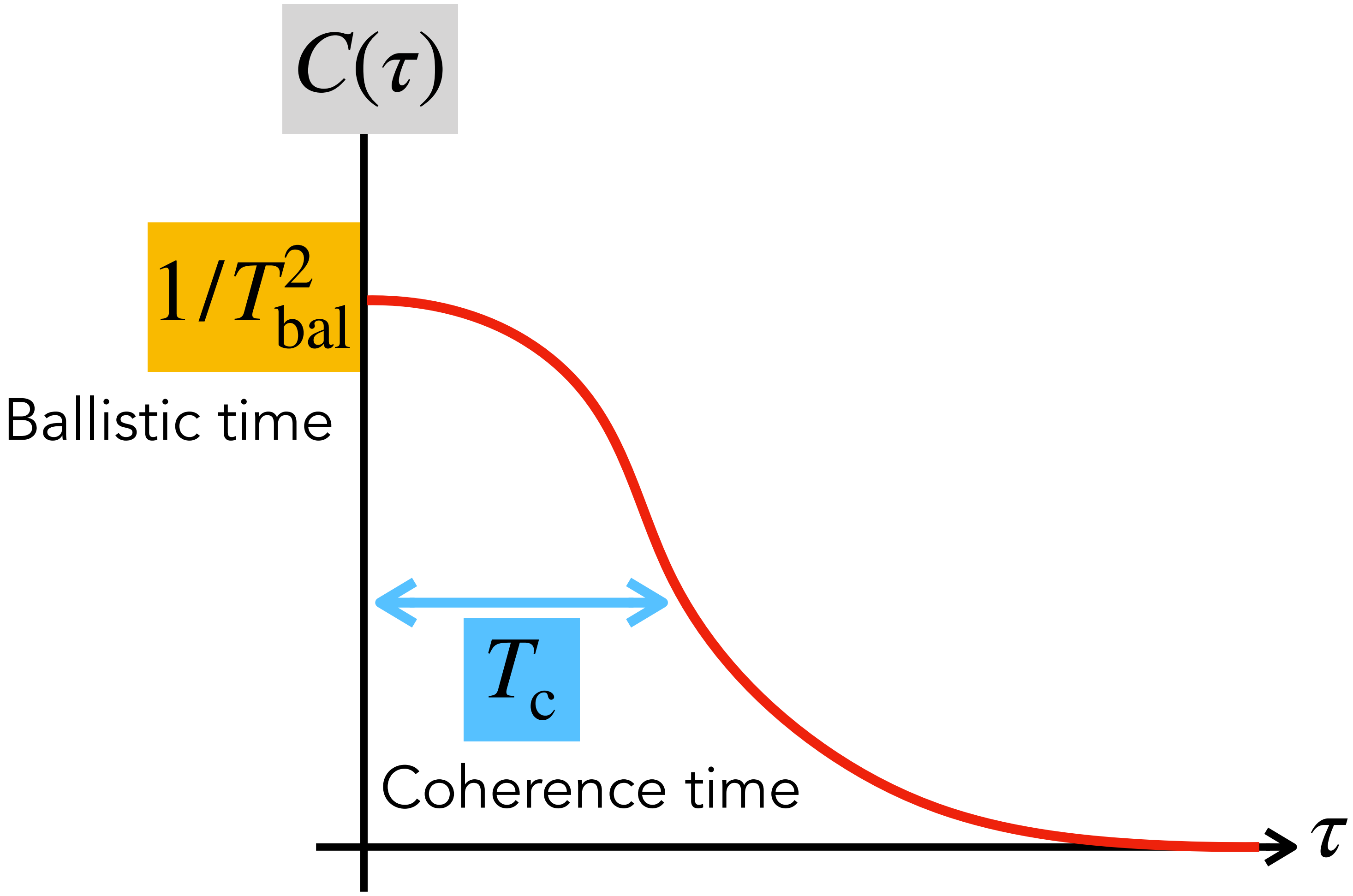
$$\eta[t, \mathbf{w}(t)]$$

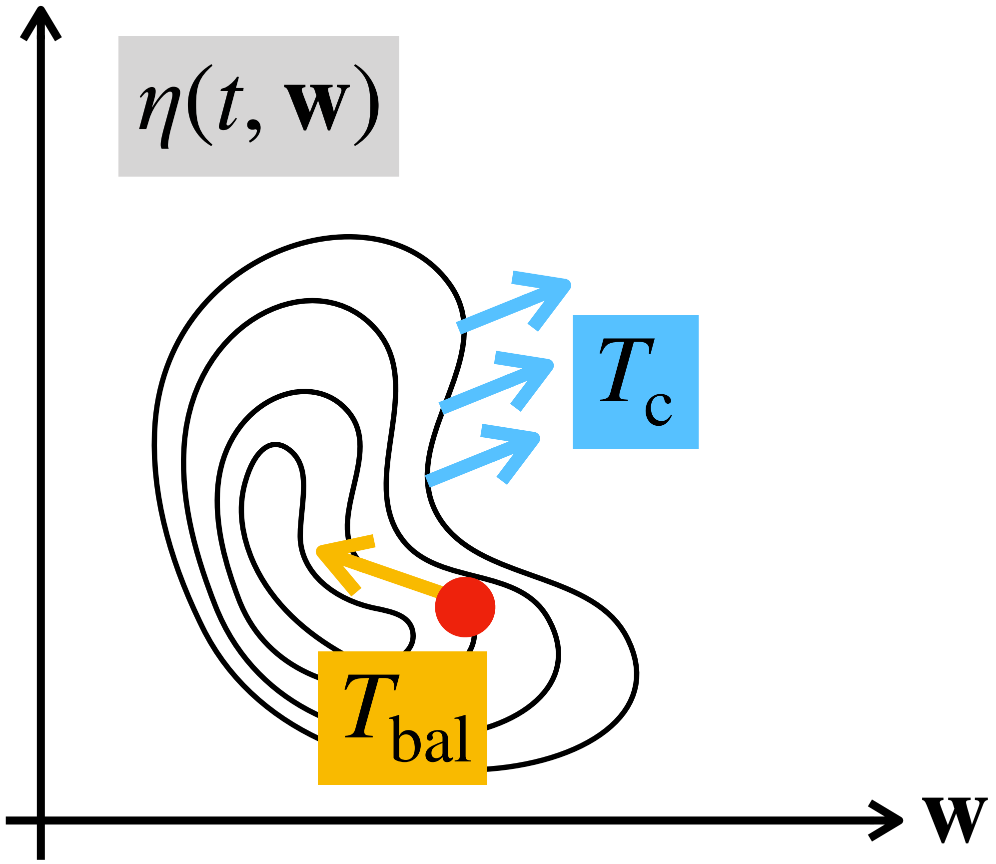
Time correlation

$$C(\tau) = \left\langle \eta(t, \mathbf{w})\, \eta(t+\tau, \mathbf{w}) \right\rangle$$

Eulerian correlation

Two **timescales**





Diffusion

Diffusion of **a test particle**

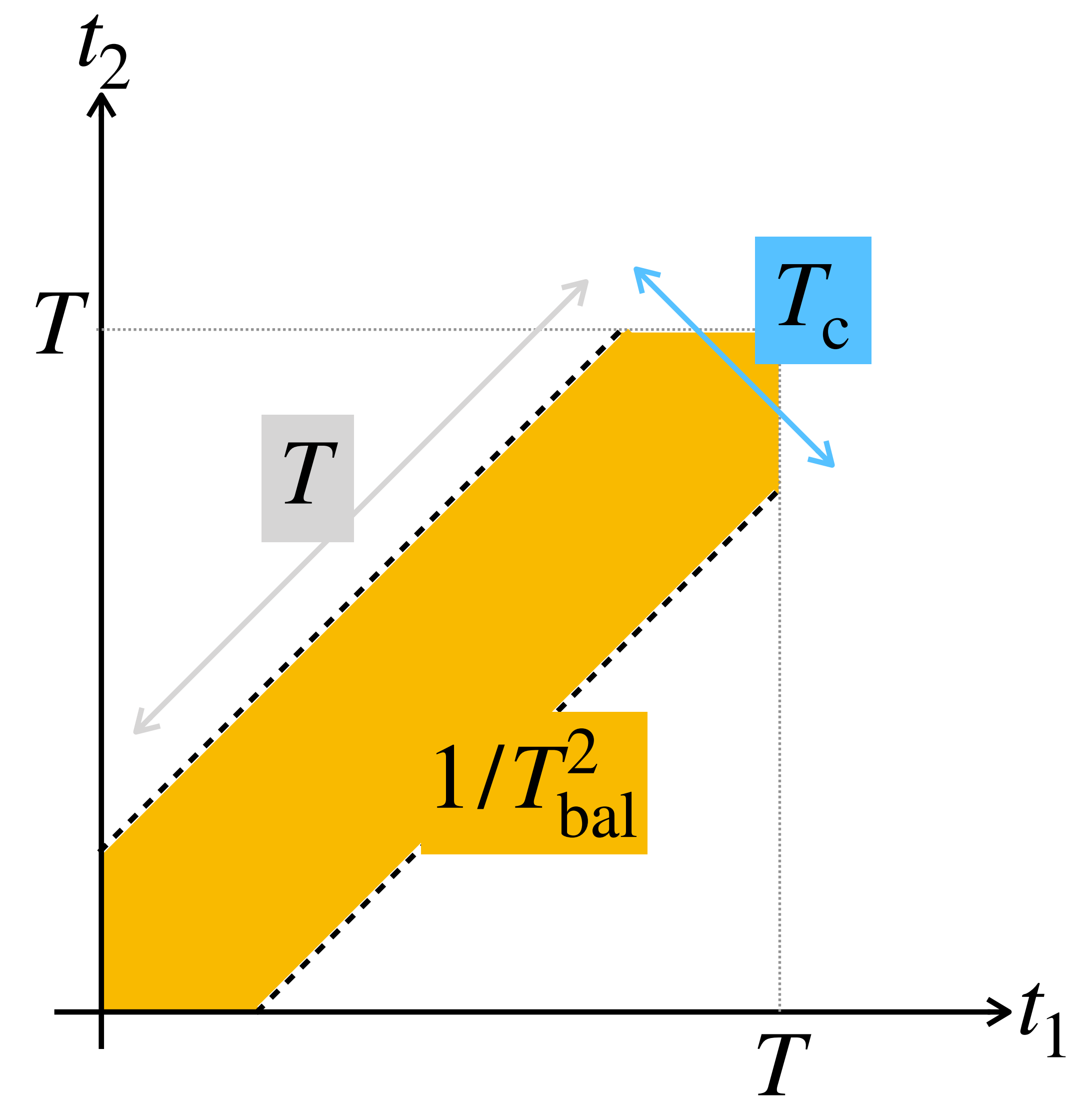
$$\left\langle \Delta \mathbf{w}^2(T) \right\rangle = \int_0^T \mathrm{d}t_1 \int_0^T \mathrm{d}t_2 \left\langle \eta(t_1, \mathbf{w}(t_1))\, \eta(t_2, \mathbf{w}(t_2)) \right\rangle$$

Lagrangian correlation

$$\simeq \int_0^T \mathrm{d}t_1 \int_0^T \mathrm{d}t_2 \left\langle \eta(t_1, \mathbf{w}_0)\, \eta(t_2, \mathbf{w}_0) \right\rangle$$

Eulerian correlation

$$\simeq \frac{T\, T_{\mathrm{c}}}{T_{\mathrm{bal}}^2}$$



Diffusion timescale

$$T_{\mathrm{diff}} \simeq T_{\mathrm{bal}}^2 / T_{\mathrm{c}}$$

Coherent/Resonant processes

drive faster relaxations

Diffusion in galactic nuclei

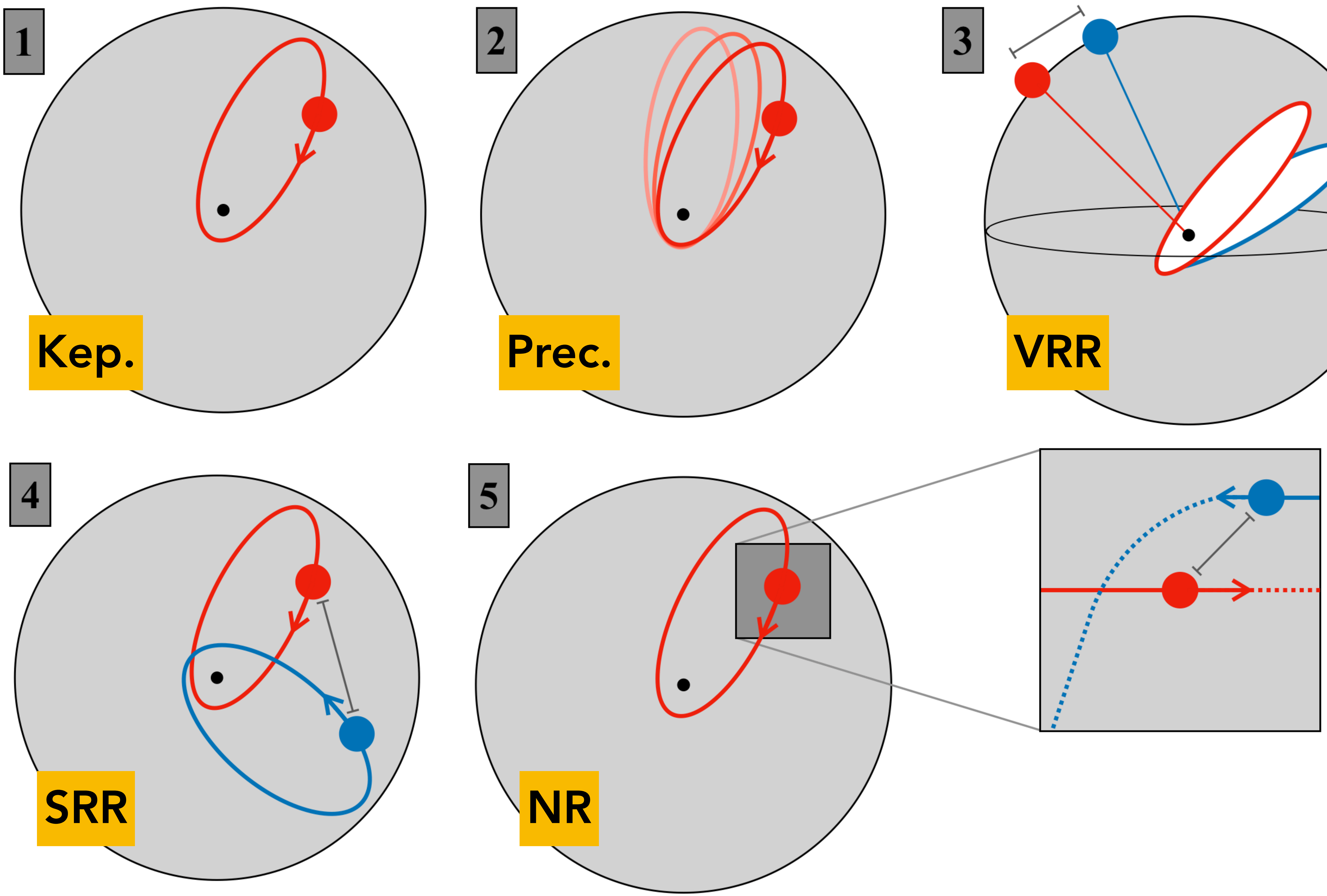
Two **large numbers**

$$Q = \frac{M_\bullet}{M_\star} \gg 1$$ Quasi-Keplerian system

$$N \gg 1$$ Statistical system

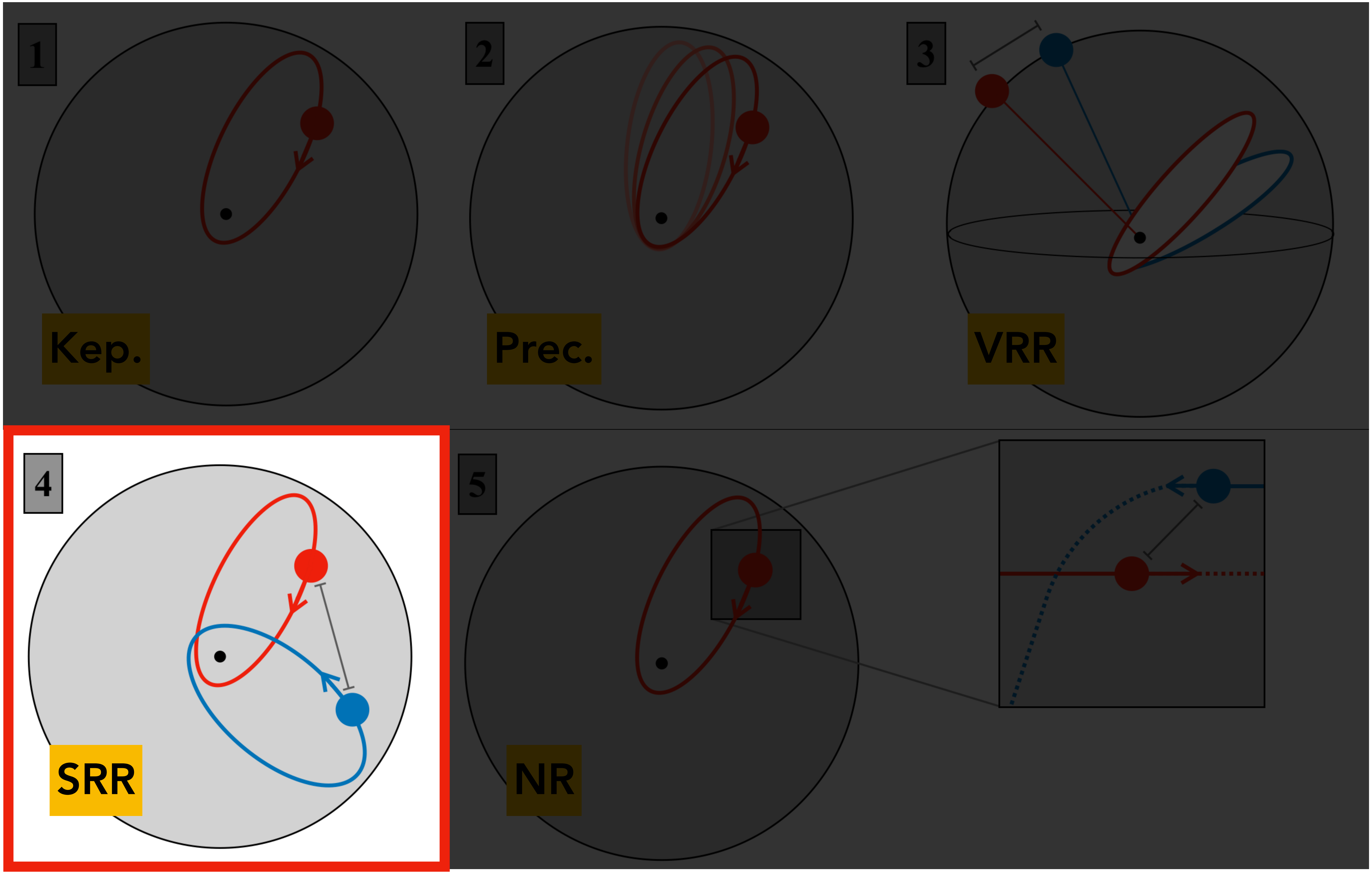
	Dynamical process	
Vector Resonant Relaxation **VRR**	Scalar Resonant Relaxation **SRR**	Non-Resonant Relaxation **NR**
	Object	
Annulus	Wire	Star
	Coherence time	
T_{bal}	T_{prec}	T_{Kep}
	Diffusion time	
$\sqrt{N}\, Q\, T_{\mathrm{Kep}}$	$N\, Q\, T_{\mathrm{Kep}}$	$N\, Q^2\, T_{\mathrm{Kep}}$

A wealth of dynamical processes



An extremely **hierarchical system**

Scalar Resonant Relaxation

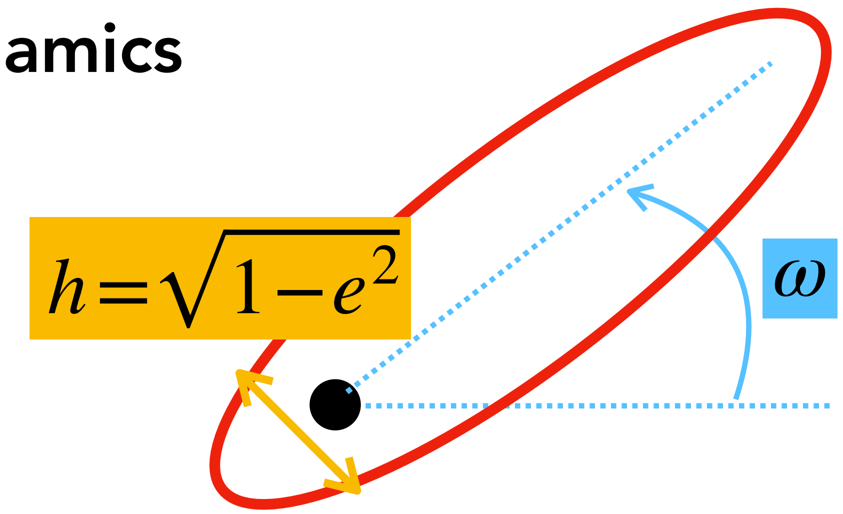


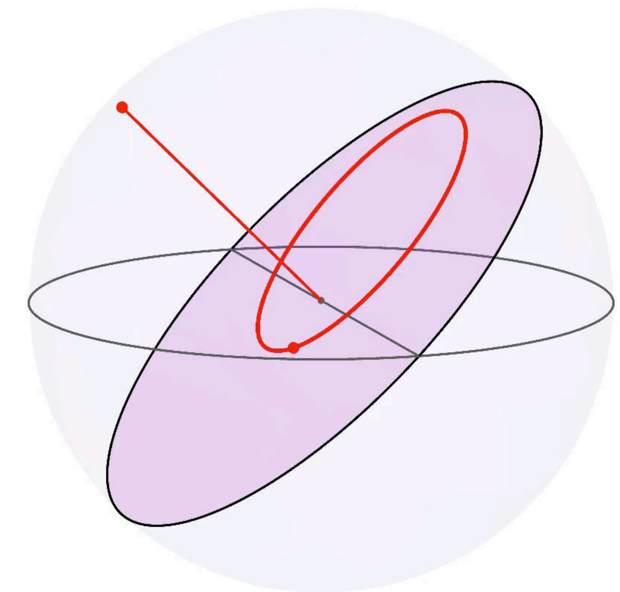
The (resonant) dynamics of **eccentricities**

Scalar Resonant Relaxation

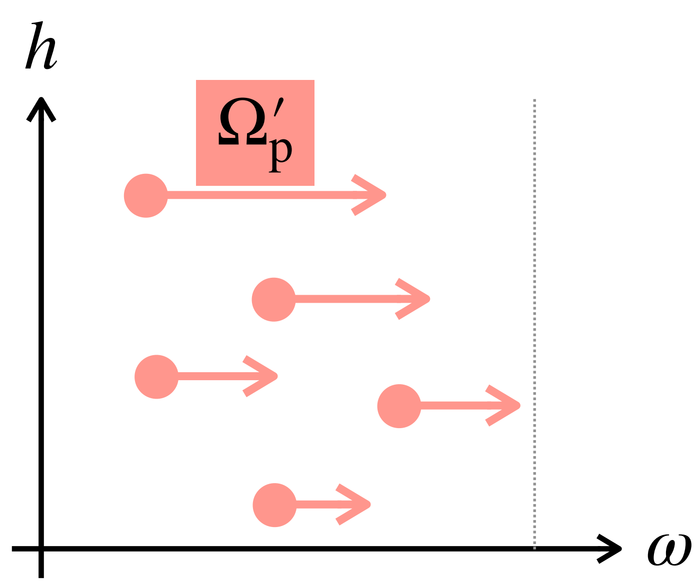
A simple **unperturbed dynamics**

$$\begin{cases} \dot{\omega} \simeq \Omega_{\mathrm{p}}(h) \\ \dot{h} \simeq \eta(t, \omega, h) \end{cases}$$

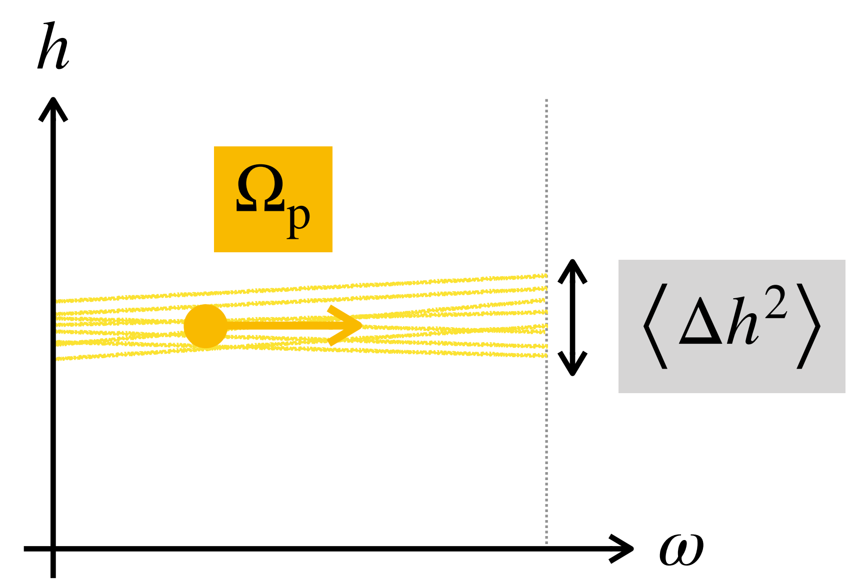




Phase-space dynamics



Background cluster



Test particle

Relaxation occurs at **resonance**

$$k\,\Omega_{\mathrm{p}}(a, h) = k'\,\Omega_{\mathrm{p}}(a', h')$$

Kinetic theory for SRR

Fokker-Planck diffusion equation $h = \sqrt{1-e^2}$

PDF of the test star

$$\frac{\partial P(h,t\,|\,a)}{\partial t} = \frac{1}{2}\frac{\partial}{\partial h}\left[h\, D_{hh}(a,h)\frac{\partial}{\partial h}\left(\frac{P(h,t\,|\,a)}{h}\right)\right]$$

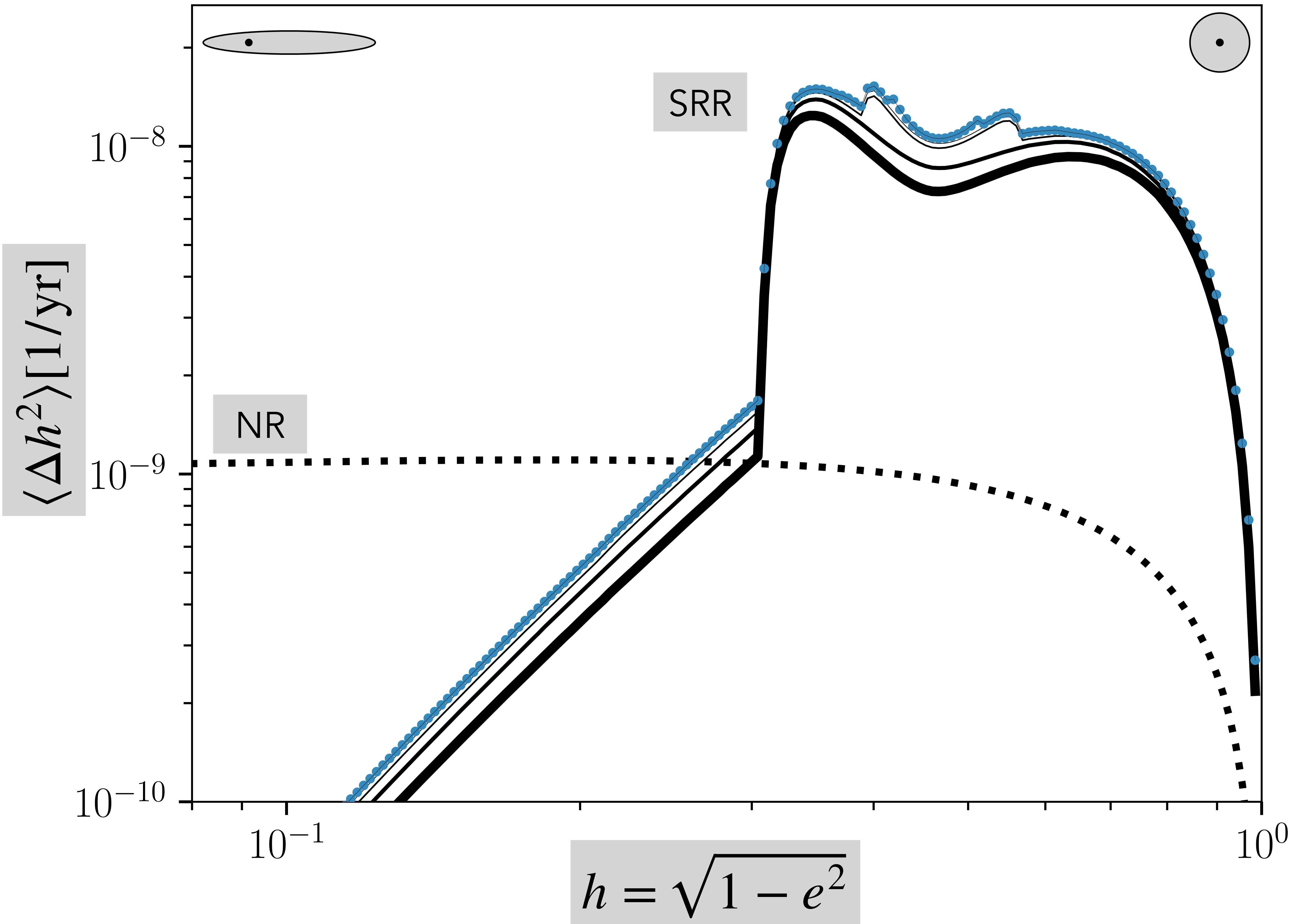
Diffusion coefficients

$$D_{hh}^{\mathrm{RR}}(a,h) = \frac{1}{N}\sum_{k,k'}\int \mathrm{d}a'\mathrm{d}h'\, F_{\mathrm{tot}}(a',h')\, |A_{kk'}(a,h,a',h')|^2\, \delta_{\mathrm{D}}\left[k\Omega_{\mathrm{p}}(a,h) - k'\Omega_{\mathrm{p}}(a',h')\right]$$

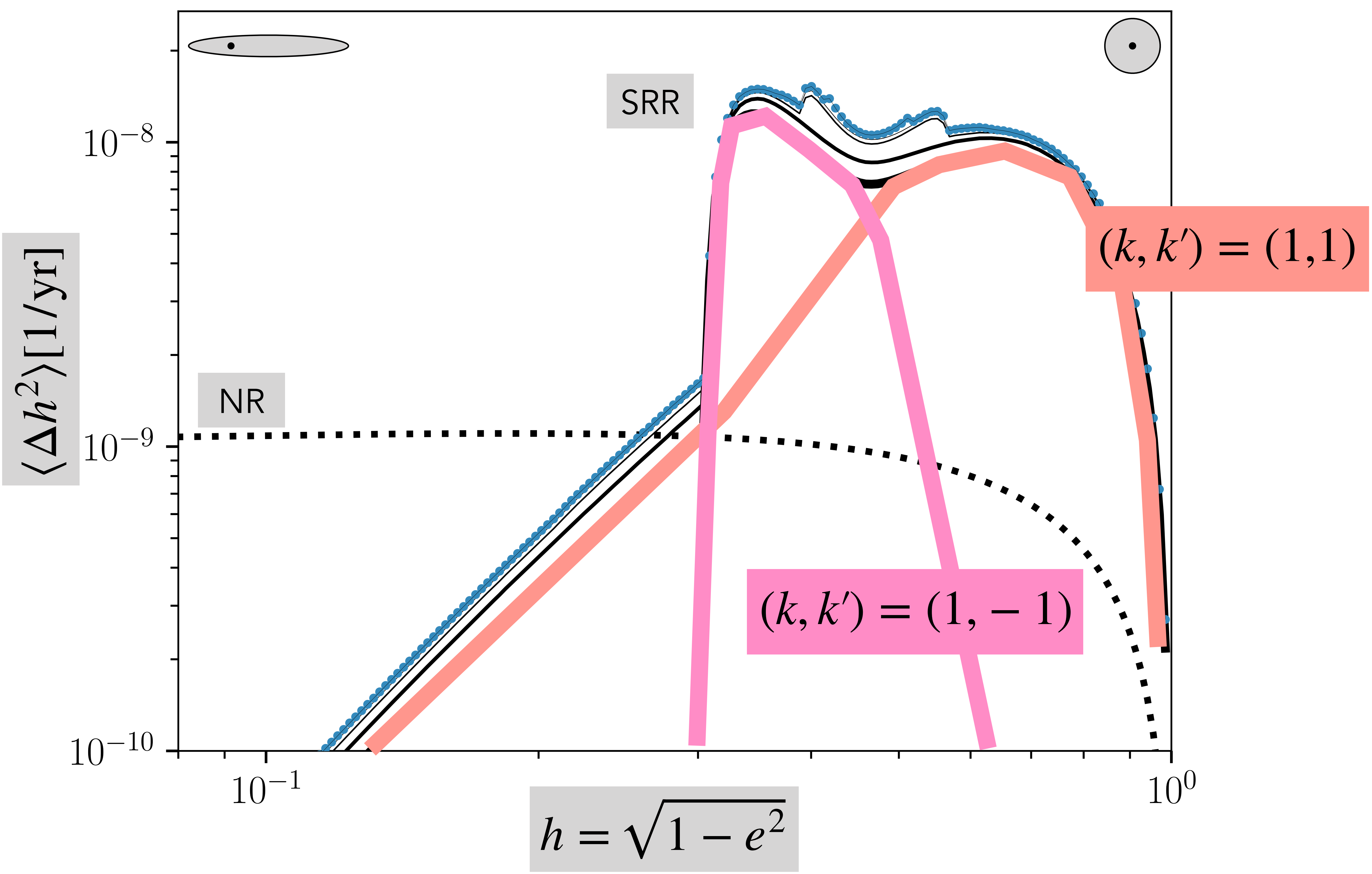
Some properties

- $\partial/\partial h$ *Adiabatic invariance*
- $D_{hh}(a,h)$ *Anisotropic diffusion*
- $1/N$ *Finite-N effects*
- $F_{\mathrm{tot}}(a',h')$ *Background cluster*
- k, k' *Resonance numbers*
- $\int \mathrm{d}a'\mathrm{d}h'$ *Scan of orbital space*
- $\delta_{\mathrm{D}}[k\Omega_{\mathrm{p}} - k'\Omega'_{\mathrm{p}}]$ *Resonance condition*
- $|A_{kk'}(a,h,a',h')|^2$ *Coupling coefficients*

The diffusion coefficients in eccentricity

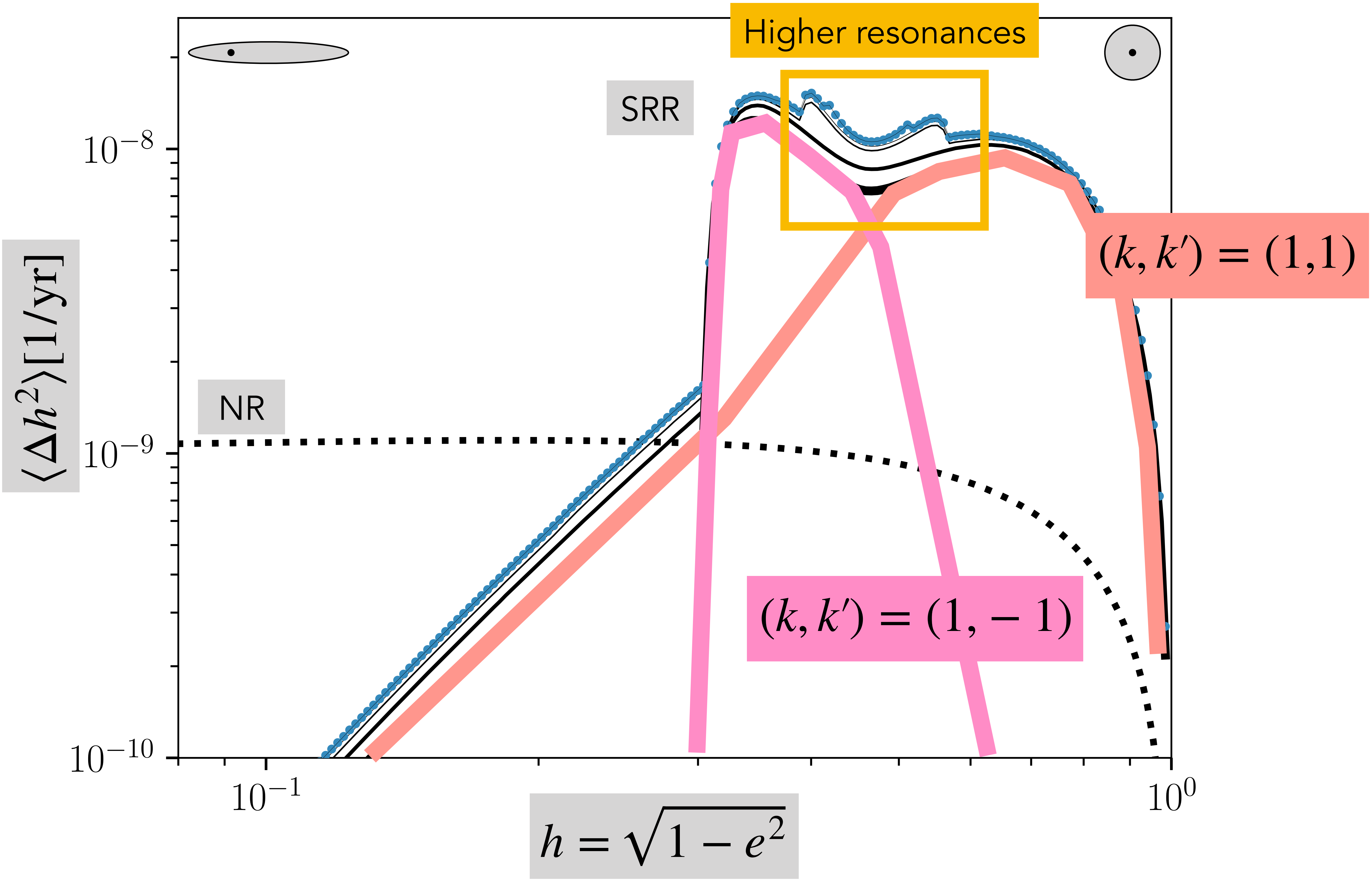


Non-local resonances $\delta_{\mathrm{D}}[k\,\Omega_{\mathrm{p}}(a,h) - k'\,\Omega_{\mathrm{p}}(a',h')]$

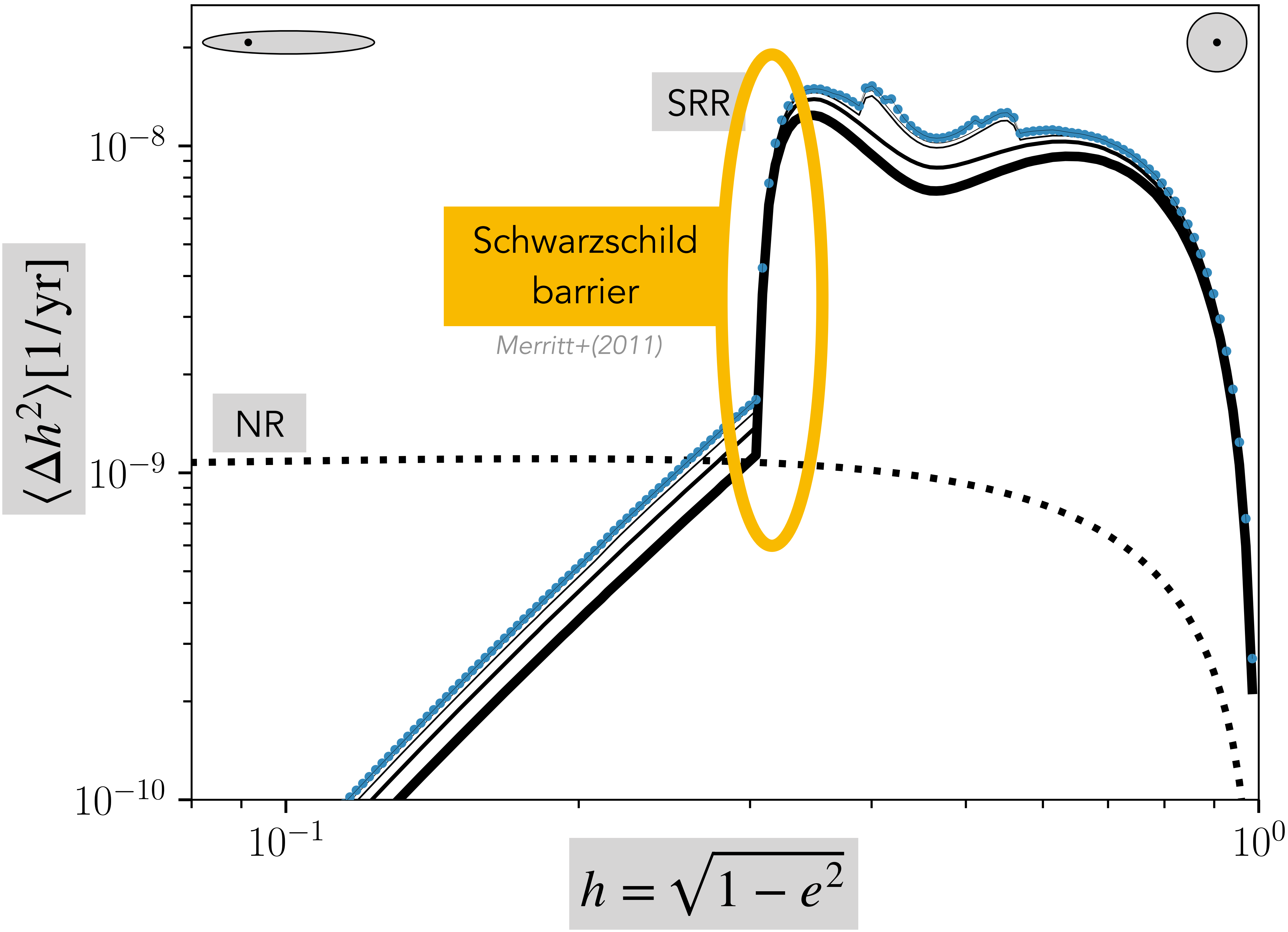


Non-local resonances

$\delta_{\mathrm{D}}[k\,\Omega_{\mathrm{p}}(a,h) - k'\,\Omega_{\mathrm{p}}(a',h')]$



The diffusion coefficients in eccentricity

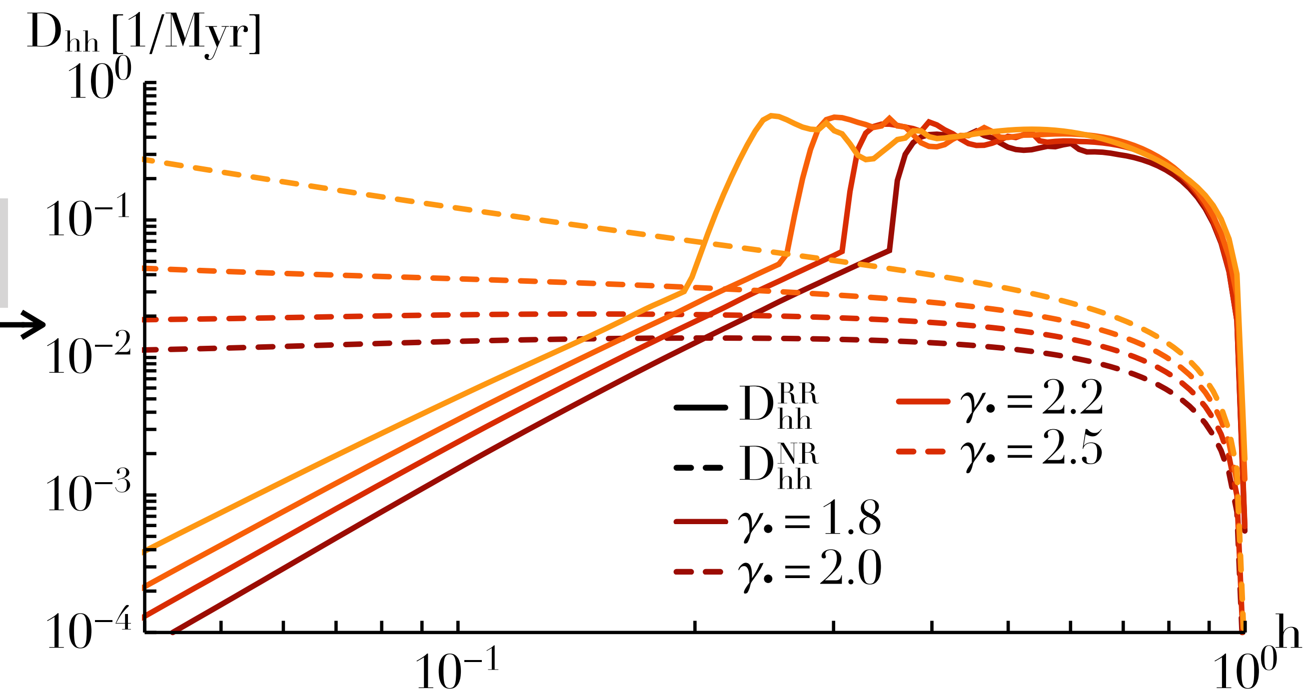


SRR around SgrA*

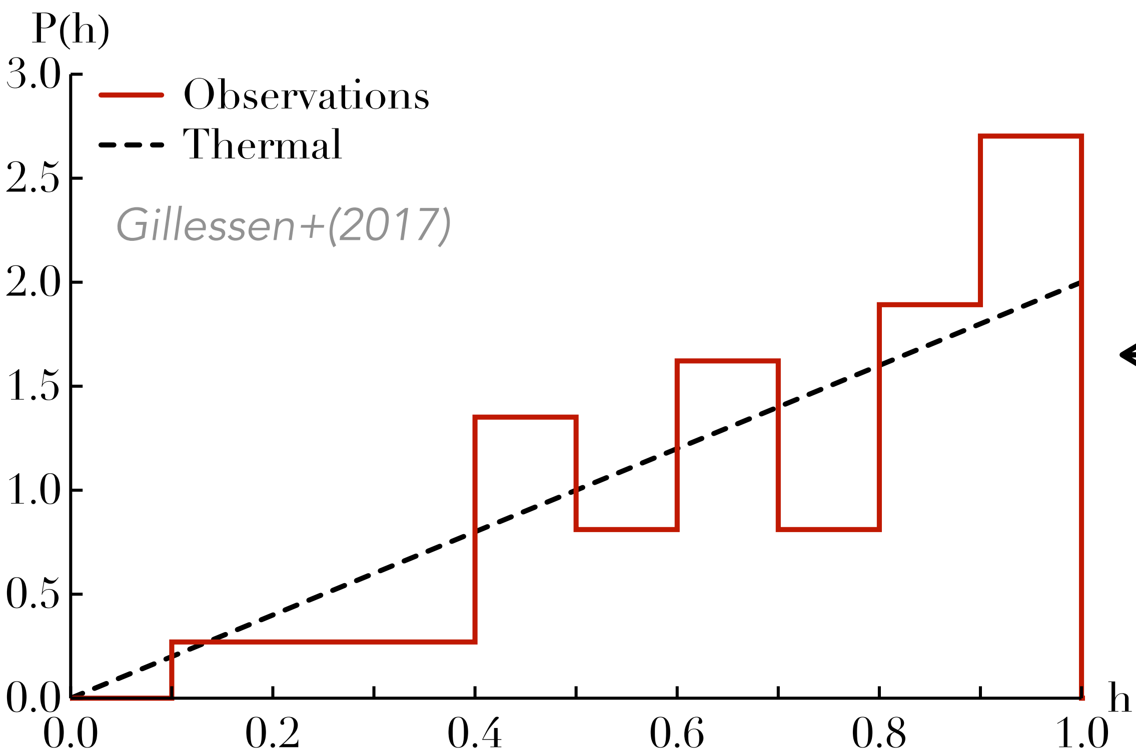
Model

- Old stars
(unresolved but relaxed)
- **IMBHs**
(strong source of Poisson noise)
- S-stars ICs
(Tidal disruption vs disc formation)

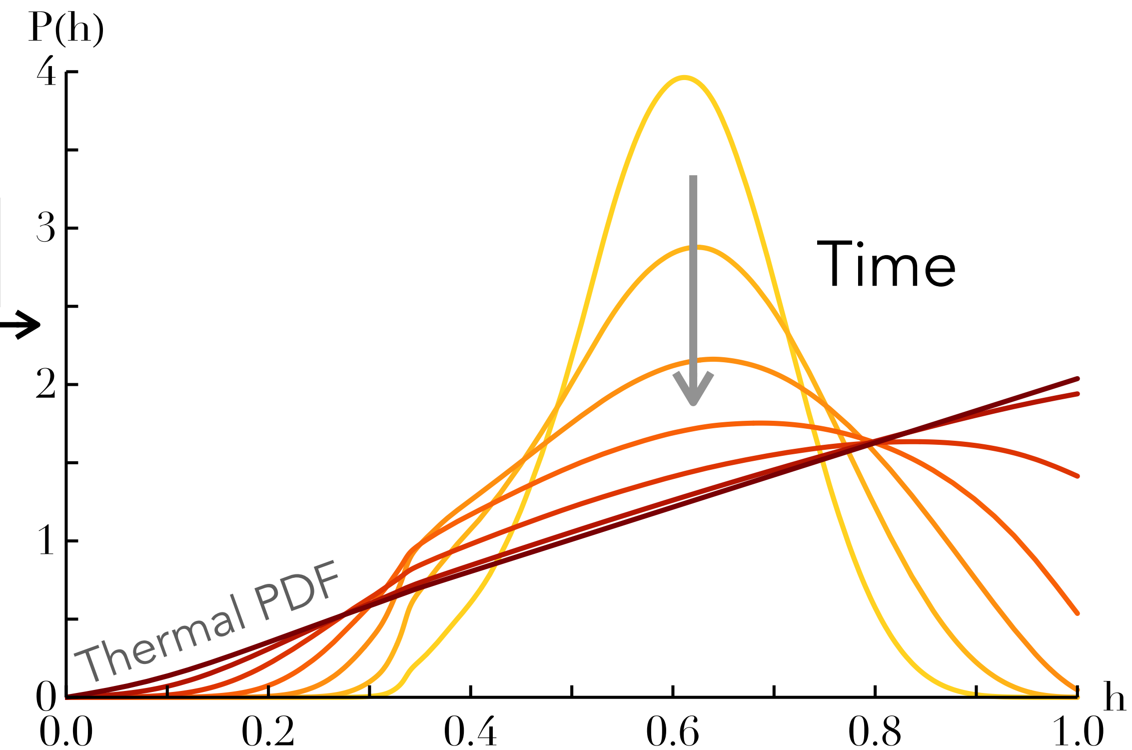
Kinetic theory



Relaxation

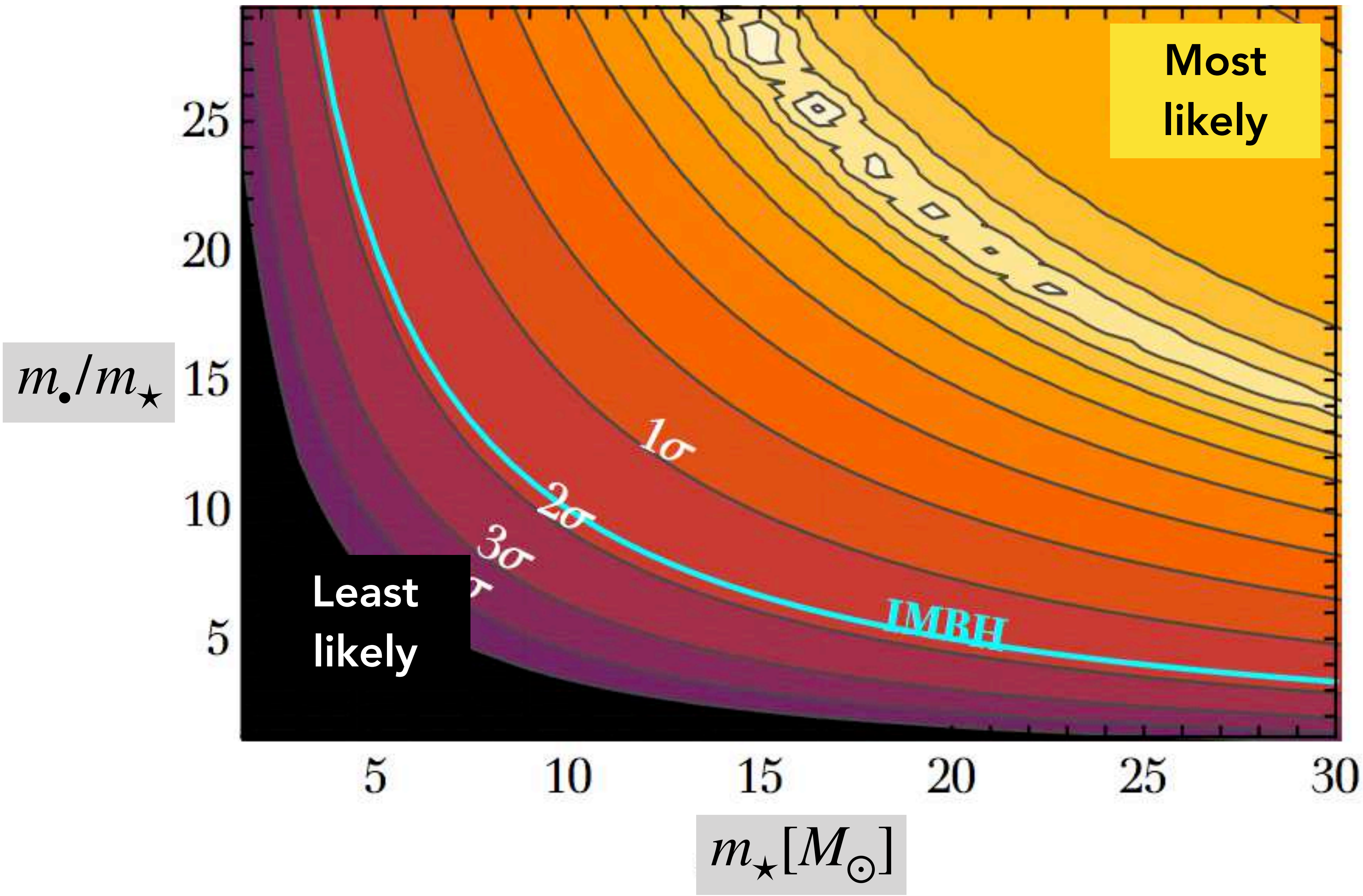


Likelihood



An example of likelihood

2-population model (stars+IMBHs)



Questions to address:

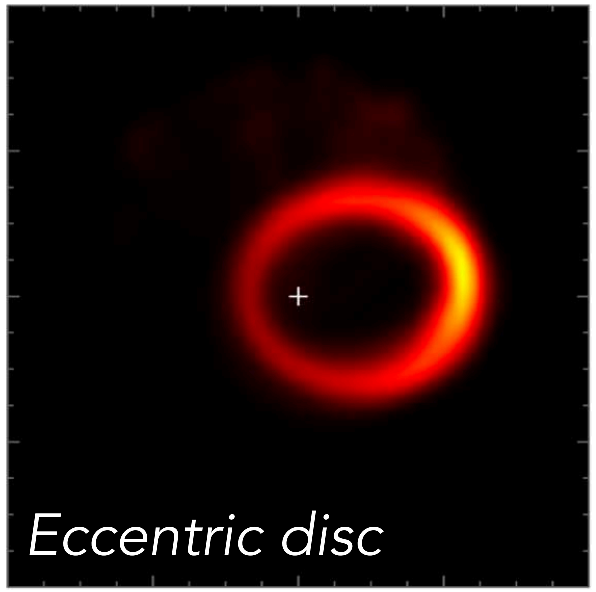
+ Are **IMBHs** mandatory?

+ Where do the **S-stars** come from?

How to do better

Lopsided equilibria

Touma+(2009)



Linear response

Tremaine(2004)

$$M(\omega) \propto \sum_k \int \mathrm{d}a\mathrm{d}h\, h \frac{k\, \partial[F/h]/\partial h}{\omega - k\Omega_{\mathrm{p}}(a,h)}$$

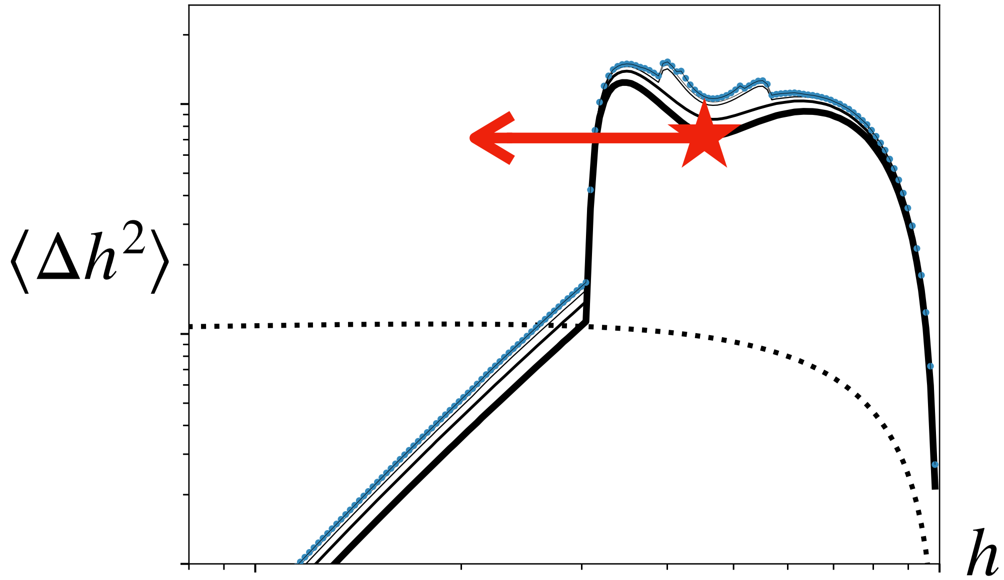
Impact of the loss cone

Resonant dynamical friction

$$F_{\mathrm{pol}}(a,h) \propto \int \mathrm{d}a'\mathrm{d}h'\, h' \frac{\partial[F'/h']}{\partial h'}$$

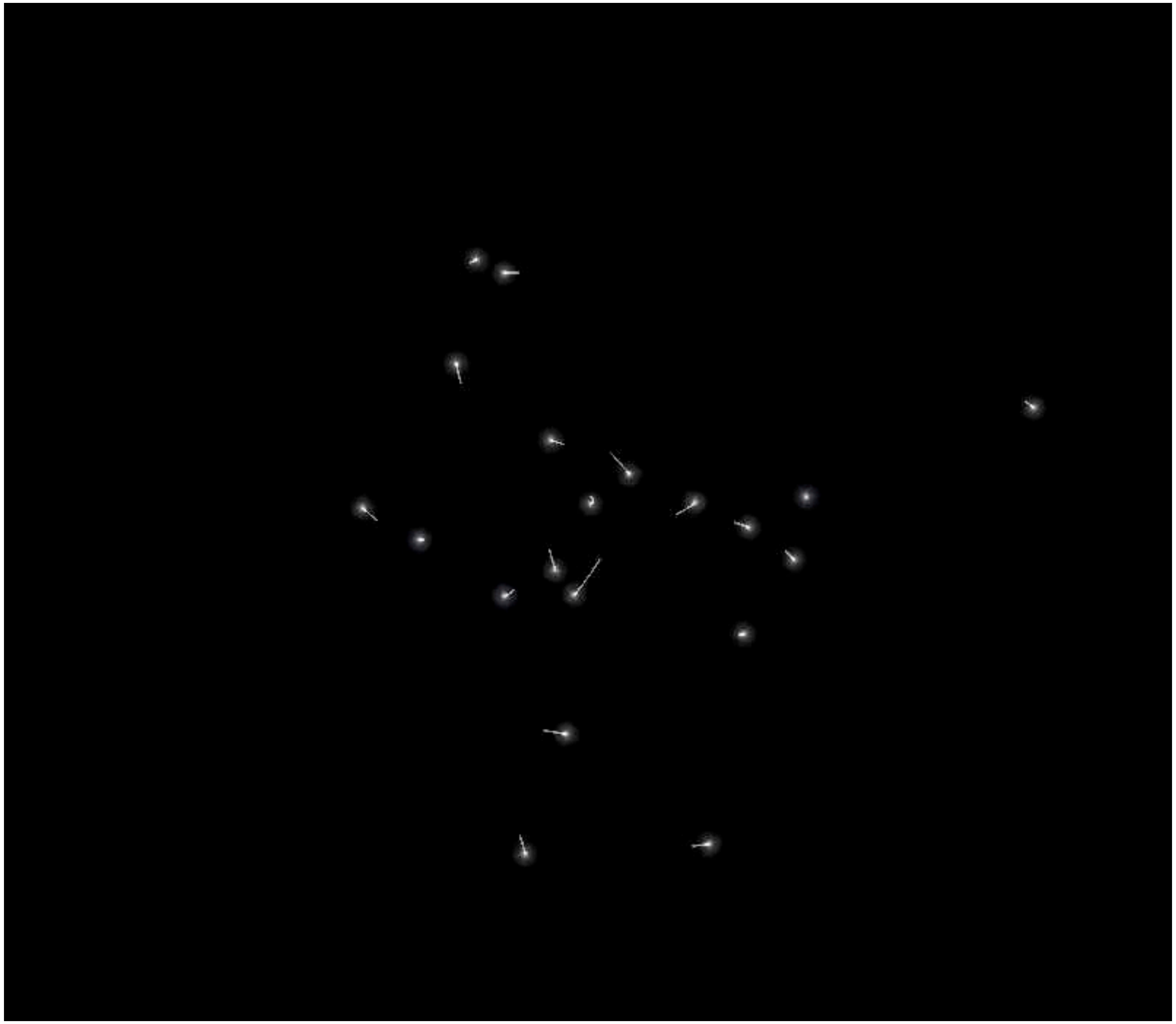
Sinking of heavy objects

Rates of TDE



N-body simulations

Orbits are present on **numerous scales**



How to simulate these **hierarchical systems**?

Orbit average

Orbit-average over **unperturbed** orbits

$$\langle H \rangle = \int \frac{\mathrm{d}M_1}{2\pi} \cdots \frac{\mathrm{d}M_N}{2\pi} \, H$$

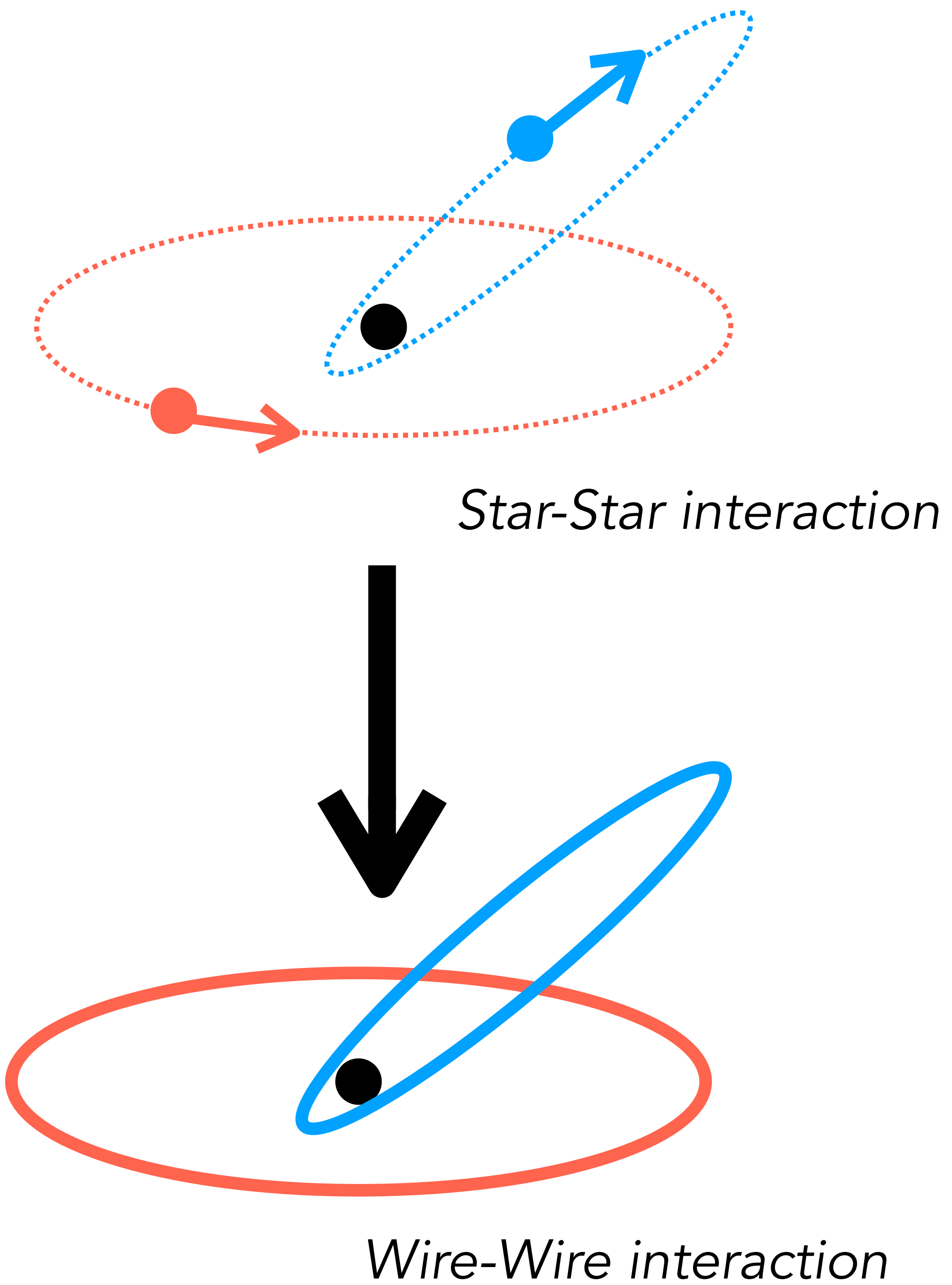
Averaged Hamiltonian

$$\langle H \rangle = \langle H_{\mathrm{GR}} \rangle + \langle H_{\star} \rangle$$

Pairwise couplings

$$\langle H_{\star} \rangle = -\sum_{i<j}^{N} \left\langle \frac{G m_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|} \right\rangle$$

Term difficult to compute



Representing wires

Orbital elements

$$(M,\ \omega,\ \Omega,\ \Lambda,\ L,\ L_z)$$

Absent Conserved

Only **four** effective variables

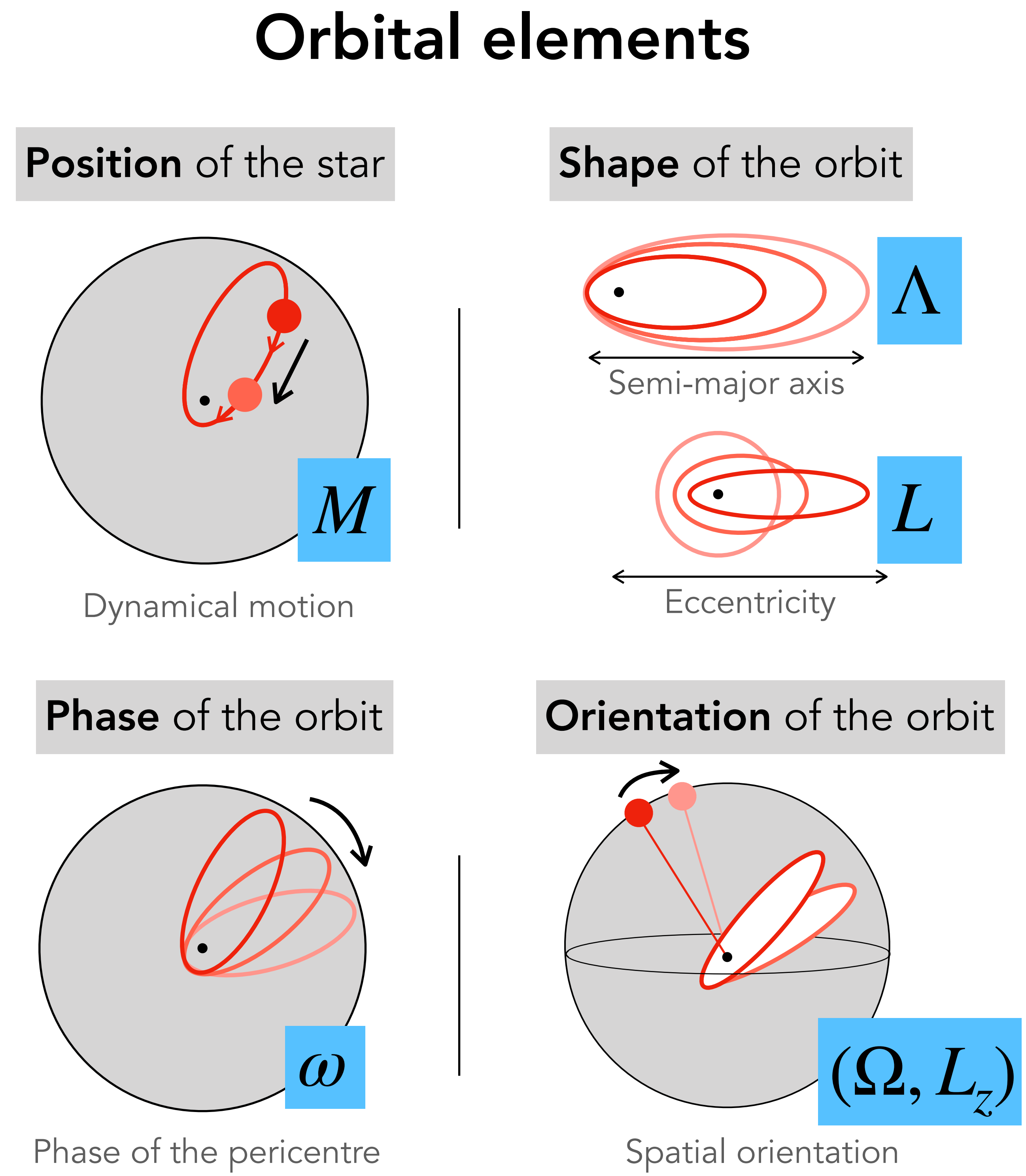
Hamilton's equations

$$\begin{cases} \dot{\omega} = \cdots, \\ \dot{\Omega} = \cdots, \\ \dot{L} = \cdots, \\ \dot{L}_z = \cdots. \end{cases}$$

Bad idea:

(i) frame-dependent

(ii) gimbal lock



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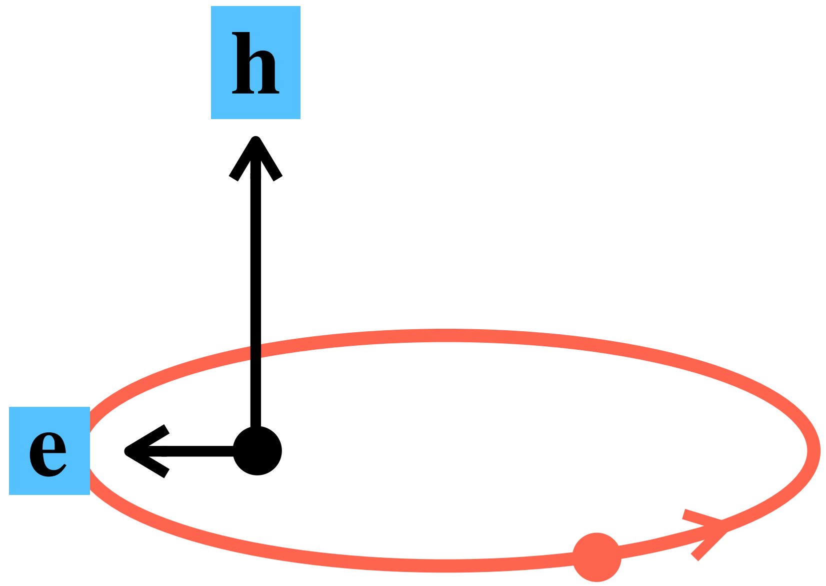
Eccentricity vectors



Six **dynamical variables**

$$(\mathbf{h},\ \mathbf{e})$$

Two **geometric** constraints

$$\mathbf{h} \cdot \mathbf{e} = 0; \quad \mathbf{h}^2 + \mathbf{e}^2 = 1$$

Good idea:

(i) frame-independent

Milankovitch's equations

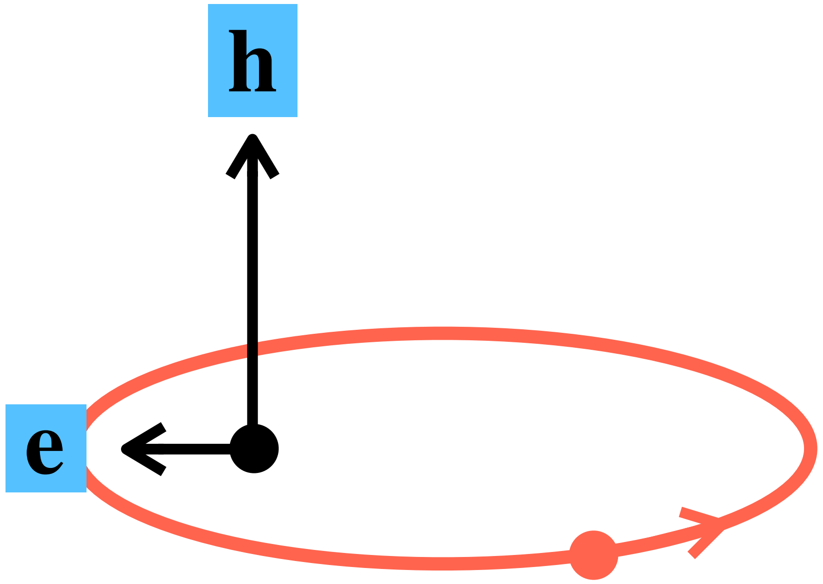
Hamilton's equations, not ideal

$$\begin{cases} \mathbf{h} = \mathbf{h}(\mathbf{r}, \mathbf{p}) \\ \mathbf{e} = \mathbf{e}(\mathbf{r}, \mathbf{p}) \end{cases}$$

Milankovitch equations *Milankovitch(1939)*

$$\dot{\mathbf{h}} = -\frac{1}{\Lambda}\left(\mathbf{h} \times \frac{\partial \langle H \rangle}{\partial \mathbf{h}} + \mathbf{e} \times \frac{\partial \langle H \rangle}{\partial \mathbf{e}}\right)$$

$$\dot{\mathbf{e}} = -\frac{1}{\Lambda}\left(\mathbf{h} \times \frac{\partial \langle H \rangle}{\partial \mathbf{e}} + \mathbf{e} \times \frac{\partial \langle H \rangle}{\partial \mathbf{h}}\right)$$



Conserves all the constraints, for any $\langle H \rangle$

$$\frac{\mathrm{d}(\mathbf{h}^2 + \mathbf{e}^2)}{\mathrm{d}t} = 0; \quad \frac{\mathrm{d}(\mathbf{h} \cdot \mathbf{e})}{\mathrm{d}t} = 0; \quad \frac{\mathrm{d}\langle H \rangle}{\mathrm{d}t} = 0$$

Self-consistent Hamiltonian approach

(1) Discretise $\langle H \rangle$ (2) Compute $(\dot{\mathbf{h}}, \dot{\mathbf{e}})$

Radial discretisation & Multipole Expansion

Time integration

Classical integration

$$\dot{y} = F(y)$$

$$y_n \xrightarrow{\tau} y_{n+1}$$

Time integration

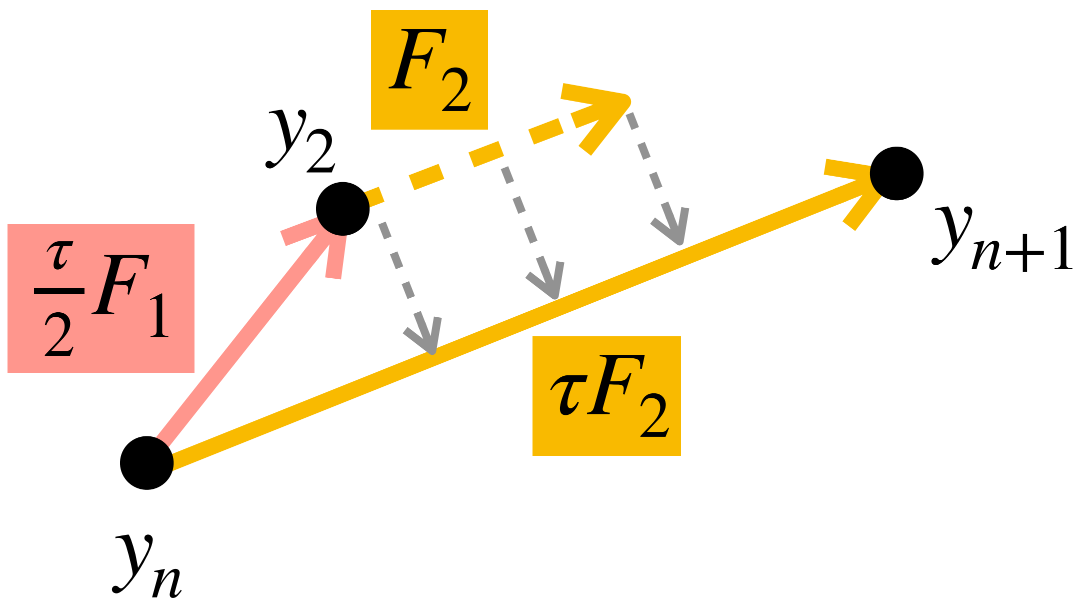
Classical integration

$$\dot{y} = F(y)$$

$$y_n \xrightarrow{\tau} y_{n+1}$$

Explicit Midpoint rule

$$\begin{aligned} F_1 &= F(y_n) \\ y_2 &= y_n + \frac{1}{2}\tau F_1 \\ F_2 &= F(y_2) \\ y_{n+1} &= y_n + \tau F_2 \end{aligned}$$



Time integration

Classical integration

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$$y_n \xrightarrow{\tau} y_{n+1}$$

Explicit Midpoint rule

$$F_1 = F(y_n)$$
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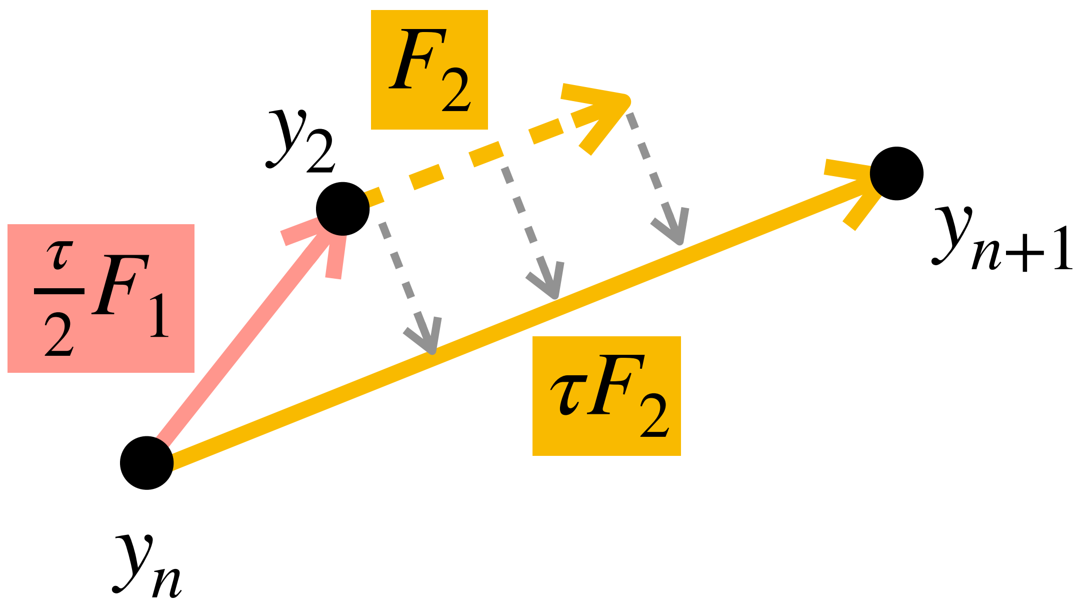
$$y_{n+1} = y_n + \tau\,F_2$$



Fourth-order **Runge-Kutta**

$$F_1 = F(y_n)$$
$$y_2 = y_n + \frac{1}{2}\tau F_1$$
$$F_2 = F(y_2)$$
$$y_3 = y_n + \frac{1}{2}\tau F_2$$
$$F_3 = F(y_3)$$
$$y_4 = y_n + \tau F_3$$
$$F_4 = F(y_4)$$
$$F = \frac{1}{6}F_1 + \frac{1}{3}F_2 + \frac{1}{3}F_3 + \frac{1}{6}F_4$$
$$y_{n+1} = y_n + \tau F$$

Time integration

Classical integration

$$\dot{y} = F(y)$$

$$y_n \xrightarrow{\tau} y_{n+1}$$

Explicit Midpoint rule

$$
\begin{aligned}
F_1 &= F(y_n) \\
y_2 &= y_n + \frac{1}{2}\tau F_1 \\
F_2 &= F(y_2) \\
y_{n+1} &= y_n + \tau F_2
\end{aligned}
$$

How to comply with constraints?

$$
\begin{aligned}
y' &= y + \tau F \\
F &= F_1 + F_2
\end{aligned}
$$

Fourth-order **Runge-Kutta**

$$
\begin{aligned}
F_1 &= F(y_n) \\
y_2 &= y_n + \frac{1}{2}\tau F_1 \\
F_2 &= F(y_2) \\
y_3 &= y_n + \frac{1}{2}\tau F_2 \\
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y_4 &= y_n + \tau F_3 \\
F_4 &= F(y_4) \\
F &= \frac{1}{6}F_1 + \frac{1}{3}F_2 + \frac{1}{3}F_3 + \frac{1}{6}F_4 \\
y_{n+1} &= y_n + \tau F
\end{aligned}
$$

Klein variables

Eccentricity vectors

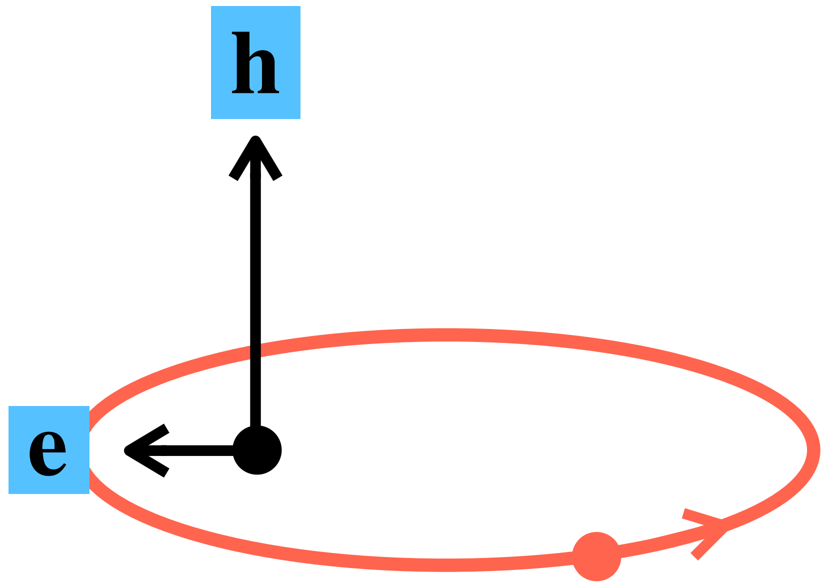
Six **dynamical variables**

$$(\mathbf{h}, \mathbf{e})$$

Two **geometric** constraints

$$\mathbf{h} \cdot \mathbf{e} = 0; \quad \mathbf{h}^2 + \mathbf{e}^2 = 1$$

Intricate vector dynamics



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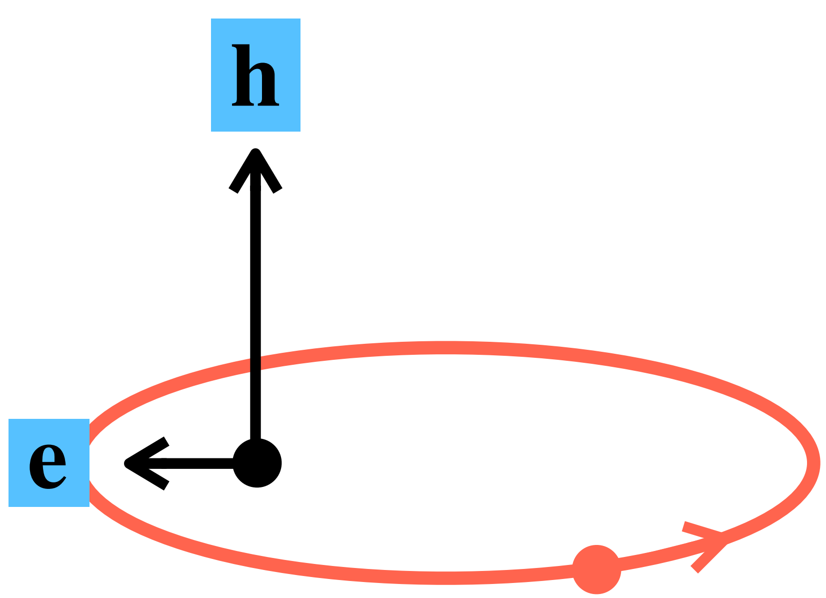


Klein variables *Klein(1924)*

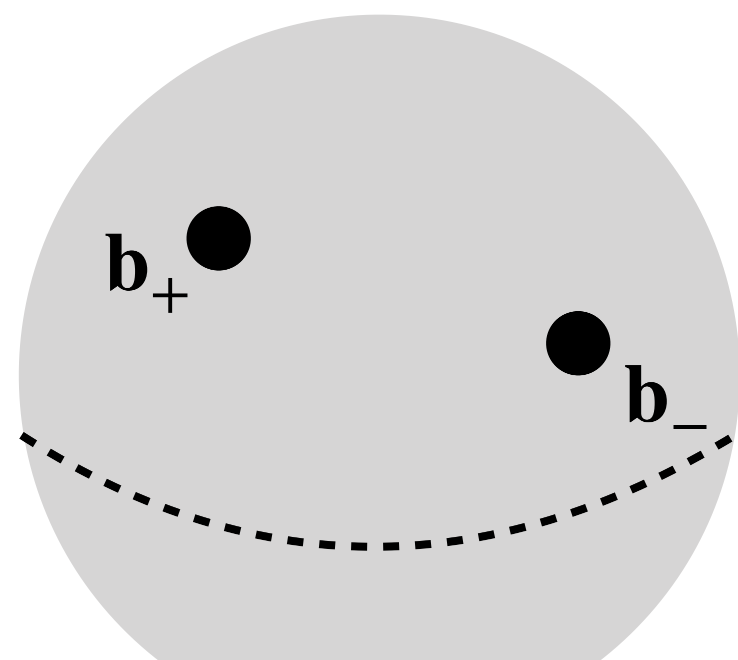
Six **dynamical variables**

$$\mathbf{b}_+ = \mathbf{h} + \mathbf{e}$$
$$\mathbf{b}_- = \mathbf{h} - \mathbf{e}$$

Two **simple** constraints

$$|\mathbf{b}_+| = |\mathbf{b}_-| = 1$$

Dynamics on the **unit sphere**



Like a **classical spin system**

Structure-preserving integration

Dynamics on the **unit sphere**

$$\dot{\mathbf{b}} = \mathbf{B}(\mathbf{b}) \quad \text{with} \quad \mathbf{B}(\mathbf{b})\cdot\mathbf{b} = 0$$

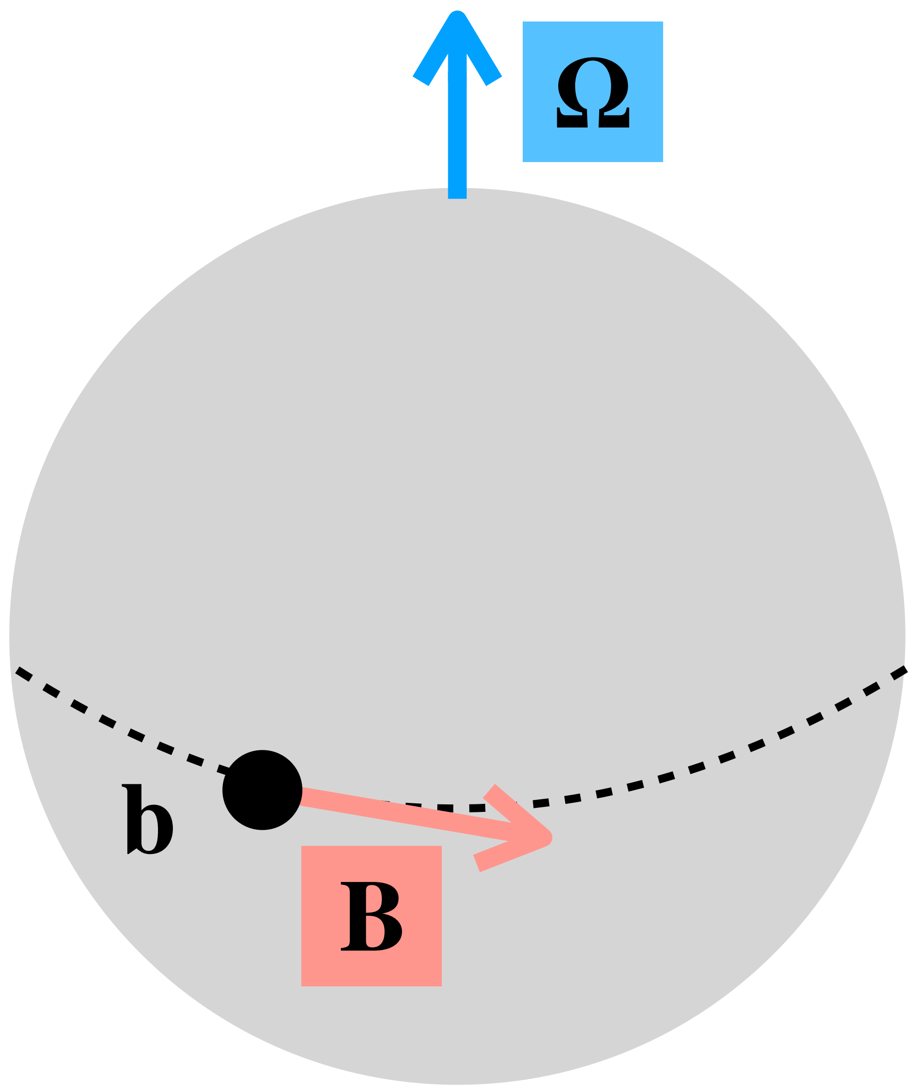
Rotation along **great circle**

$$\dot{\mathbf{b}} = \boldsymbol{\Omega} \times \mathbf{b} \quad \text{with} \quad \boldsymbol{\Omega} = \mathbf{b} \times \dot{\mathbf{b}}$$

Exact solution for fixed $\boldsymbol{\Omega}$

$$\mathbf{b}(t) = \phi[t\,\boldsymbol{\Omega}] \circ \mathbf{b}(0)$$

Rodrigues' rotation formula



Explicit scheme

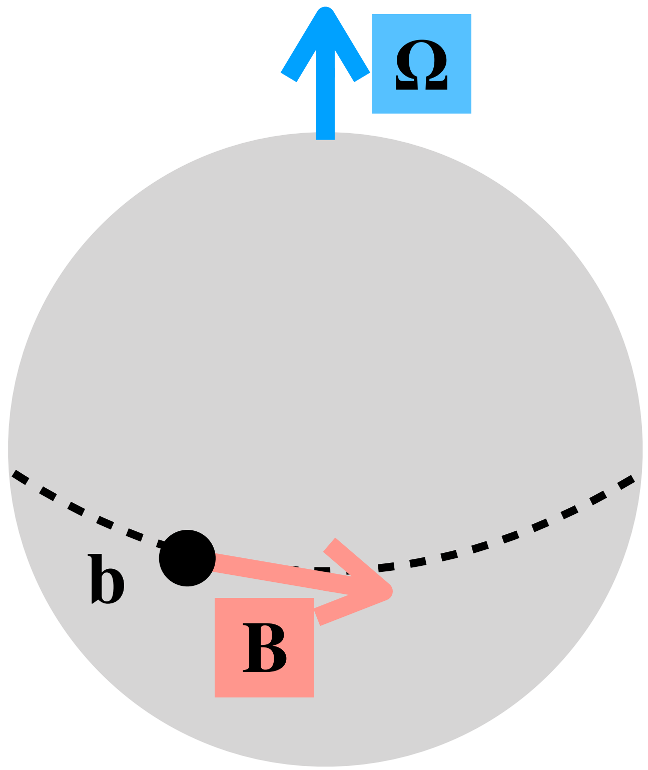
Explicit Midpoint via **rotations**

$$F_1 = F(y_n)$$
$$y_2 = y_n + \frac{1}{2}\tau F_1$$
$$F_2 = F(y_2)$$
$$y_{n+1} = y_n + \tau F_2$$

→

MK2

$$\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}(\mathbf{b}_n)$$
$$\mathbf{b}_2 = \phi[\tfrac{1}{2}\tau\,\boldsymbol{\Omega}_1] \circ \mathbf{b}_n$$
$$\boldsymbol{\Omega}_2 = \boldsymbol{\Omega}(\mathbf{b}_2)$$
$$\mathbf{b}_{n+1} = \phi[\tau\boldsymbol{\Omega}_2] \circ \mathbf{b}_n$$

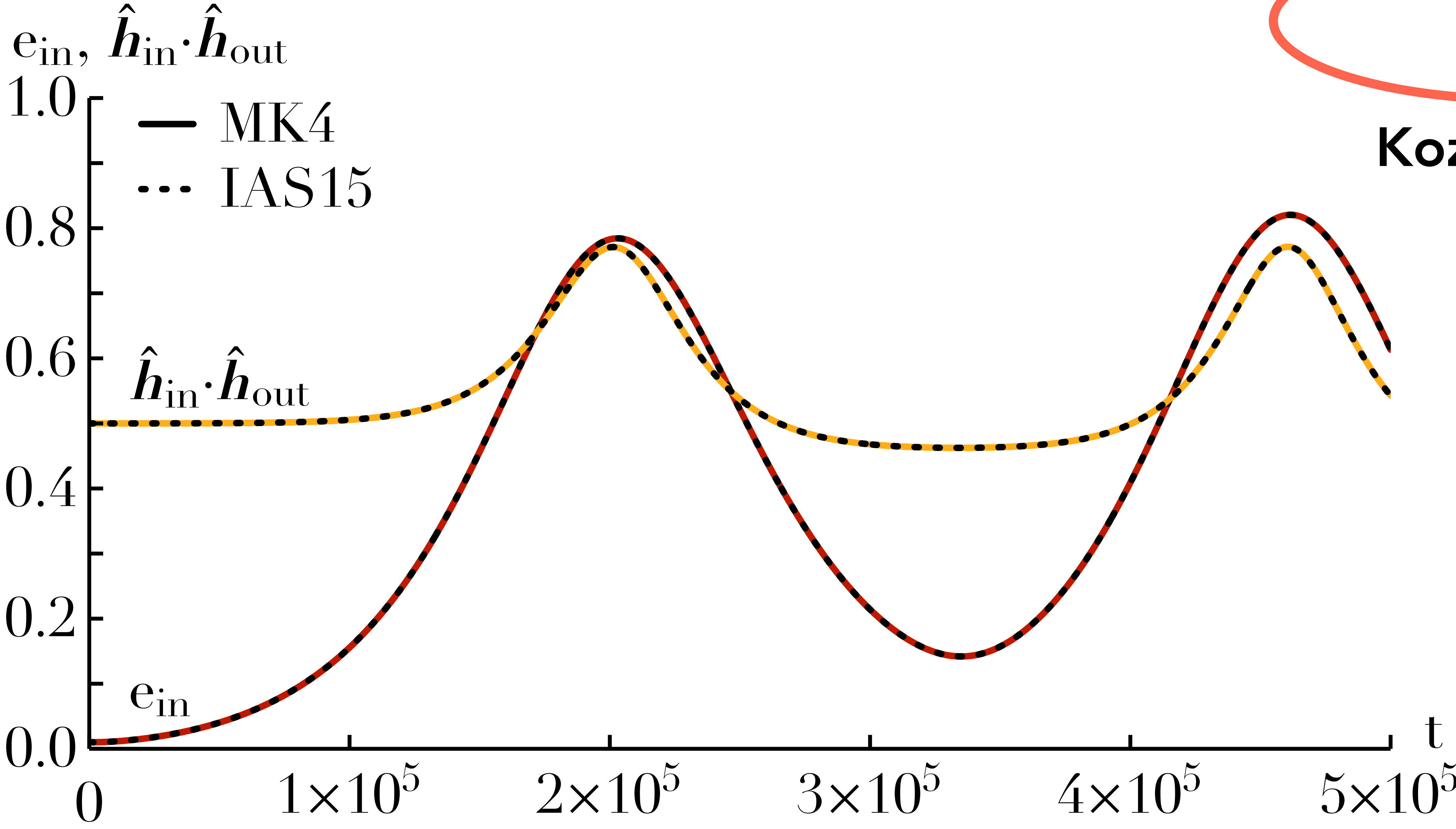


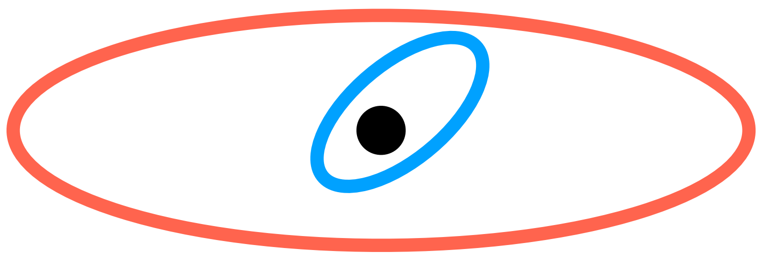
Properties:
(i) explicit
(ii) intrinsic
(ii) exactly conserves |b|
(iii) second-order accurate
(iv) two-stage

Up to **commutations**, can be used for high-order schemes *Munthe-Kaas(1999)*

Convergence of the time integration

Comparison with **direct integration**



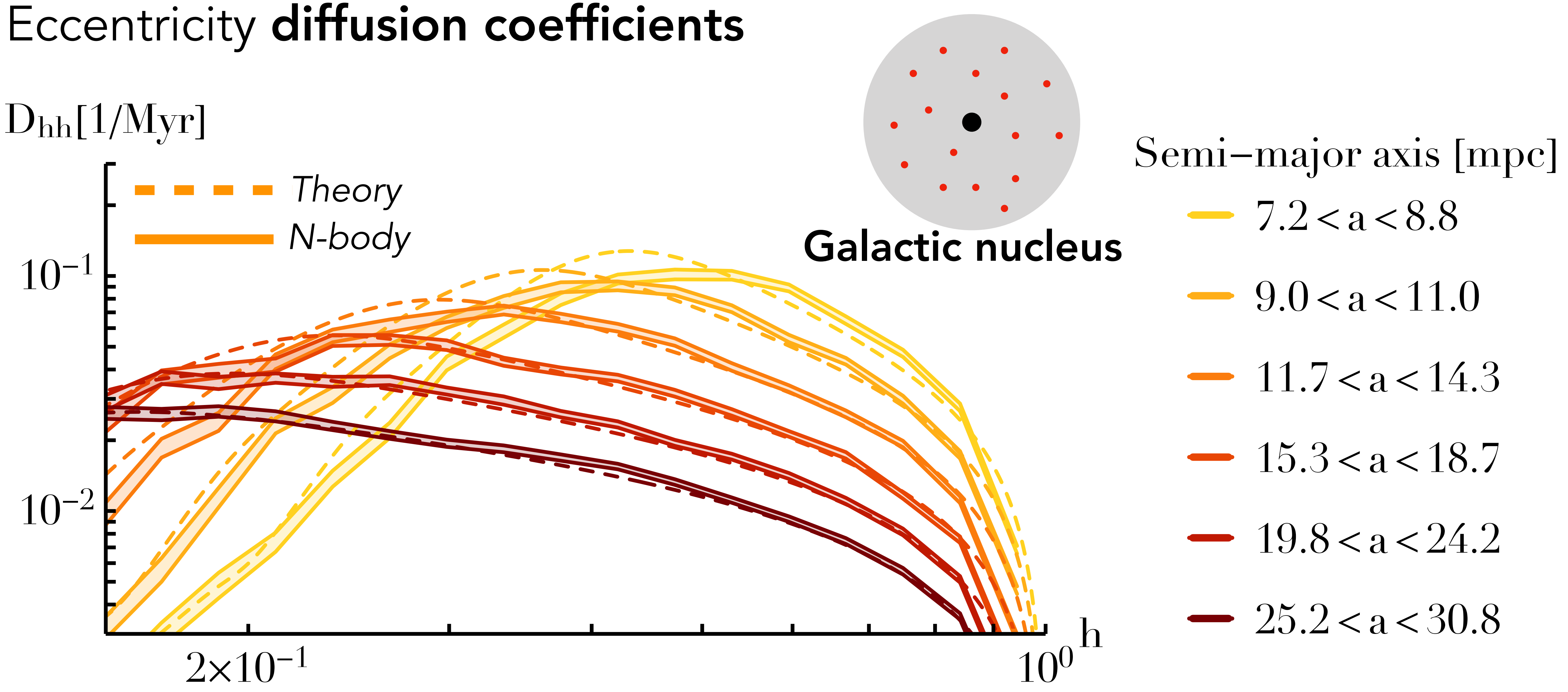


Kozai-Lidov

Eccentricity and inclination oscillations matched
using a **significantly** larger timestep

Matching kinetic theory

Eccentricity **diffusion coefficients**



Kinetic predictions

$$D_{hh}(T) = \int \mathrm{d}a'\mathrm{d}h'\, G_T[k\Omega_\mathrm{p} - k'\Omega_\mathrm{p}]\, \psi_{kk'}\, F'$$

with

$$G_T(\Omega) = \frac{1 - \cos(T\Omega)}{\pi\Omega^2 T}$$

How to do better

Parallelisation

Ladner+(1980)

$$P_{\ell m}(r) = \sum_{j,l;r_{jl}<r} Y_{\ell m}(\mathbf{r})$$

Parallel prefix sum

Multi-timesteps

Saha+(1994)

$$\langle H \rangle = \sum_{(i,j)\in A} \langle H_{ij} \rangle + \sum_{(i,j)\in B} \langle H_{ij} \rangle$$

Hamiltonian splitting

Softening

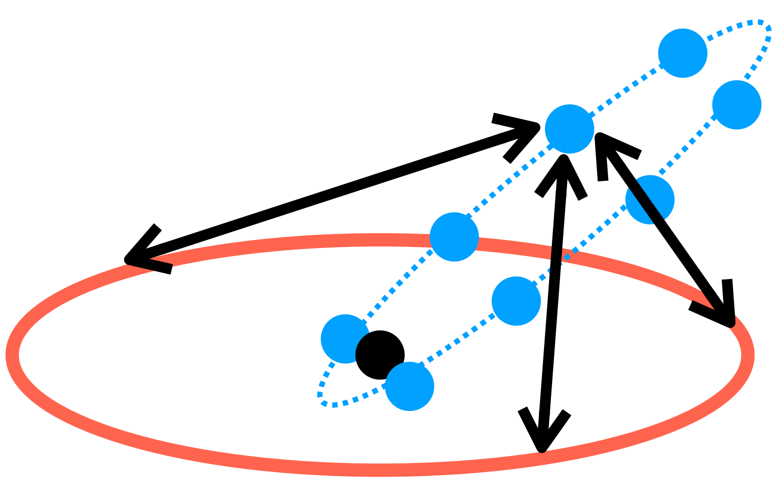
Dehnen+(2014)

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} \rightarrow \varphi_\epsilon(\mathbf{r}-\mathbf{r}')$$

Direct summation & Opening angle

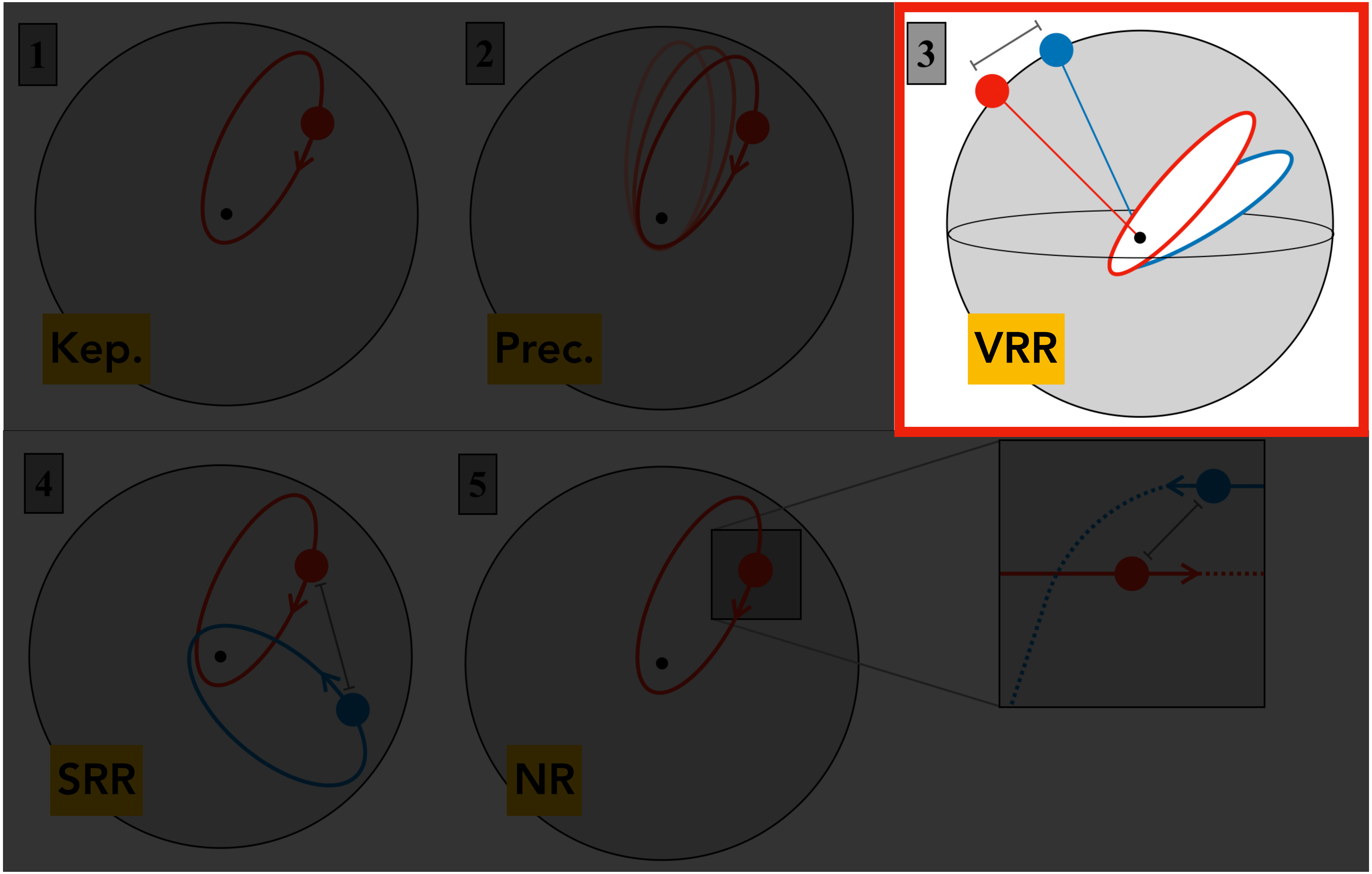
Gauss method

Touma+(2009)



Star-Wire interaction

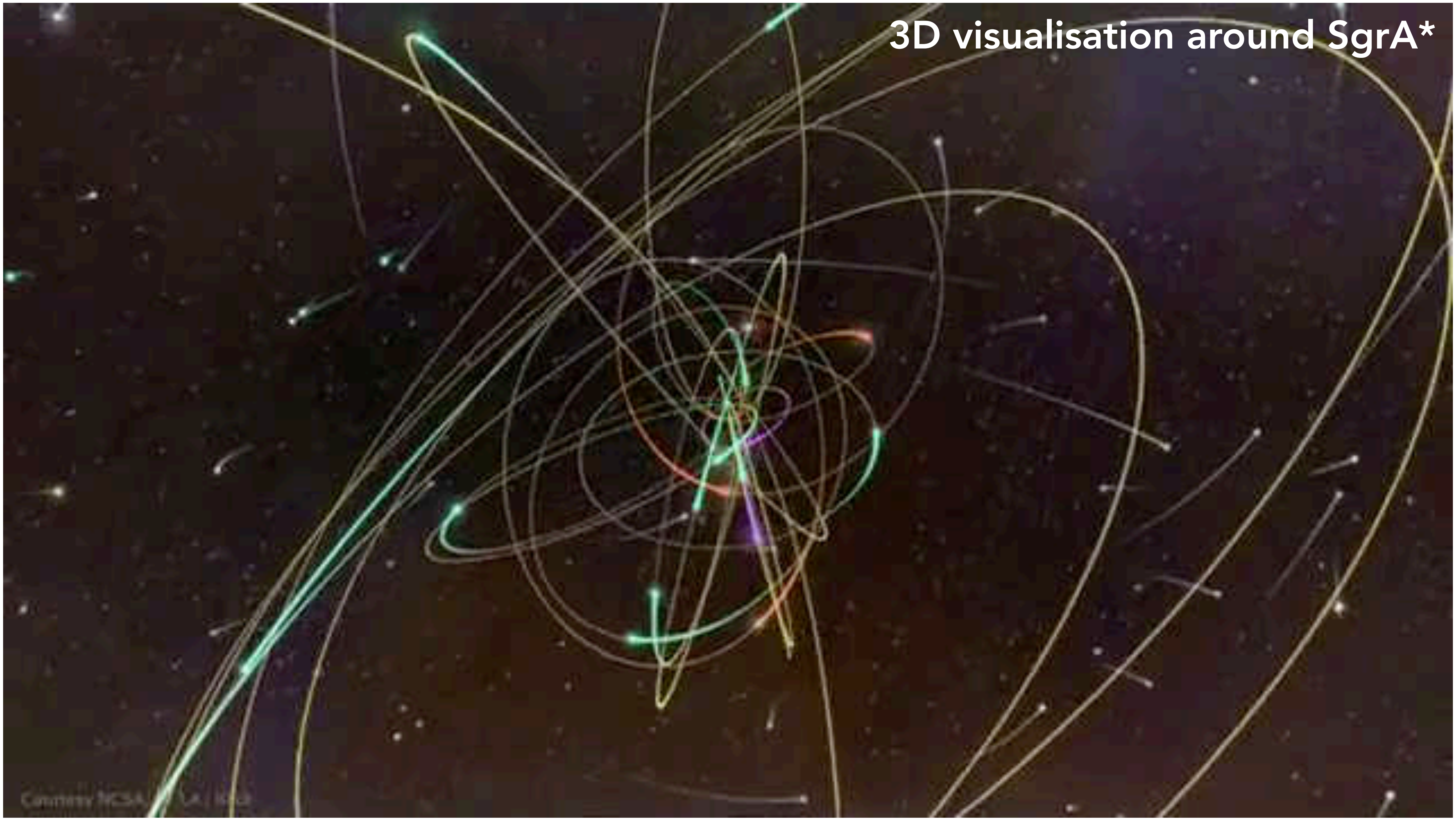
Vector Resonant Relaxation



The coherent dynamics of **orientations**

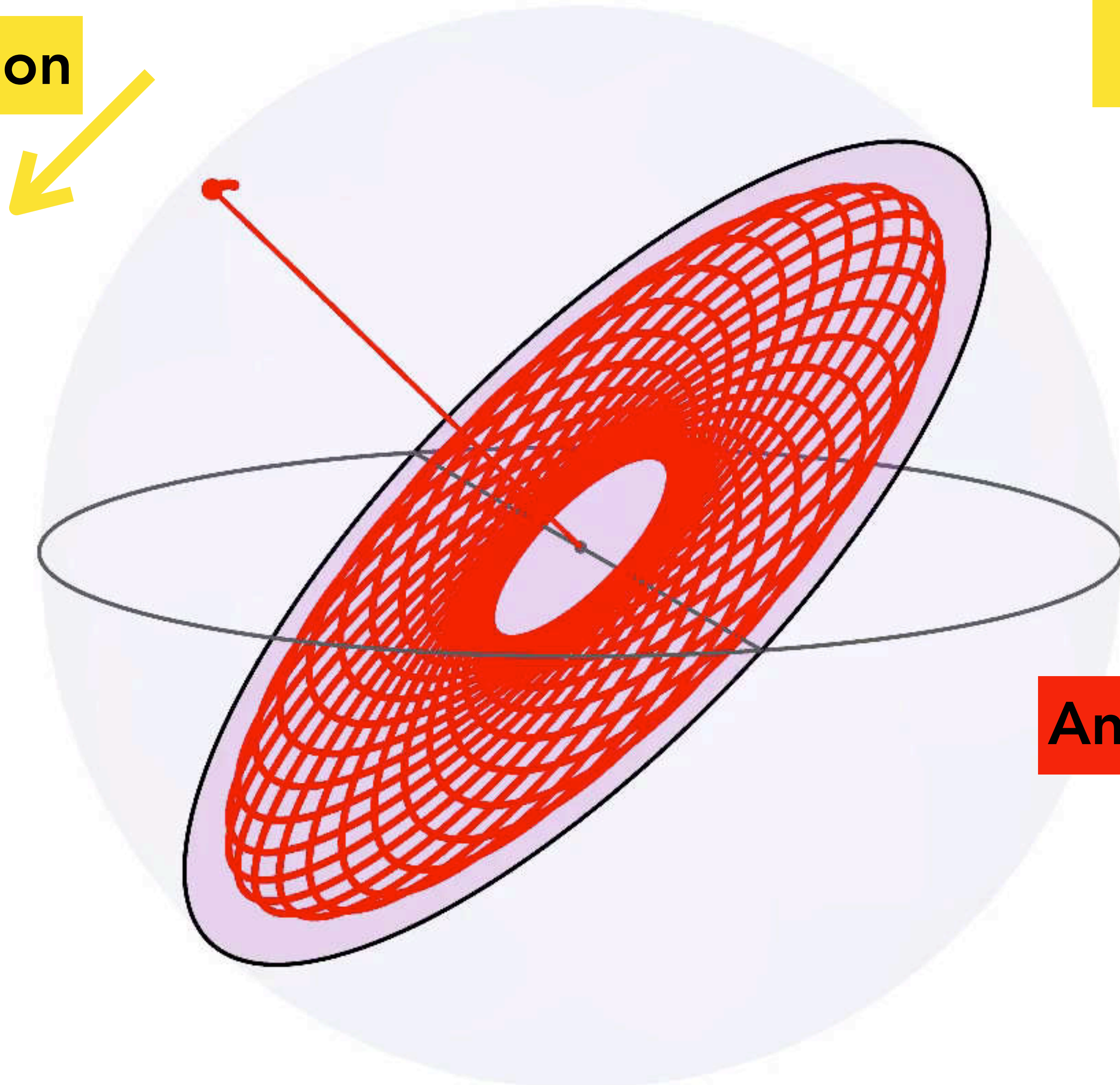
Stellar orientations

Orbits are in **all directions**



How do stars change of **orientations**?

Stellar orientations



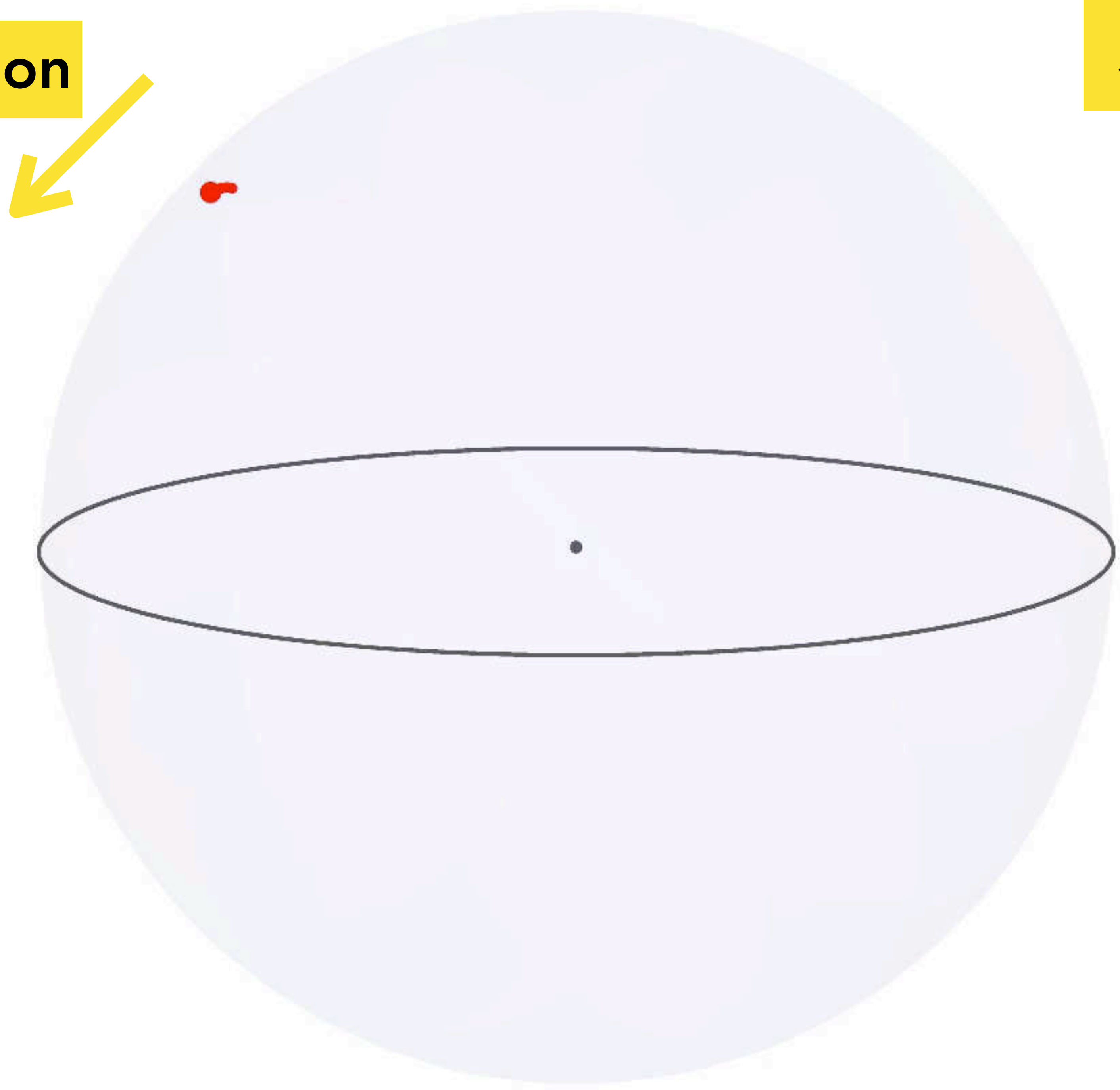
After a full precession, **ellipses** become **annuli**

Orbital orientations

Orientation

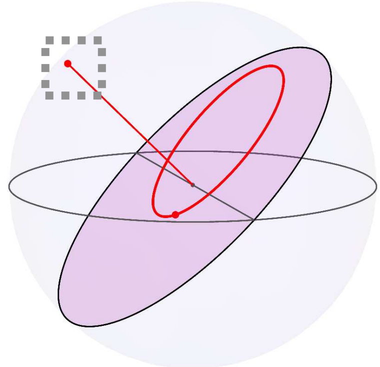
$\hat{\mathbf{L}}$

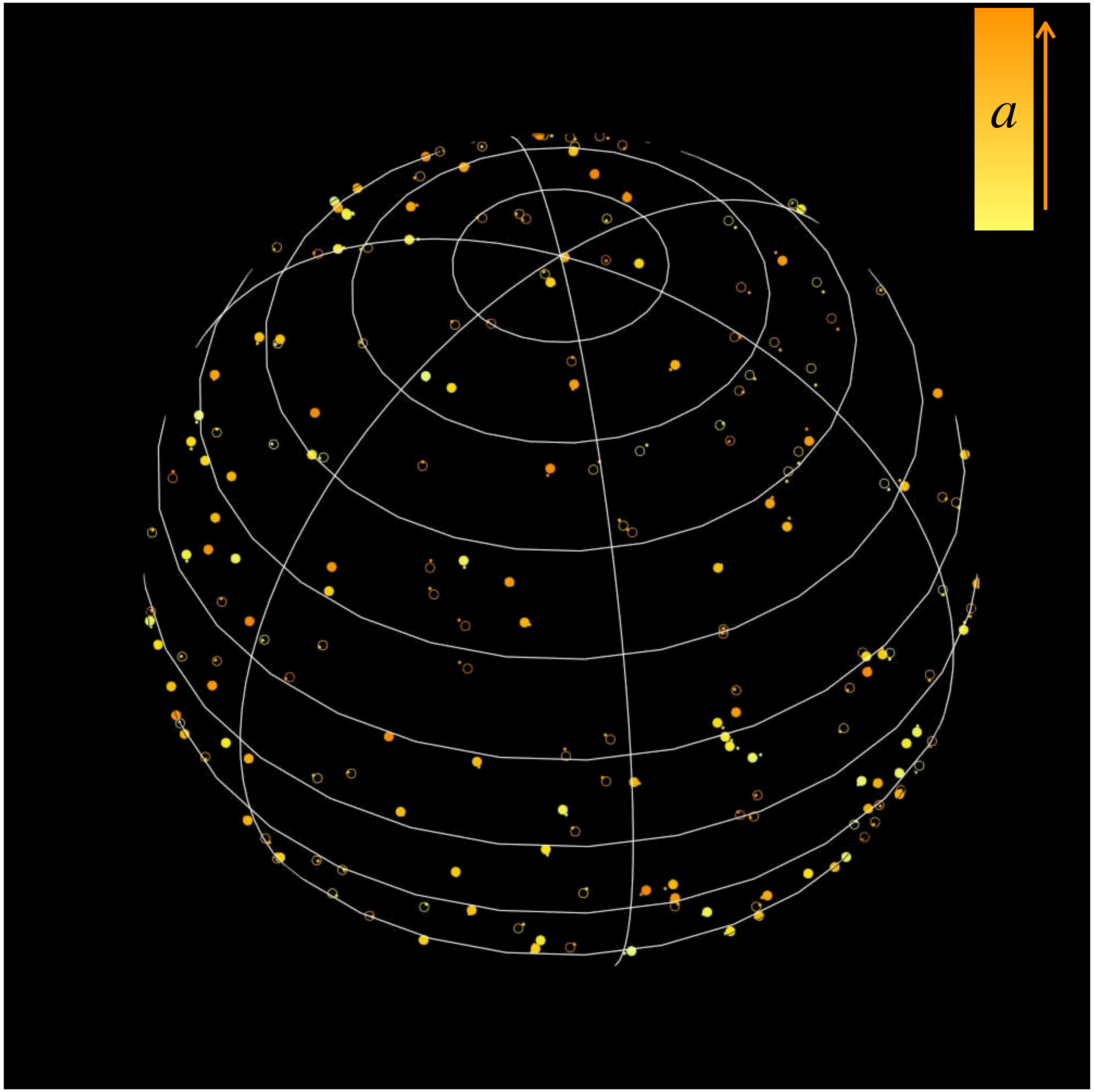
Typical timescale
~1,000,000 years



One orientation becomes a single point on the **unit sphere**

Vector Resonant Relaxation

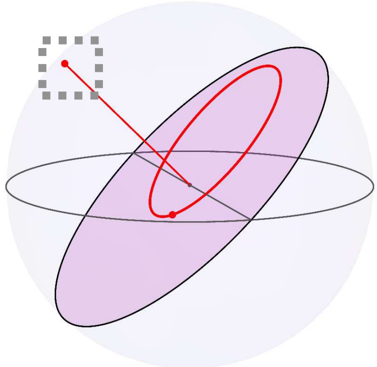


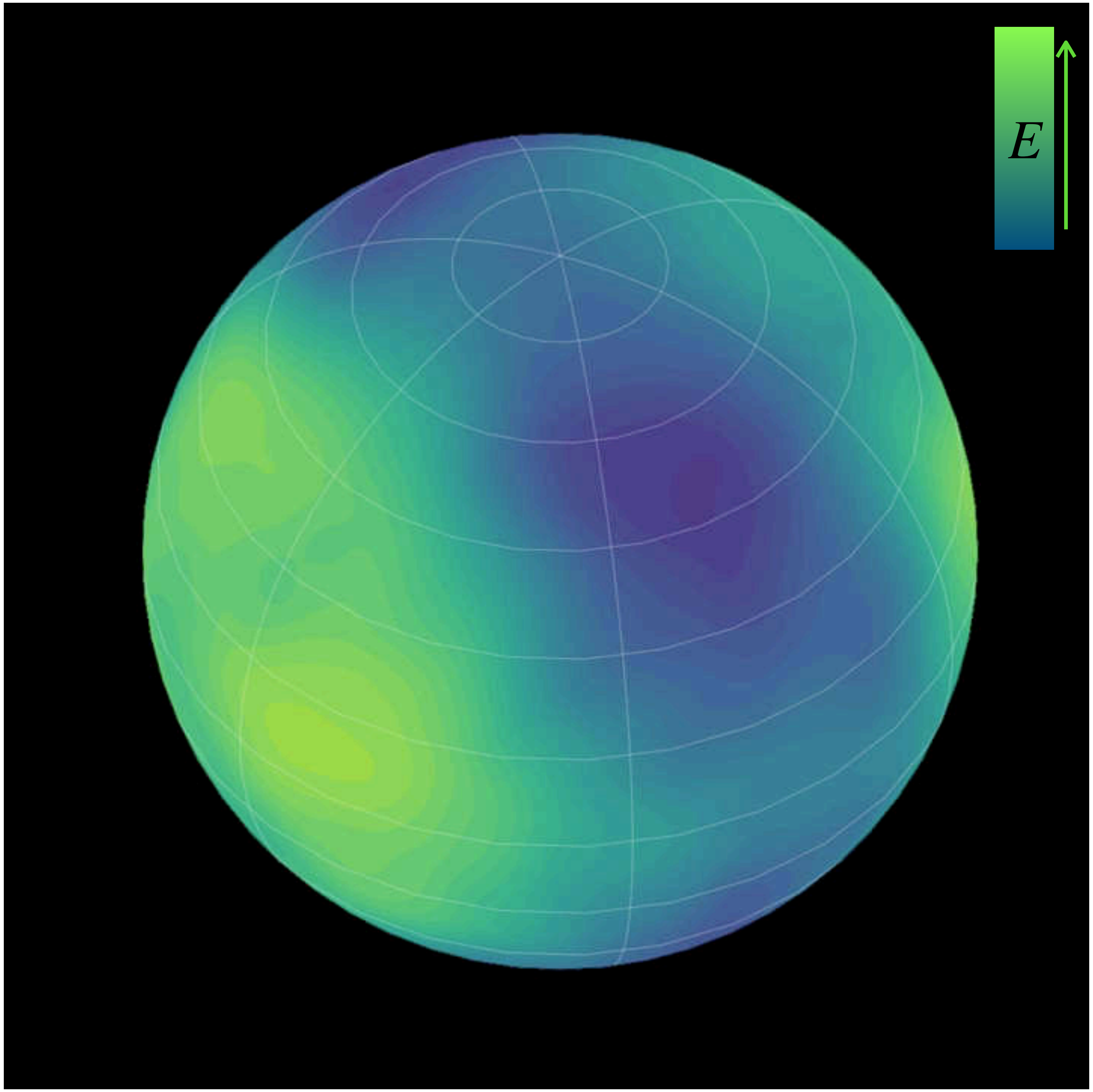


+ Motion coherent on large scales
  - **Long-range interacting system**
+ Motion smooth on short times
  - **Time-correlated noise**
+ Particles have ``preferred friends''
  - **Parametric coupling** (a, e)
+ System in statistical equilibrium
  - **Time stationarity** $(t - t')$
  - **Rotation invariance** $(\hat{\mathbf{L}} \cdot \hat{\mathbf{L}}')$

Simulations can be performed just like for wires

Vector Resonant Relaxation





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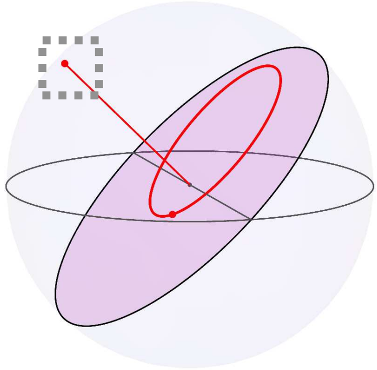
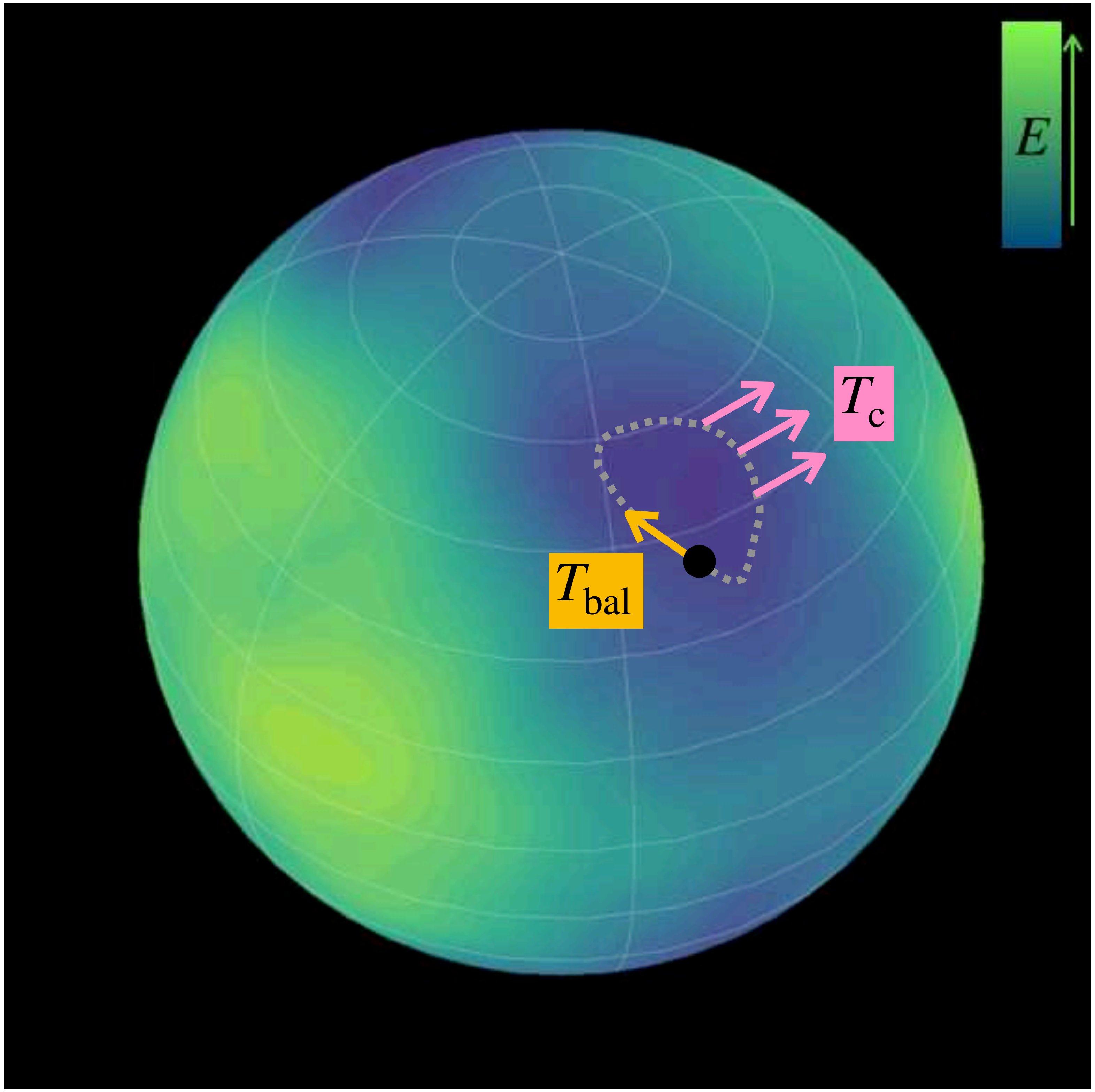
  - **Rotation invariance** $(\hat{\mathbf{L}} \cdot \hat{\mathbf{L}}')$

Vector Resonant Relaxation

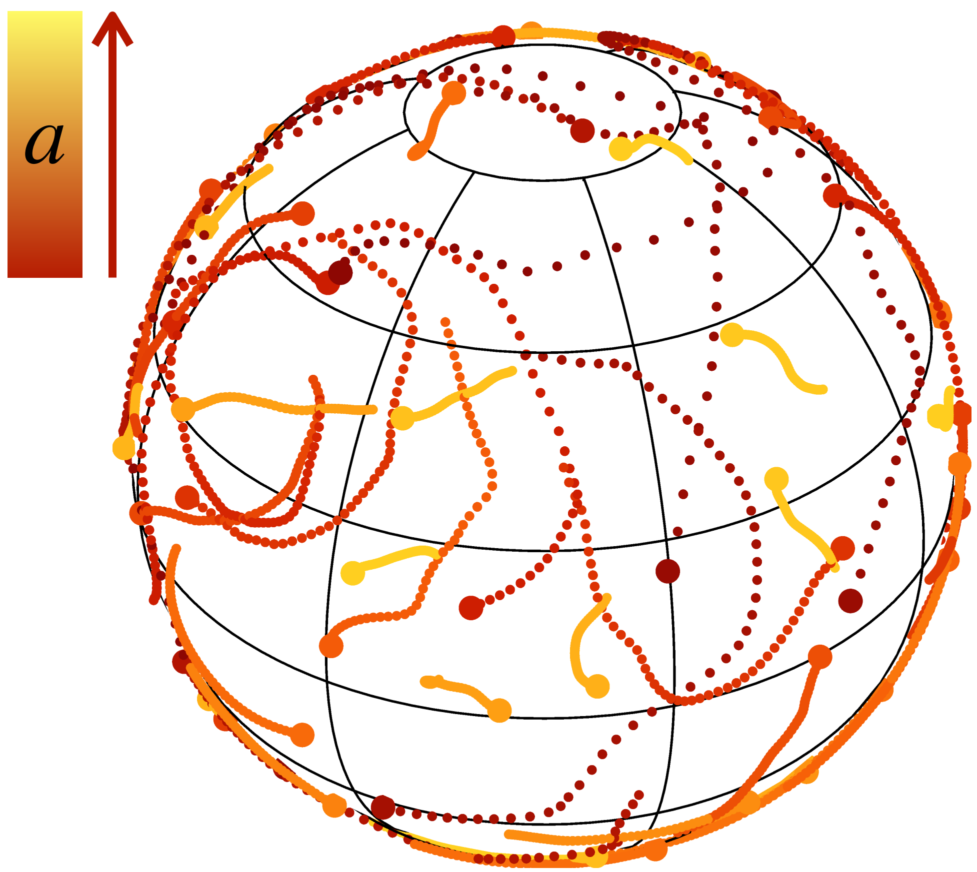




+ Motion coherent on large scales
  - **Long-range interacting system**
+ Motion smooth on short times
  - **Time-correlated noise**
+ Particles have "preferred friends"
  - **Parametric coupling** (a, e)
+ System in statistical equilibrium
  - **Time stationarity** $(t - t')$
  - **Rotation invariance** $(\hat{\mathbf{L}} \cdot \hat{\mathbf{L}}')$

Self-consistency requirement

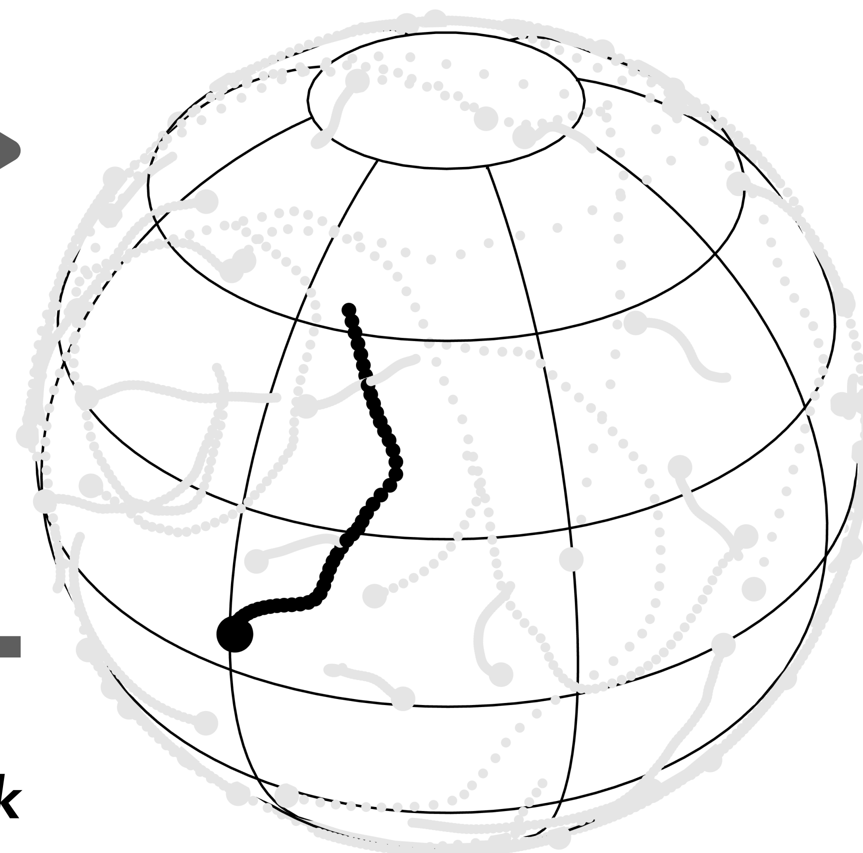
Background particles



Imposes a noisy (correlated) ***potential***

Undergoes a (correlated) ***random walk***

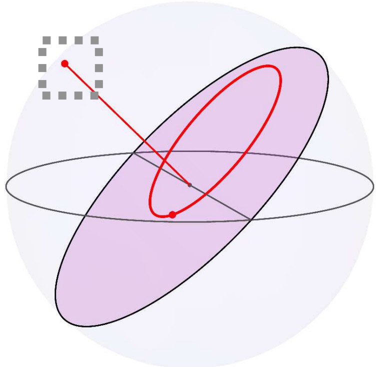
Test particle

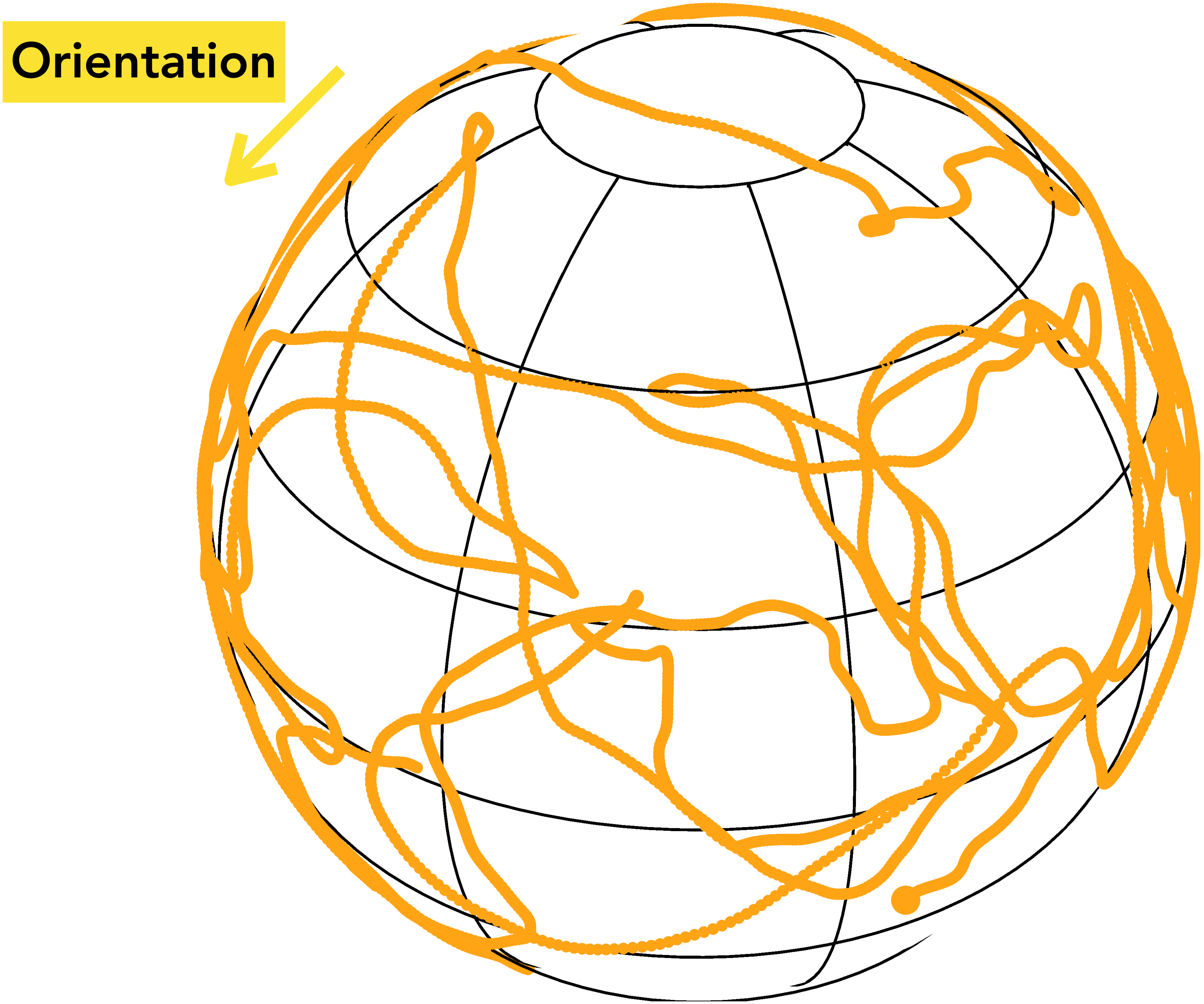


$$\hat{C}_{\mathrm{bath}} = \left\langle \eta(\hat{\mathbf{L}}, t)\, \eta(\hat{\mathbf{L}}', t') \right\rangle$$

$$\hat{C}_{\mathrm{test}} = \left\langle \hat{\mathbf{L}}_{\mathrm{test}}(t) \cdot \hat{\mathbf{L}}_{\mathrm{test}}(0) \right\rangle$$

Typical evolution of an orientation

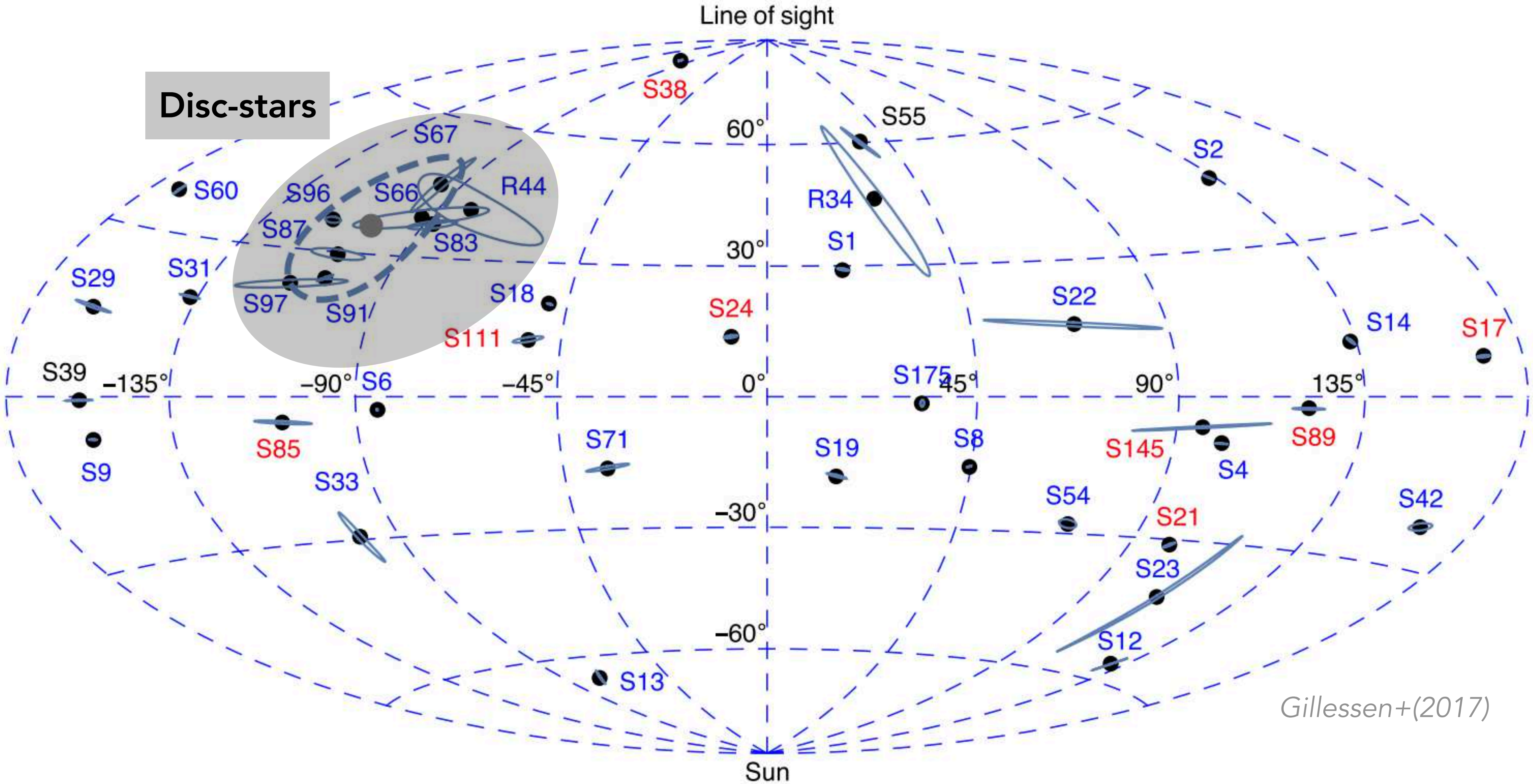




Typical timescale
~1,000,000 years

Stellar orientations follow a **correlated random walk**

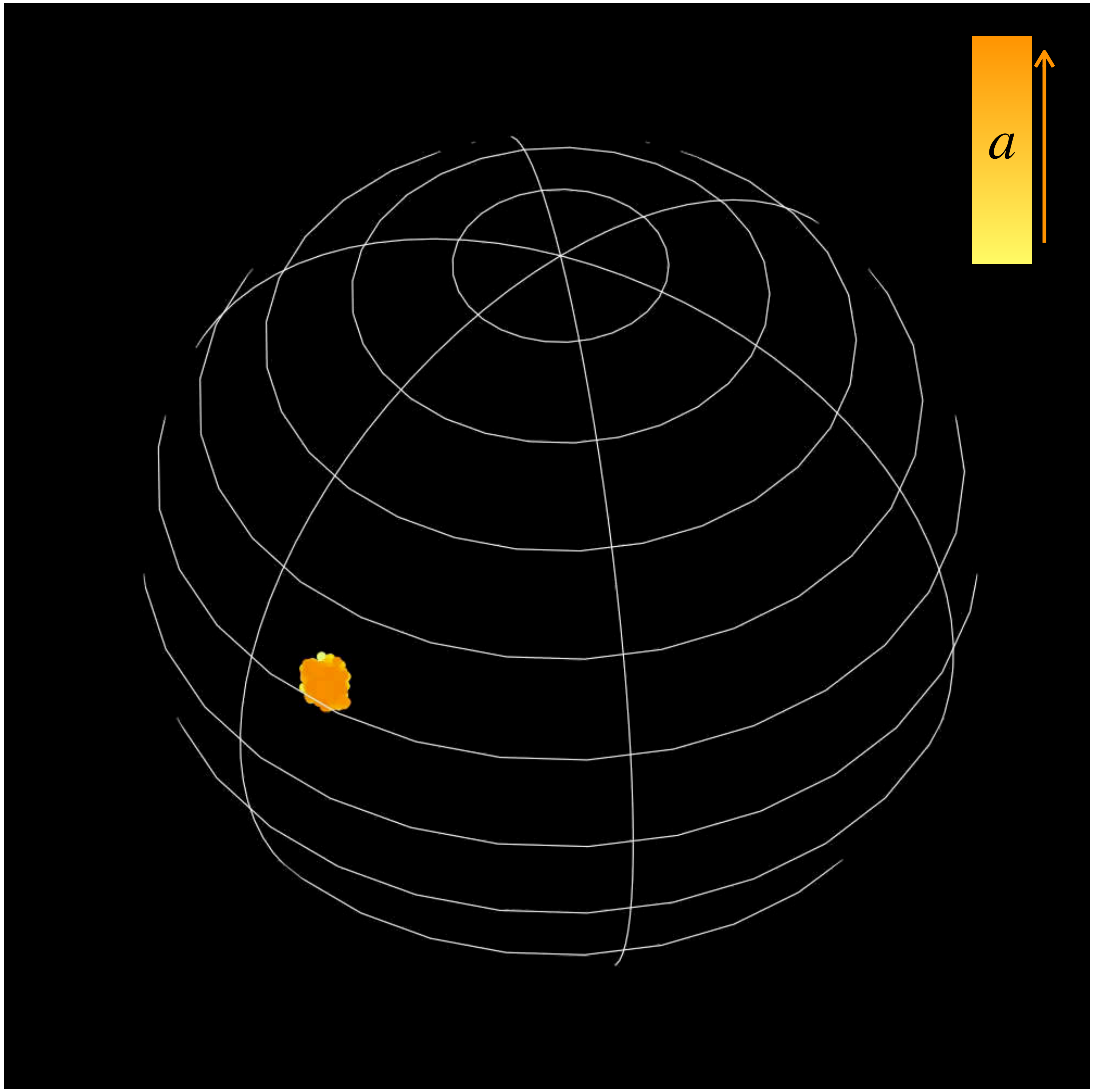
Vector Resonant Relaxation can affect the disc-stars



How long should these stars stay ``**neighbors**''?

Are they **young enough**?

Vector Resonant Relaxation can randomize disc stars



+ How ``**neighbors**'' get separated

$$\frac{\mathrm{d}\hat{\mathbf{L}}_i}{\mathrm{d}t} = \eta(\hat{\mathbf{L}}_i, t)$$

+ Evolution sourced by a **shared**, **spatially-extended** and **time-correlated** noise

$$\left\langle \eta(a_i, \hat{\mathbf{L}}_i, t)\, \eta(a_j, \hat{\mathbf{L}}_j, t') \right\rangle = C(a_i, a_j, \hat{\mathbf{L}}_i \cdot \hat{\mathbf{L}}_j, t-t')$$

+ Two joint sources of **separation**

- **Parametric** separation

$$a_i \neq a_j$$

- **Angular** separation

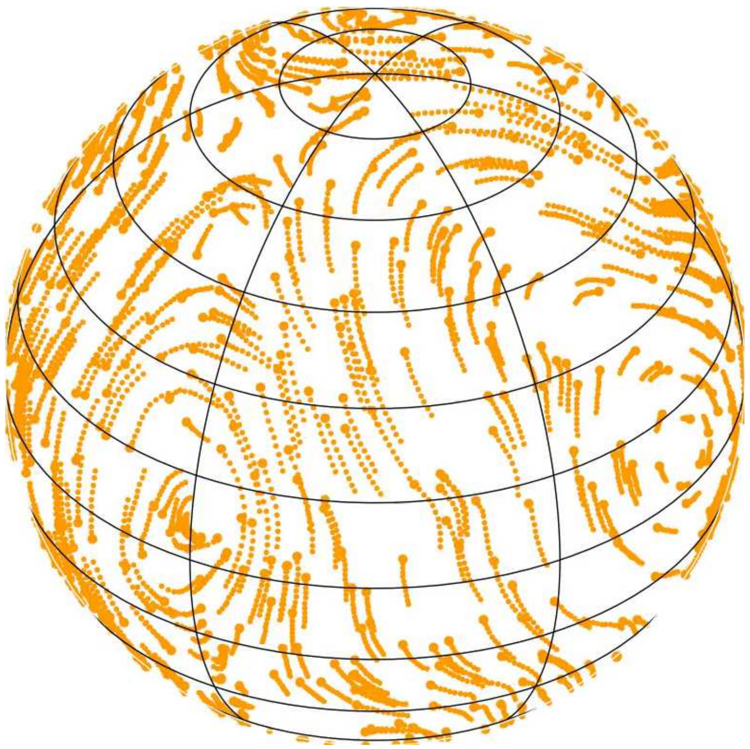
$$\hat{\mathbf{L}}_i \neq \hat{\mathbf{L}}_j$$

VRR around SgrA*

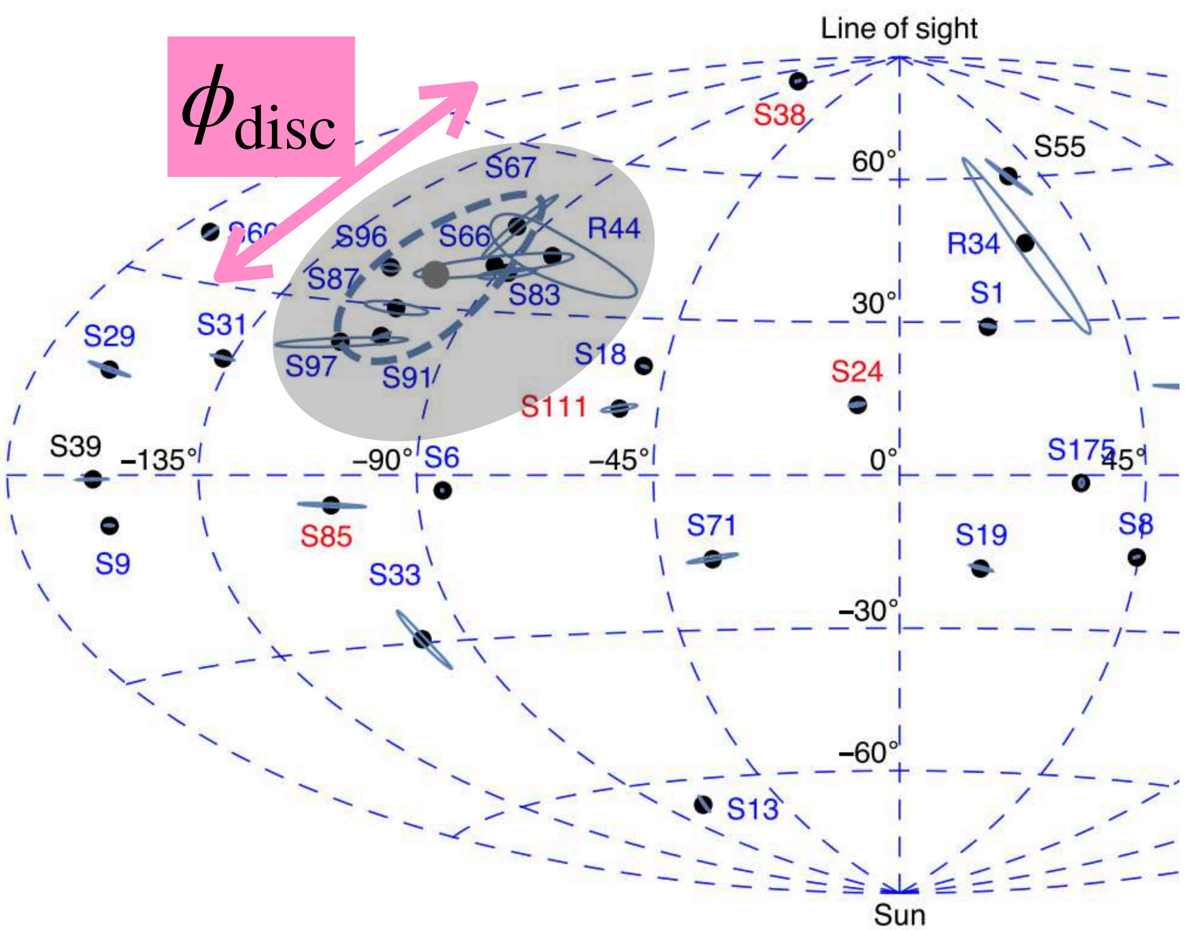
Model

- Old stars
  (unresolved but relaxed)
- **IMBHs**
  (strong source of Poisson noise)
- S-stars disc ICs
  (initial angular dispersion)

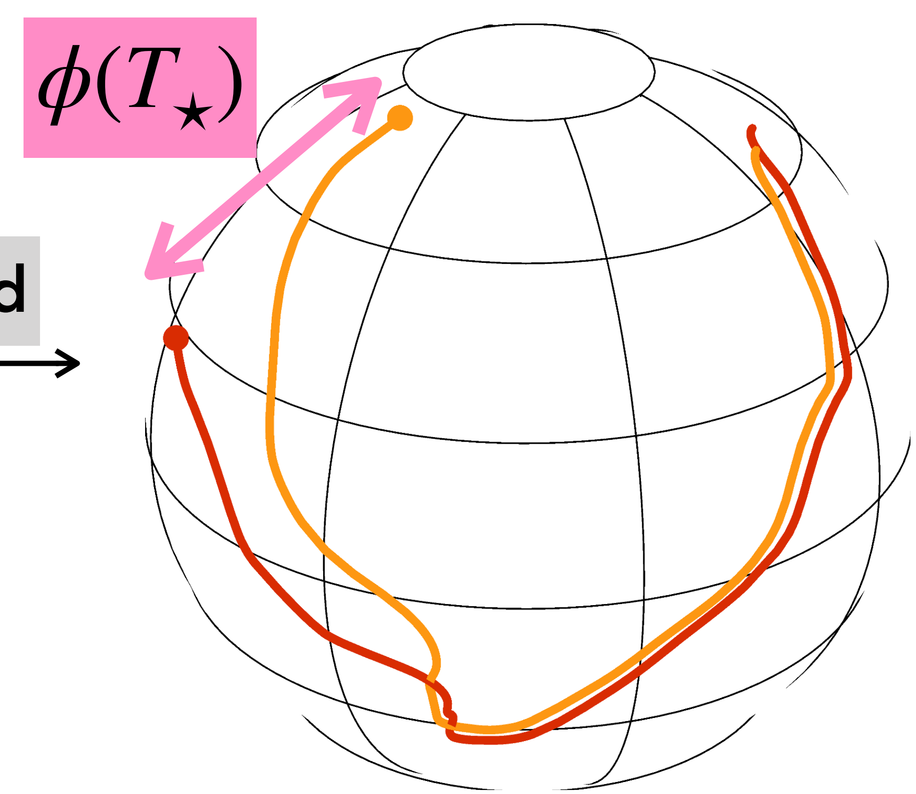
Kinetic theory →



↕ Dilution

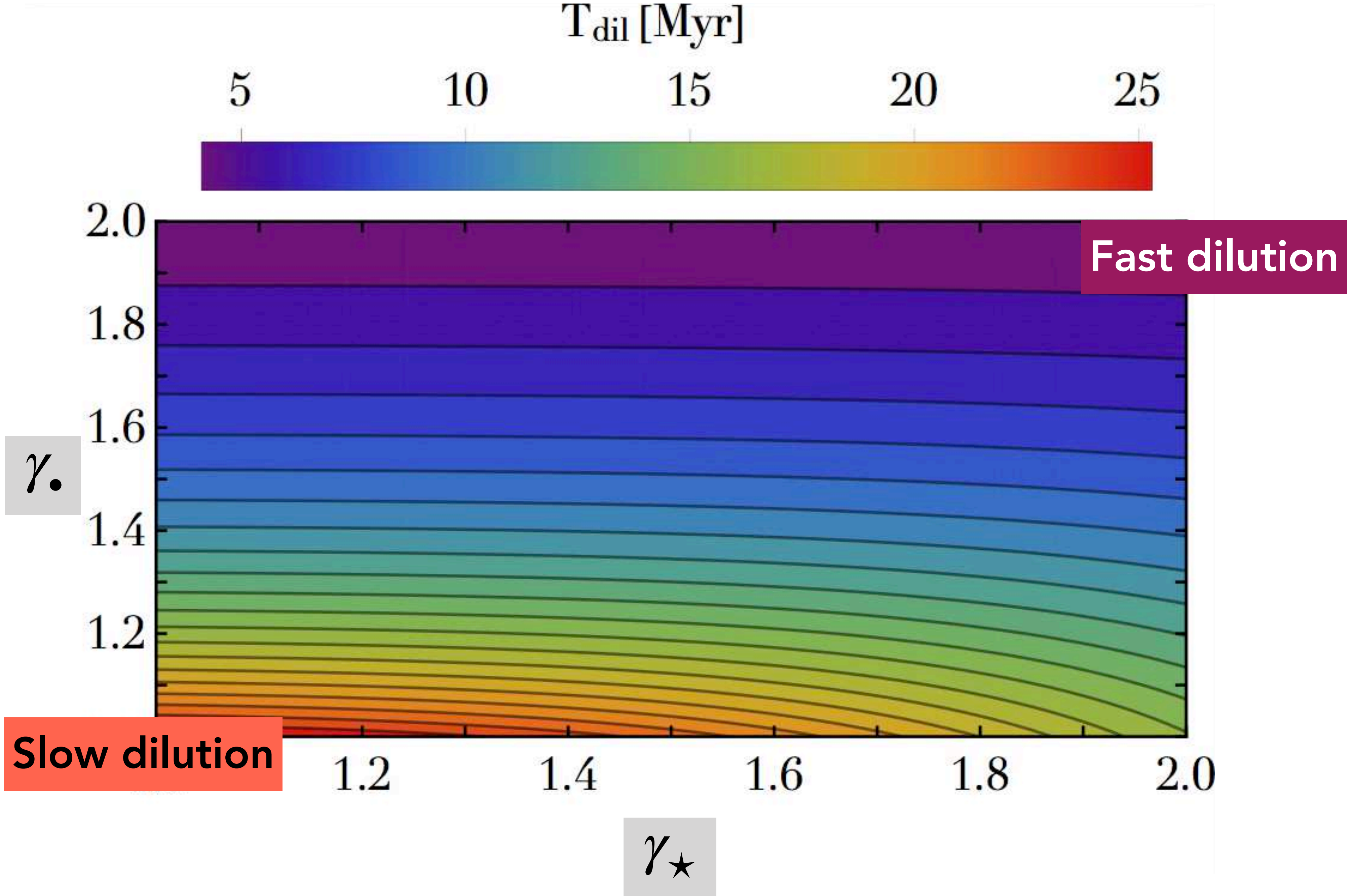


Likelihood ↔



An example of likelihood

2-population model (stars+IMBHs)



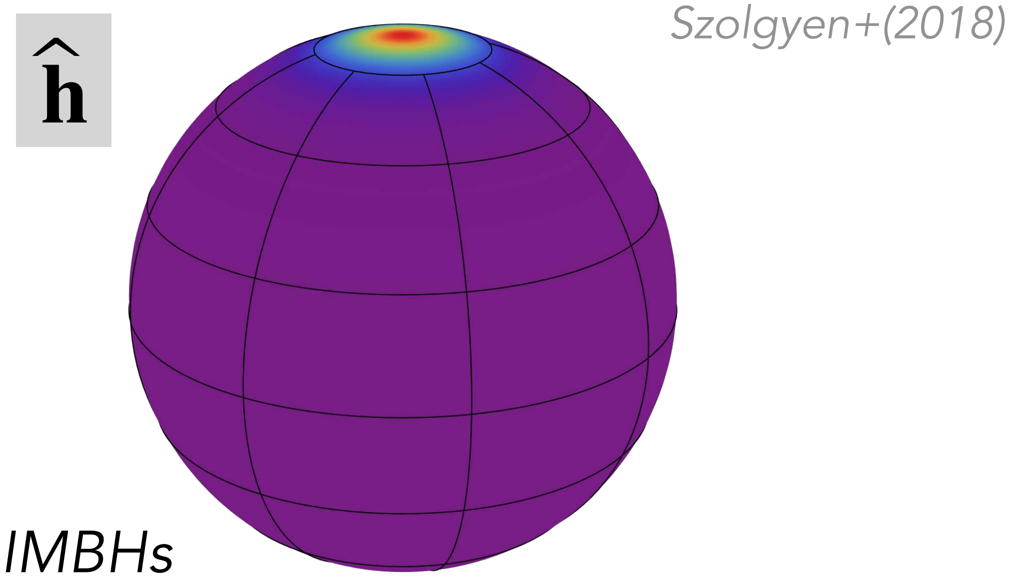
Questions to address:

+ Are **IMBHs** mandatory?

+ Where do the **S-stars** come from?

How to do better

Anisotropic orientations



Disc of IMBHs

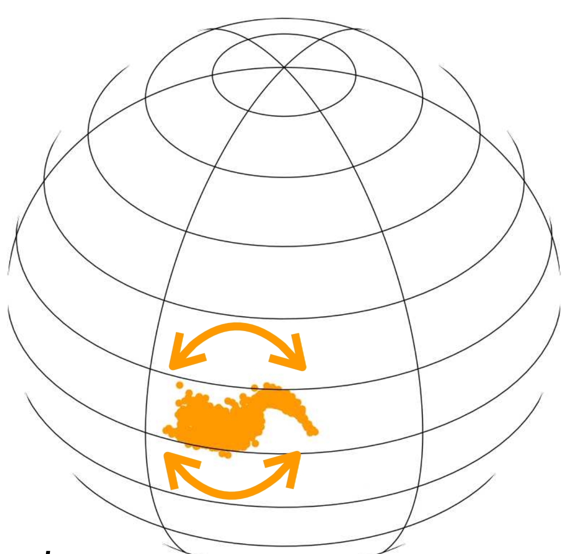
Additional relaxation

$$\frac{\mathrm{d}e}{\mathrm{d}t} \neq 0; \quad \frac{\mathrm{d}a}{\mathrm{d}t} \neq 0$$

Impact of SRR and NR

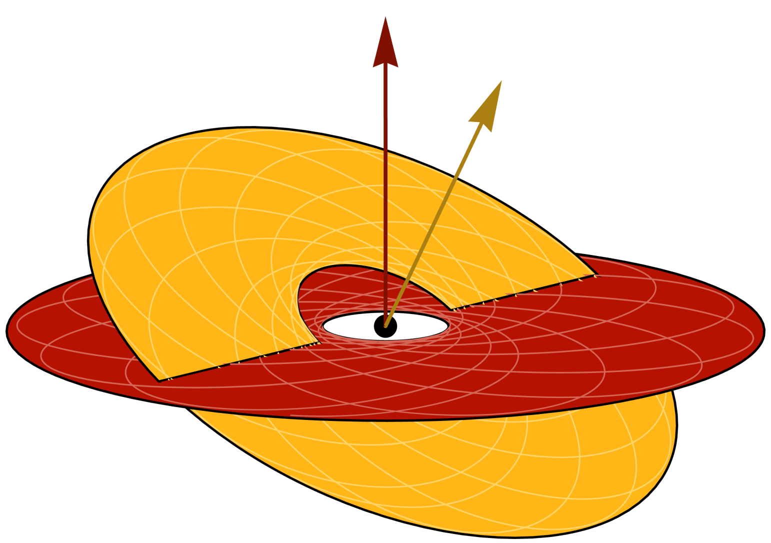
Self-gravity

Kocsis+(2011)



Pairwise couplings

Globular clusters



Orbital interactions

Thermodynamics of VRR

N-body dynamics

$$\frac{\partial F_{\mathrm{b}}}{\partial t} + \left[F_{\mathrm{b}}, H(F_{\mathrm{b}})\right] = 0$$

Quadratic, orbit-averaged, hierarchical, multi-population

Kinetic Theory

$$\frac{\partial \langle F_{\mathrm{b}} \rangle}{\partial t} = C\left[\langle F_{\mathrm{b}} \rangle, \langle F_{\mathrm{b}} \rangle\right]$$

Integrable equilibrium, small perturbations, quasi-linear expansion, collective effects, resonant couplings

Thermodynamics

$$F_{\mathrm{eq}}(\hat{\mathbf{L}}) = \lim_{t \to +\infty} \langle F_{\mathrm{b}}(\hat{\mathbf{L}}, t) \rangle$$

Ergodic principle

Global N-body invariants

$$\mathbf{K} = (a, e)$$

Annuli shape

$$\begin{cases} N(\mathbf{K}) & \textit{Sub-populations} \\ E_{\mathrm{tot}} & \textit{Total energy} \\ \hat{\mathbf{L}}(\mathbf{K}) & \textit{Total angular momentum} \end{cases}$$

Thermodynamics of VRR

Entropy maximisation

$$S \propto \int \mathrm{d}\hat{\mathbf{L}}\mathrm{d}\mathbf{K}\, F \ln[F]$$

under the conservation of the **invariants**

Generalised **Boltzmann DF**

$$F_{\mathrm{eq}}(\hat{\mathbf{L}}, \mathbf{K}) \propto \exp\big[-\beta\, \varepsilon(\hat{\mathbf{L}}, \mathbf{K}) + L(\mathbf{K})\, \boldsymbol{\gamma}\cdot\hat{\mathbf{L}}\big]$$

Temperature *Spin*

Self-consistency

$$\big[\beta, \boldsymbol{\gamma}, \langle Y_{\ell m}\rangle\big] \longrightarrow \big[E_{\mathrm{tot}}, \mathbf{L}_{\mathrm{tot}}\big] \overset{?}{=} \big[E_{\mathrm{tot}}(t\!=\!0), \mathbf{L}_{\mathrm{tot}}(t\!=\!0)\big]$$

Root finding performed by Newton iteration

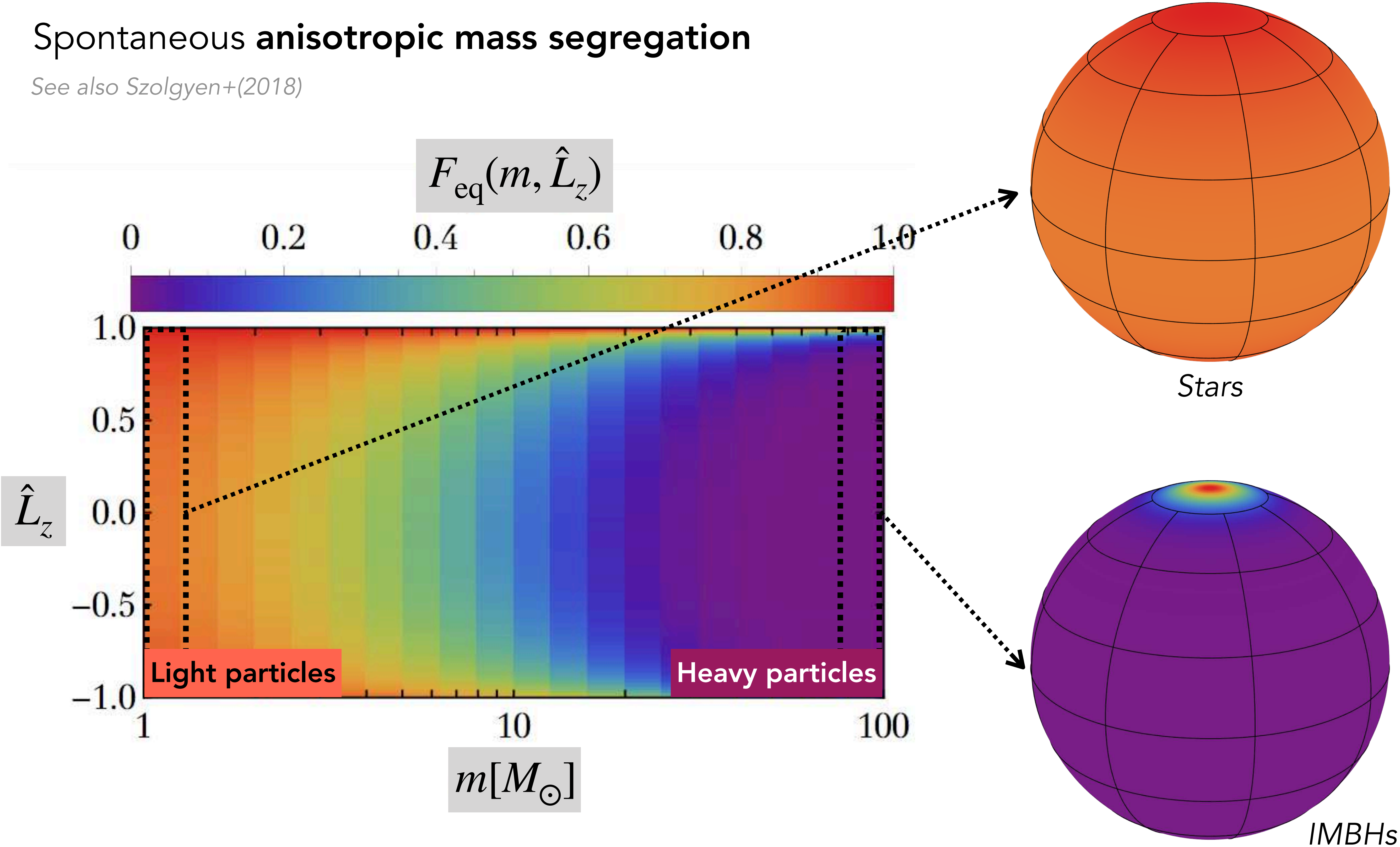
Additional **accelerations** via

Explicit gradients	Axisymmetric limit	Population discretisation	Initialisation from $\beta = 0$	Multipole integrations

An example of equilibrium

Spontaneous **anisotropic mass segregation**

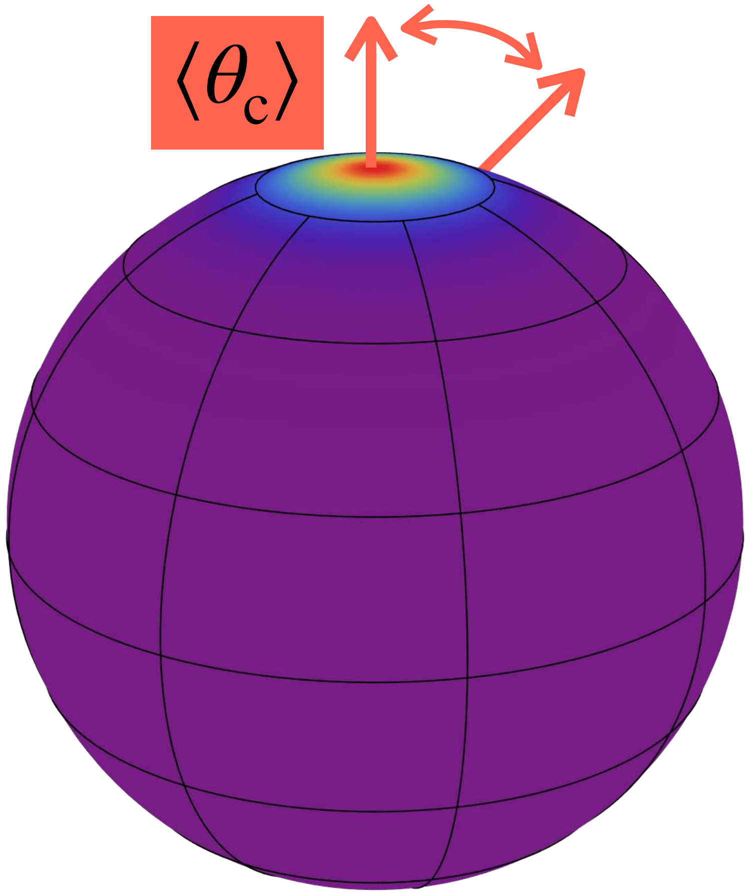
See also Szolgyen+(2018)



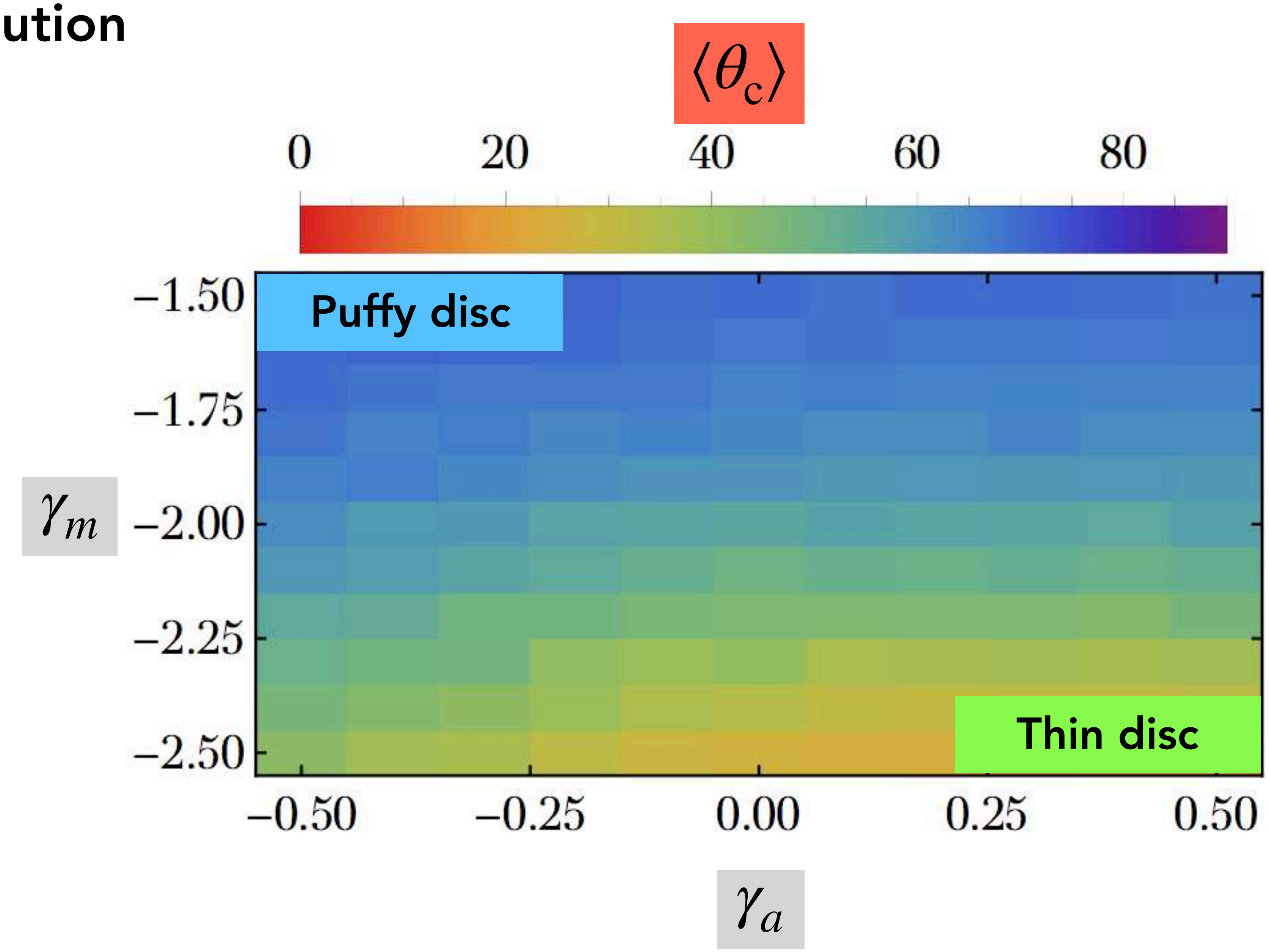
The more individually massive the population, the stronger the **alignment**

Kinematic diversity

Impact of the **orbital distribution**



Angular size of the disc



Other features driven by **long-range interactions**

Negative temperatures	Ensemble inequivalence

How to do better

Non-axisymmetry

$$\langle Y_{\ell m}\rangle \quad \text{for} \quad m \neq 0$$

Spontaneous symmetry breaking

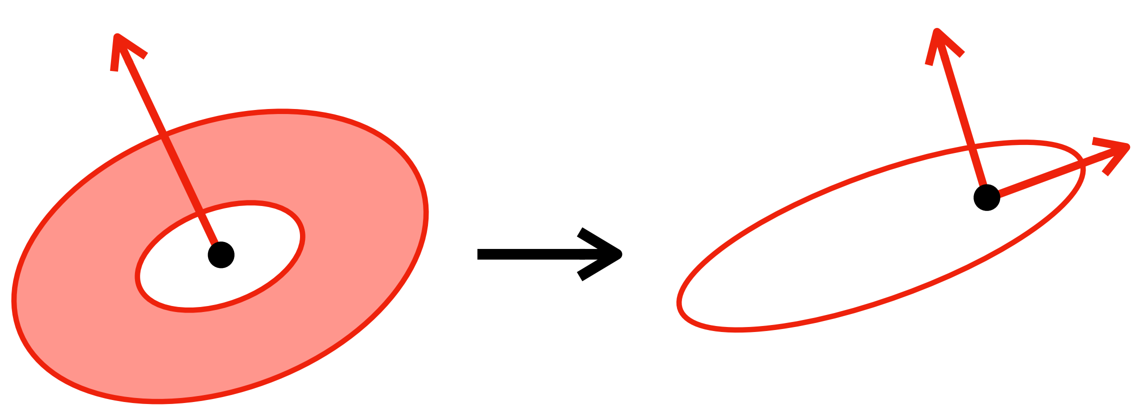
Timescale

$$F_{\mathrm{b}}(\hat{\mathbf{L}}, t) \xrightarrow{T_{\mathrm{relax}}} F_{\mathrm{eq}}(\hat{\mathbf{L}})$$

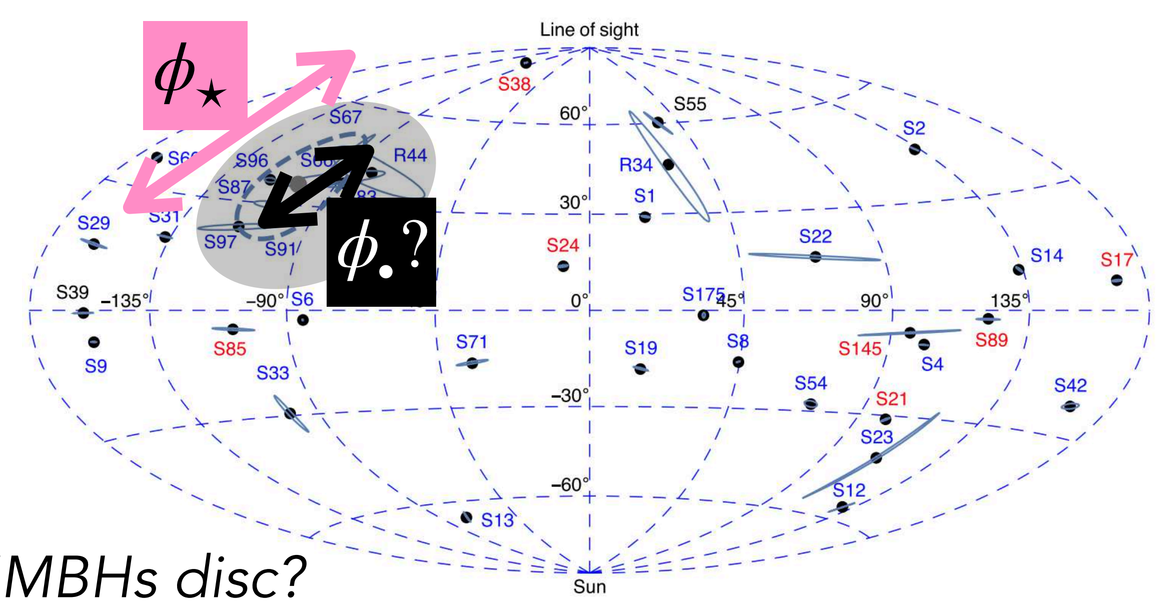
How fast to create anisotropies?

Wires thermodynamics

Gruzinov+(2020)

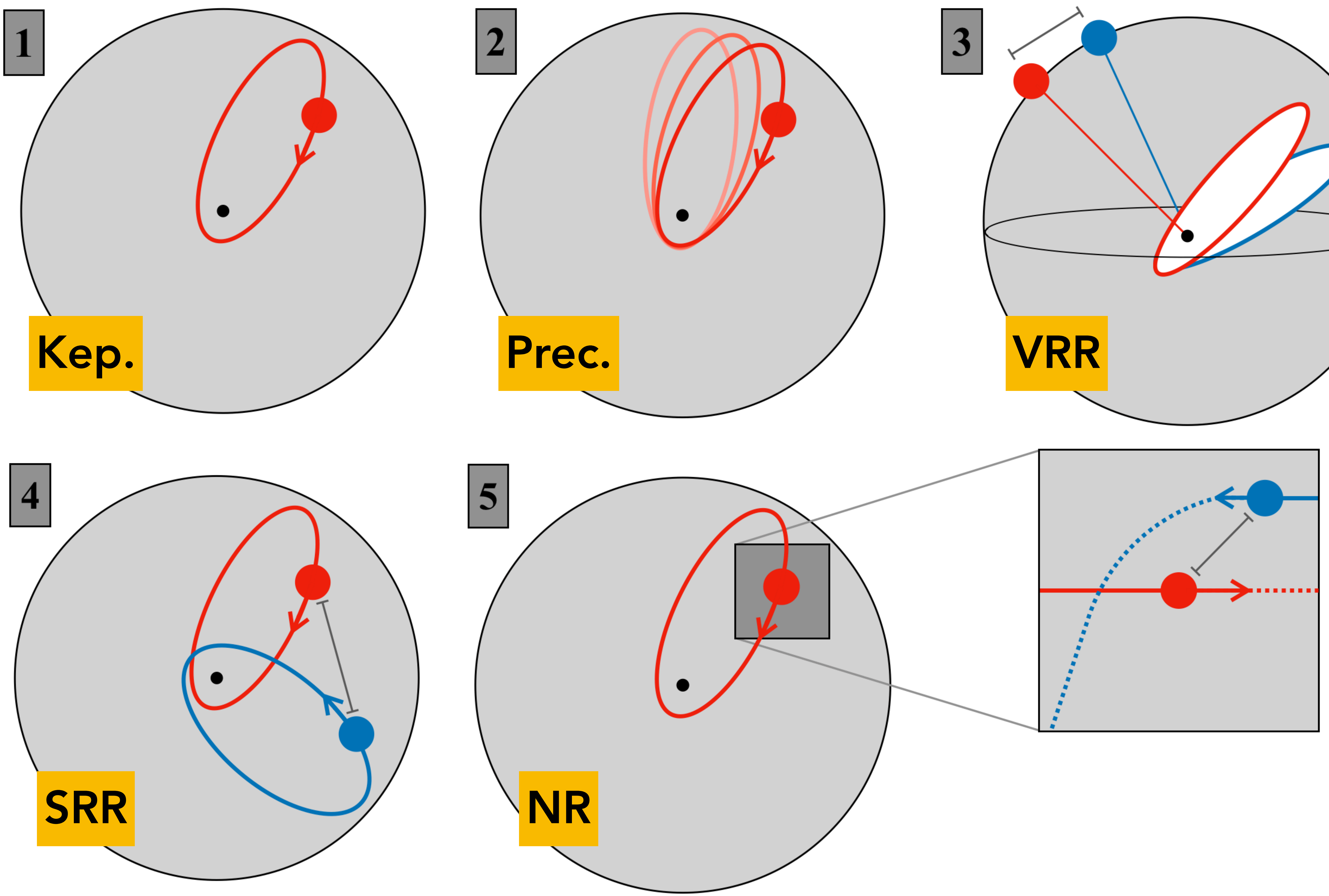


Observations



IMBHs disc?

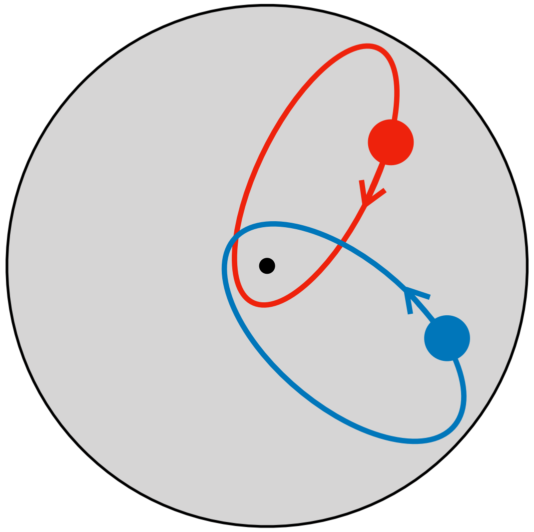
A wealth of dynamical processes



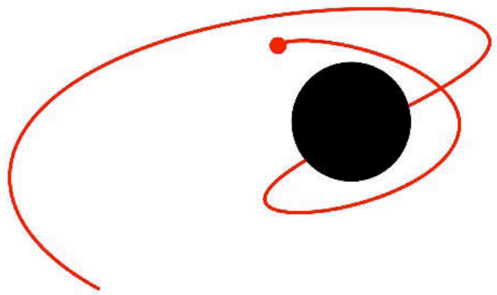
An extremely **hierarchical system**

The future of galactic nuclei

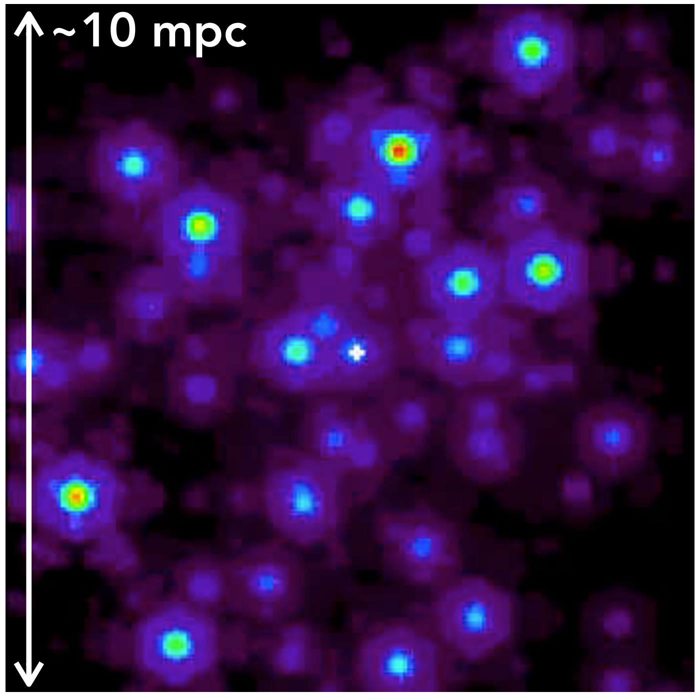
New stellar orbits



Infall of **compact objects**



TMT and ELT



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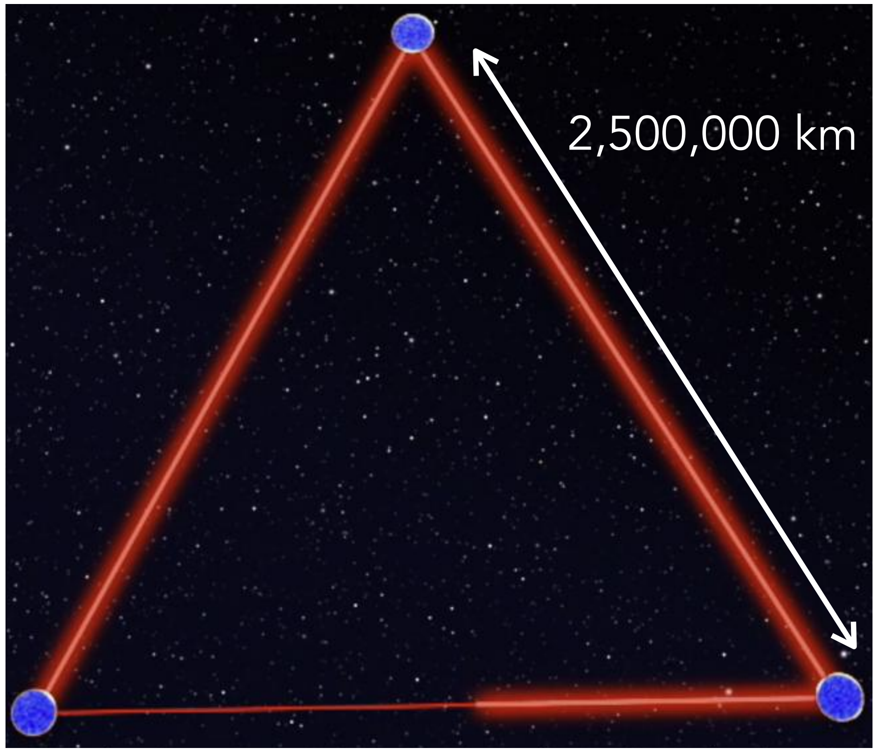
Expected observations

LISA spatial interferometer



Next steps — Theory & Numerics

Linear response

$$\mathbf{M}(\omega) = \sum_{\mathbf{k}} \int \mathrm{d}\mathbf{J} \frac{G(\mathbf{J})}{\mathbf{k}\cdot\mathbf{\Omega}(\mathbf{J}) - \omega}$$

Response matrix & Modes

Non-axisymmetry

$$F_{\mathrm{tot}} = F_{\mathrm{tot}}(a, h, \widehat{\mathbf{h}})$$

Rotation

More efficient methods

$$T_{\mathrm{Kep}} \propto a^{3/2}$$

$$T_{\mathrm{rel}} \propto a^{4/2}\,(1-e^2)$$

Range of timescales

Time integration

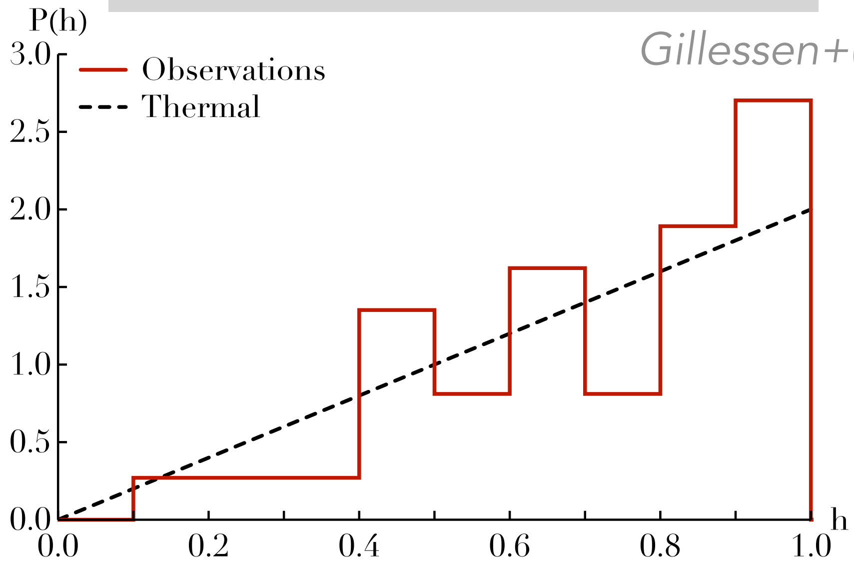
$$\frac{\partial F}{\partial t} = C[F, F]$$

Collision operator

Next steps — SgrA* & Observations

SRR & Eccentricity

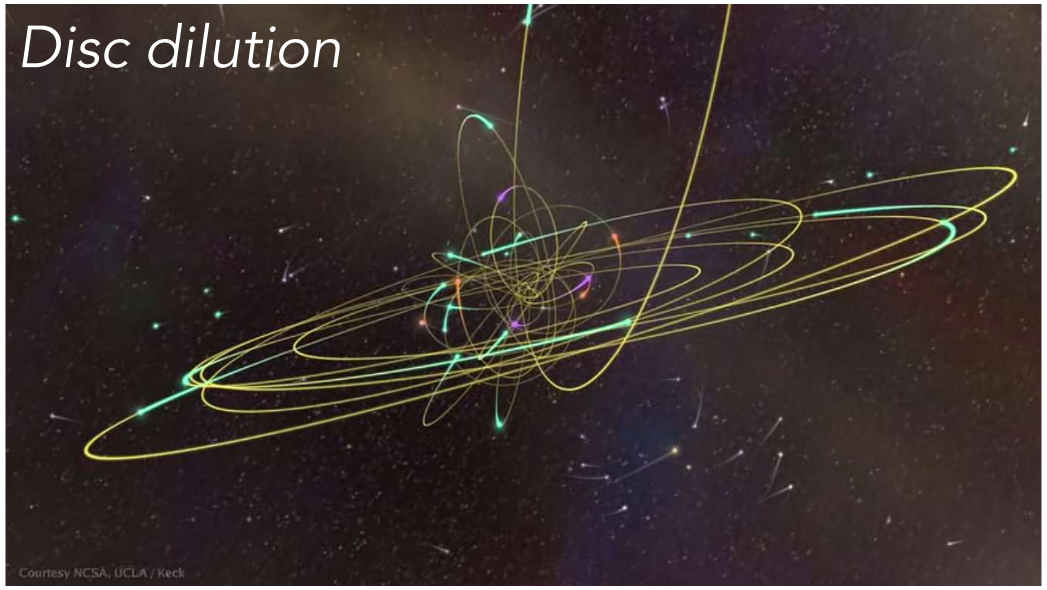
Gillessen+(2017)



Eccentricity thermalisation

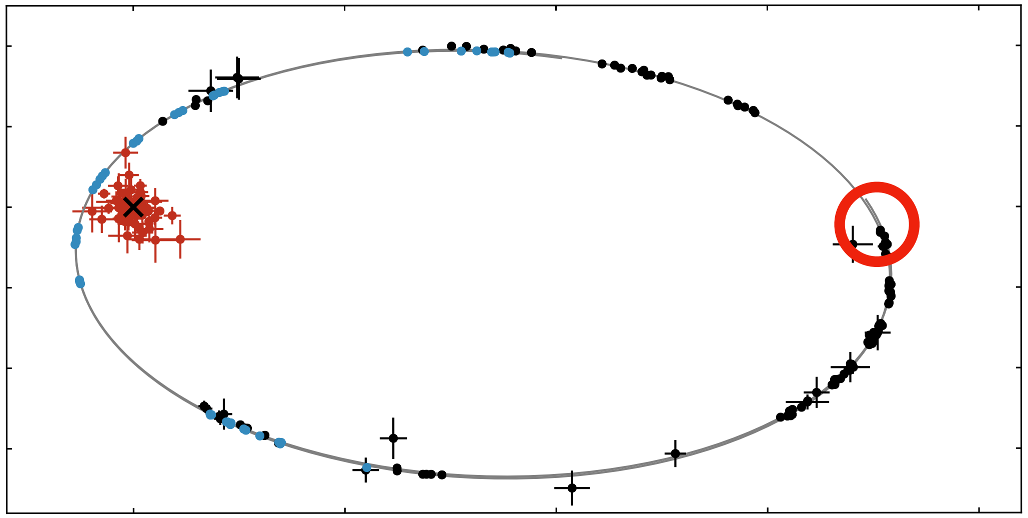
VRR & Stellar Discs

VLT, Keck



S2's kinematics

Gravity+(2020)



Local perturbations?

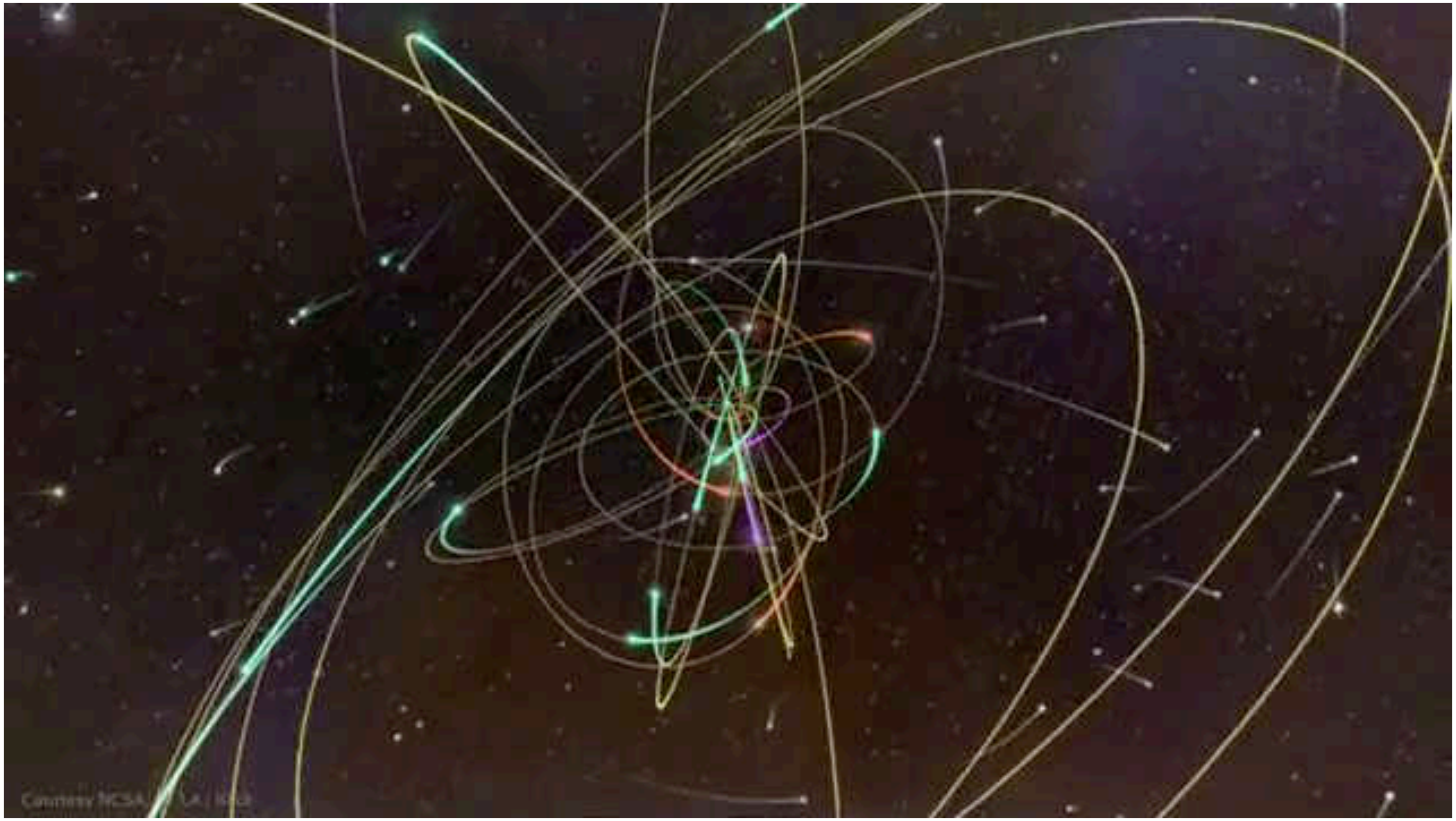
Future observations

$$P = P(a, h, \widehat{\mathbf{h}})$$

Full PDF statistics

Galactic nuclei

Visualisation of **SgrA***



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Galactic nuclei, a fantastic ``astrophysical lab''

Dense (1,000,000x more than around the Sun)

Far away (10,000,000x smaller than the Moon in the sky)

Relativistic (BH 4,000,000x heavier than the Sun)

Noisy (Great source of gravitational waves)