Linear Stability of Rotating Stellar Clusters

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PhD defense - Institut d'astrophysique de Paris - 17/09/2020

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Stellar Clusters

Orbital structure of stellar clusters

Linear Response Theory

Identifying unstable modes

Anisotropic, rotating stellar clusters

Destabilisation processes

Conclusions and prospects

Stellar Clusters

Fluid system

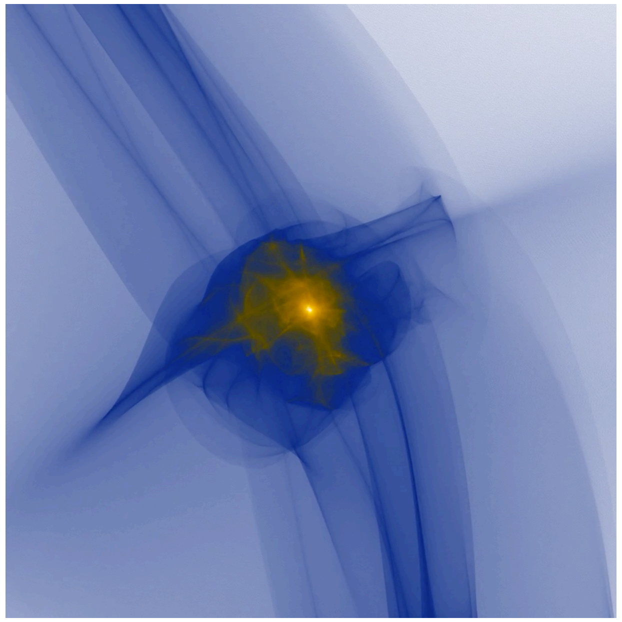
Blender IN 4K

Collisions, shocks, dissipation $\rho(\mathbf{x})$

No shell crossing $\mathbf{v}(\mathbf{x})$

$\neq$

Stellar system



WDM (R. Angulo)

No physical collisions $F(\mathbf{x}, \mathbf{v})$

Shell crossing

Disk systems

Saturn's rings (Cassini)

100 000 km

NGC 4302 - NGC 4298 (HST)

10 kpc

$\neq$

Stellar clusters

M 10 (HST)

10 pc

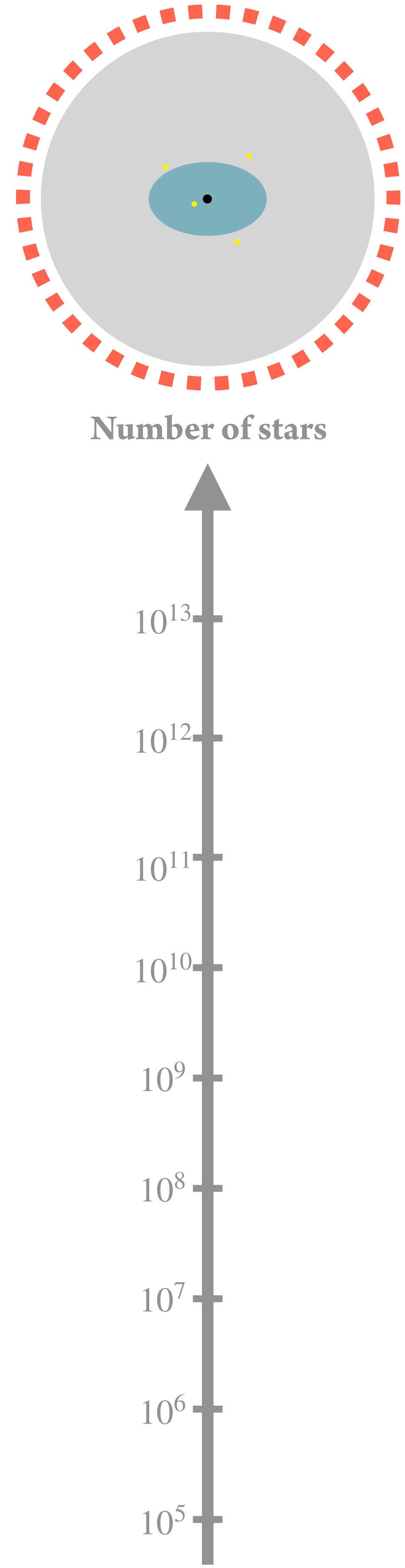
NGC 4621 (SDSS)

100 kpc

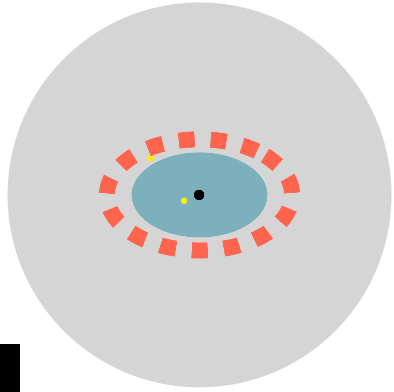
Dark matter halos

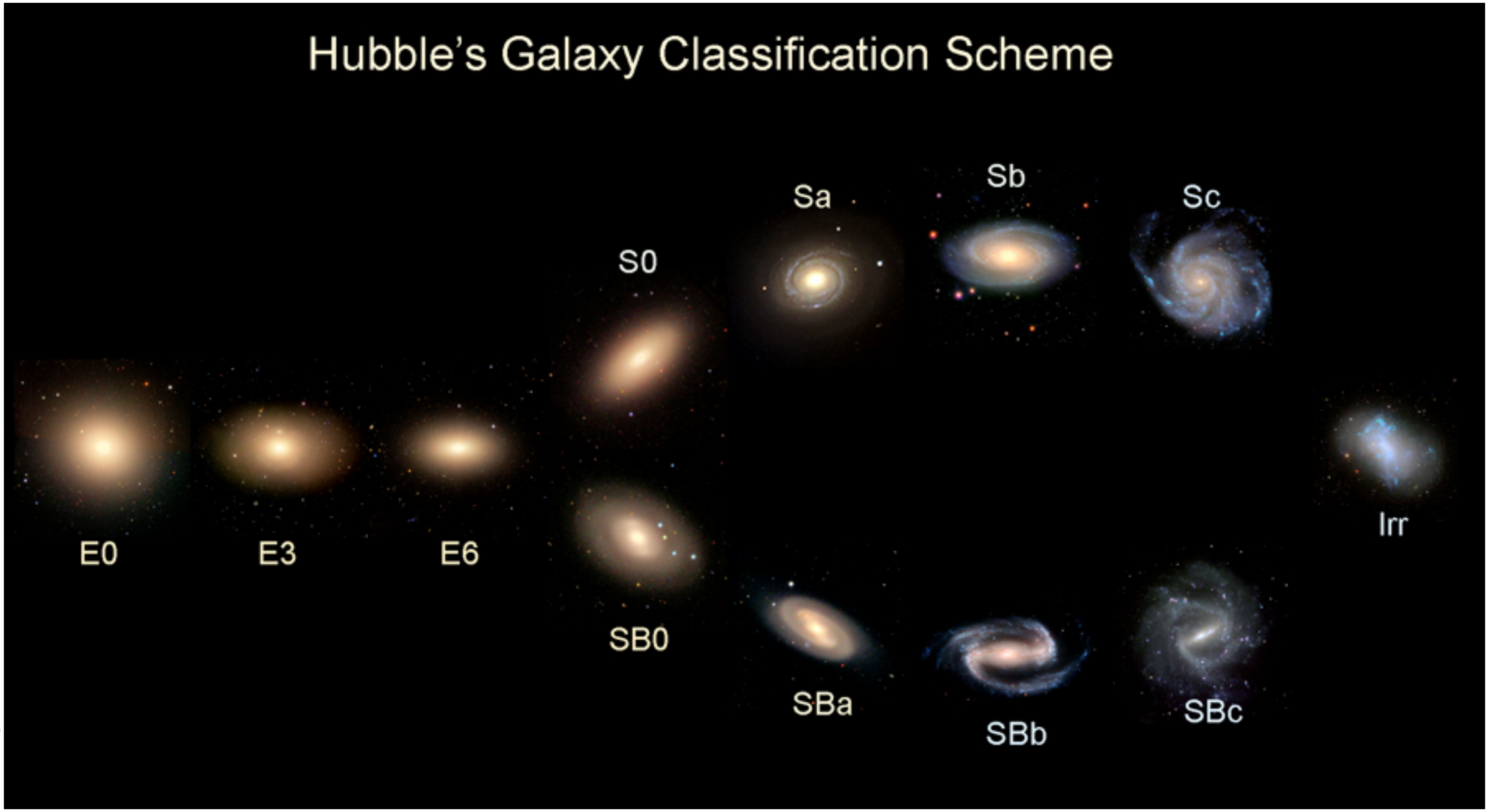
C. Pichon

10 Mpc

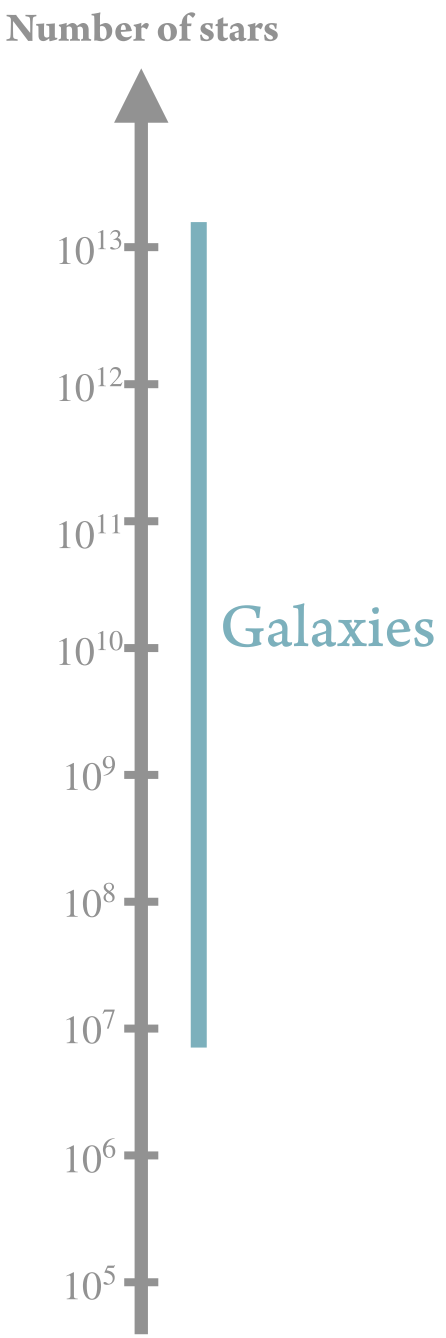


Galaxies

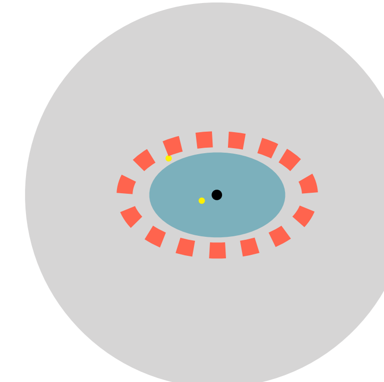


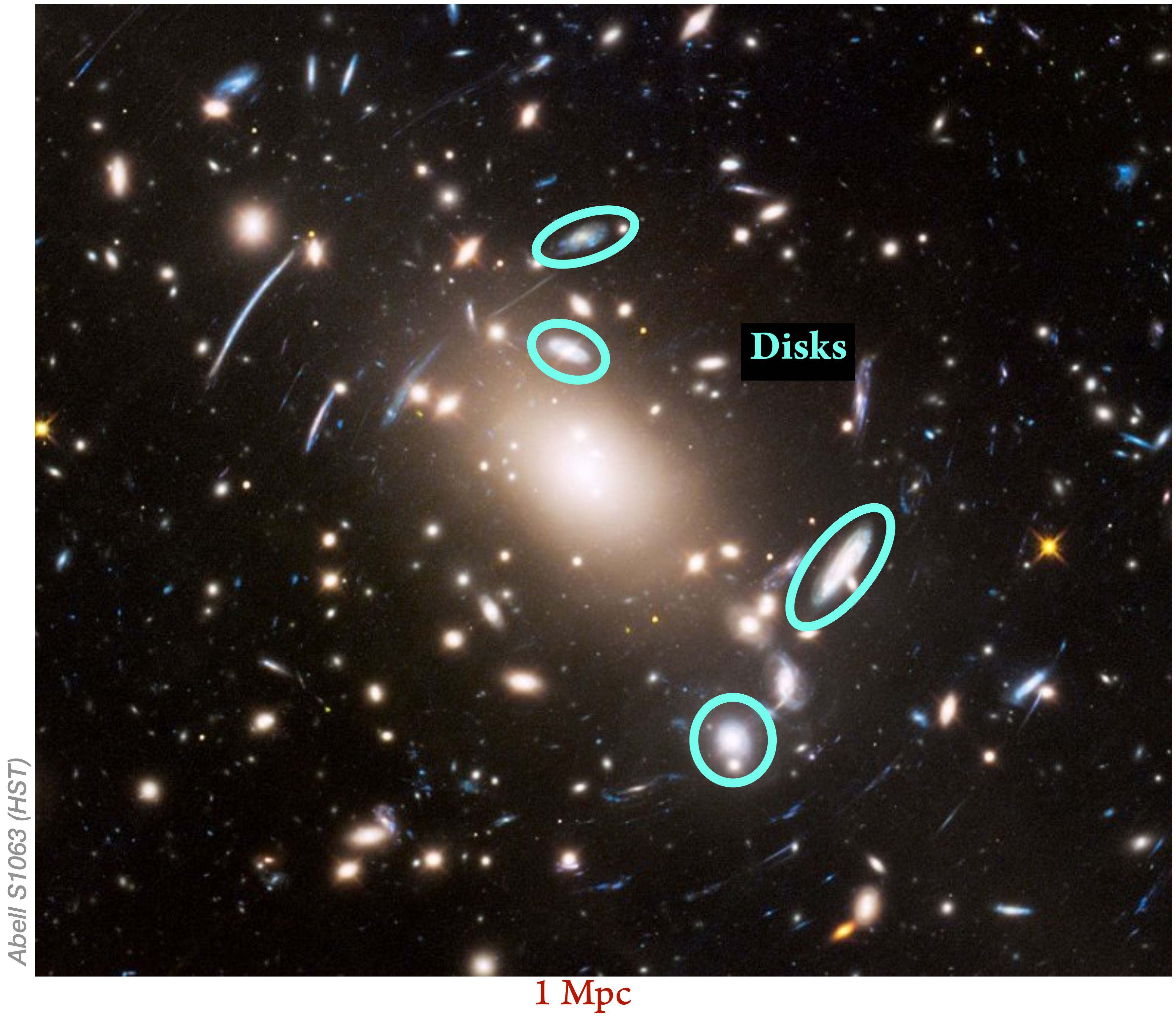


Galaxy Zoo

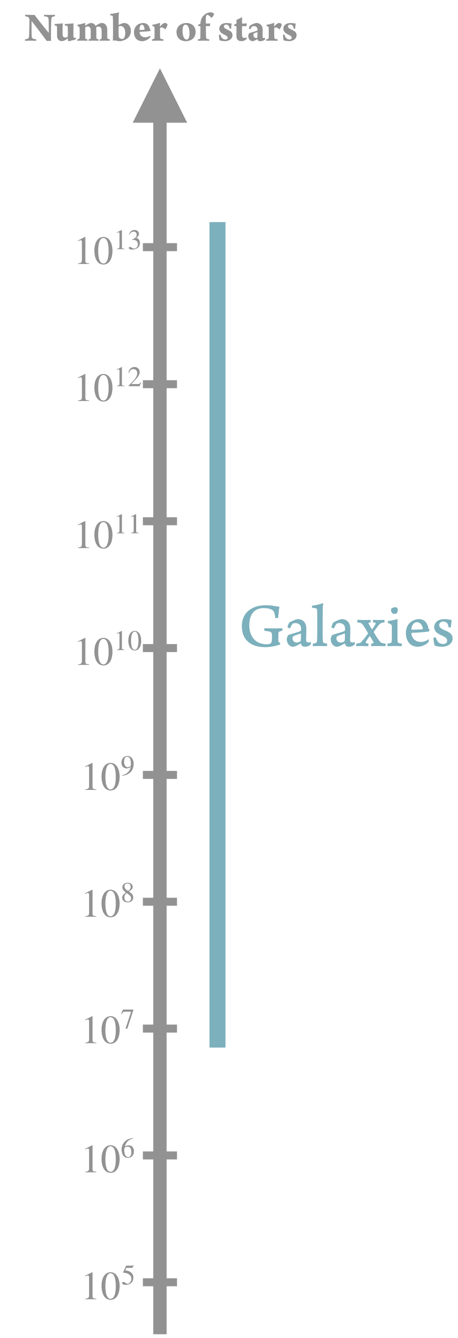


Galaxies



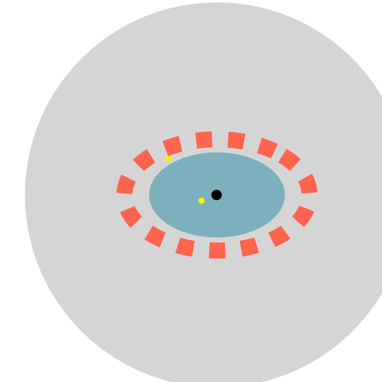


Disks
Abell S1063 (HST)
1 Mpc

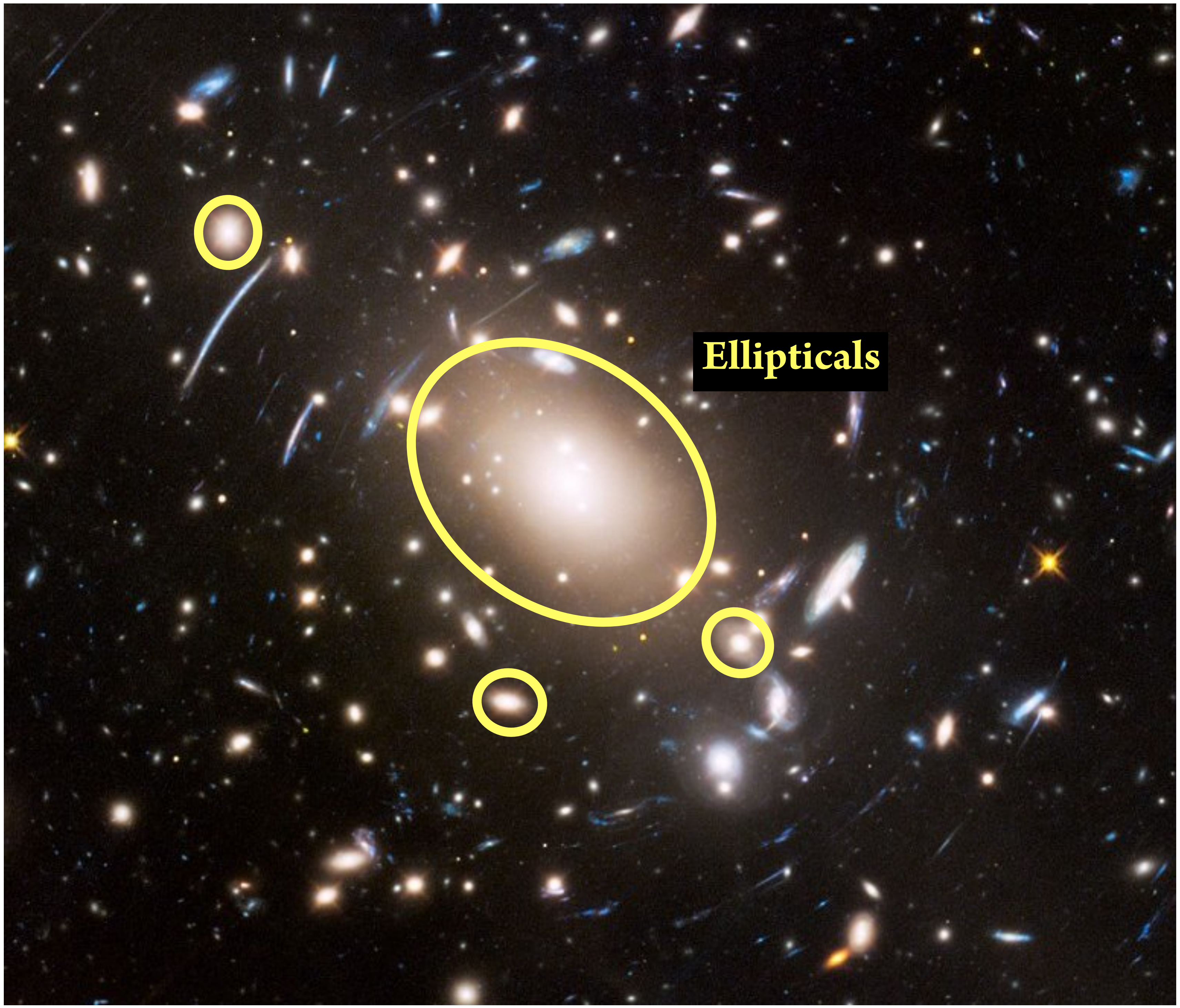


Number of stars
10^13
10^12
10^11
10^10
10^9
10^8
10^7
10^6
10^5
Galaxies

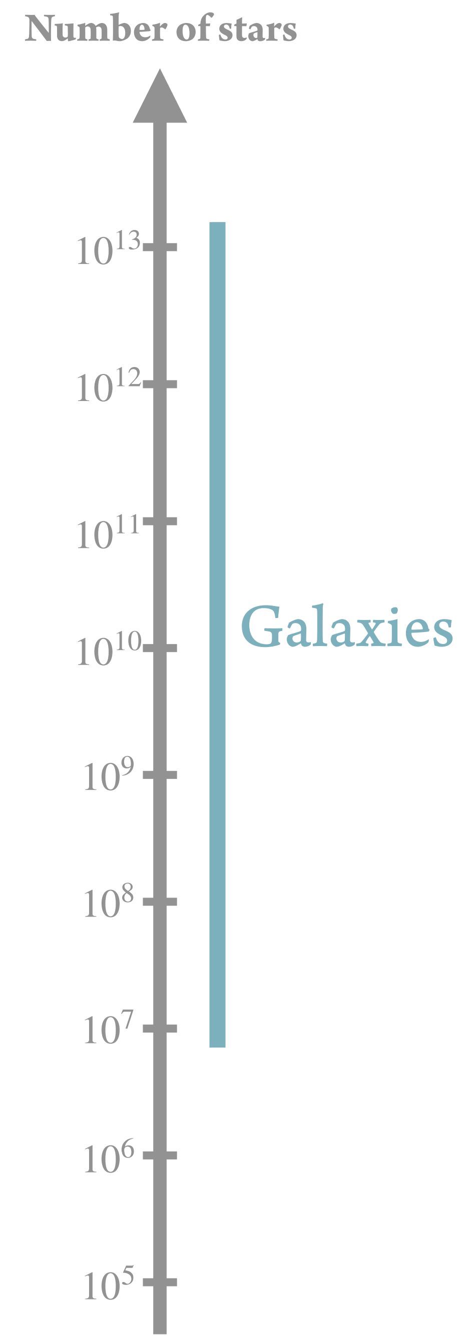
Galaxies



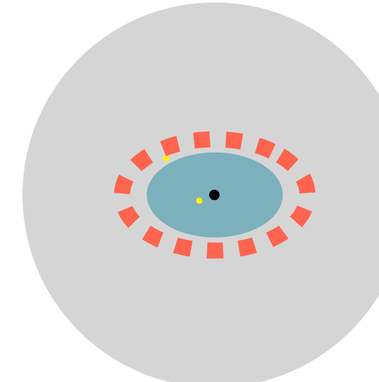
Abell S1063 (HST)

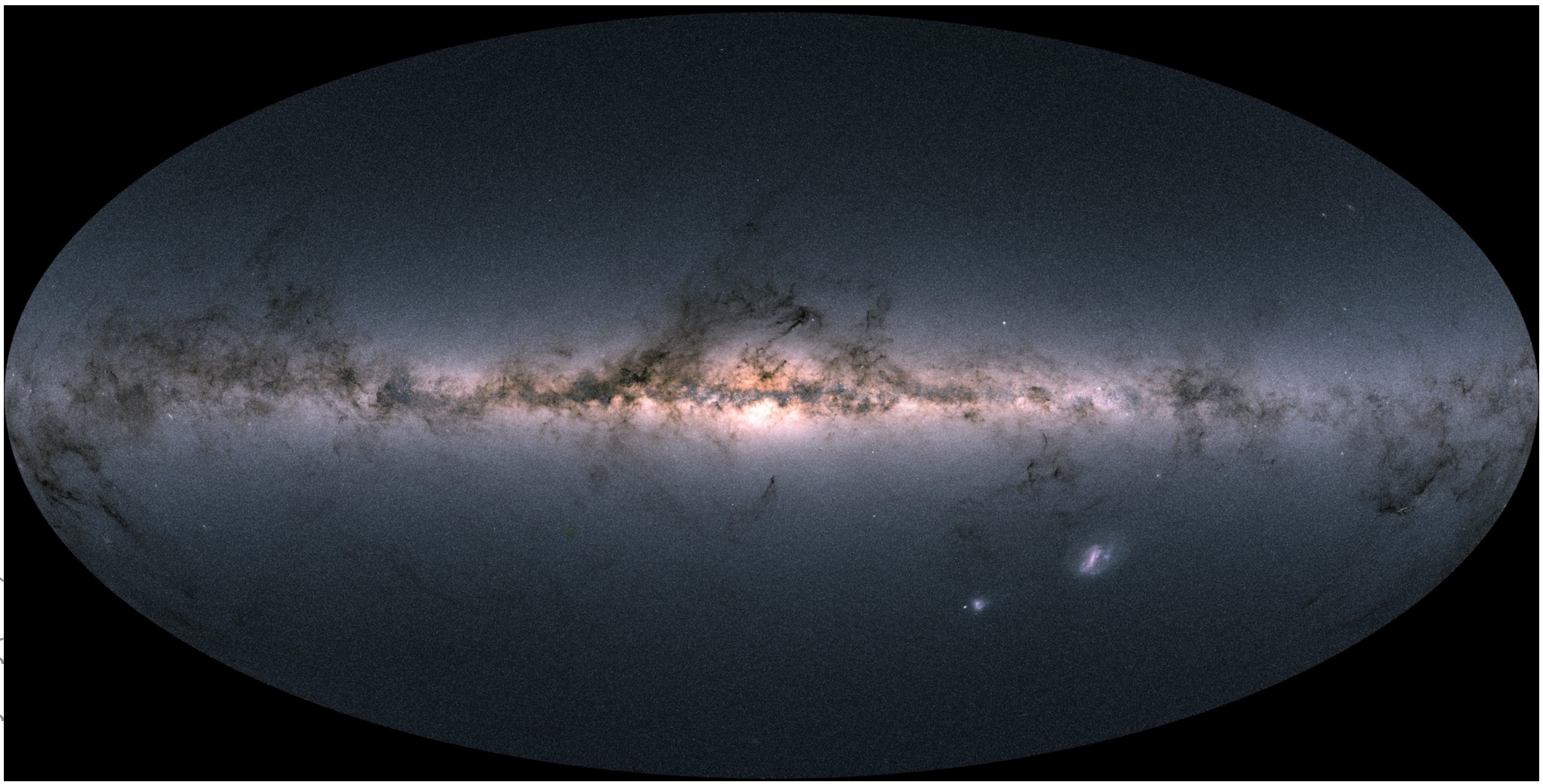


1 Mpc



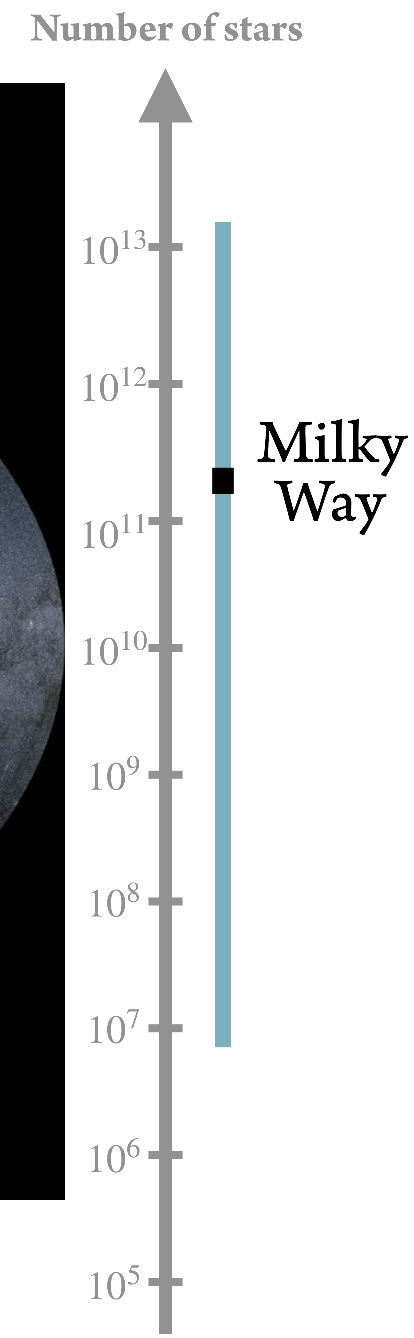
The Milky Way



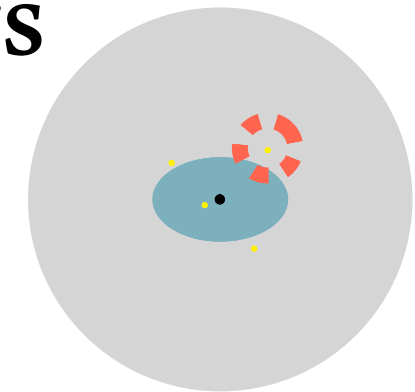


Milky Way (Gaia)

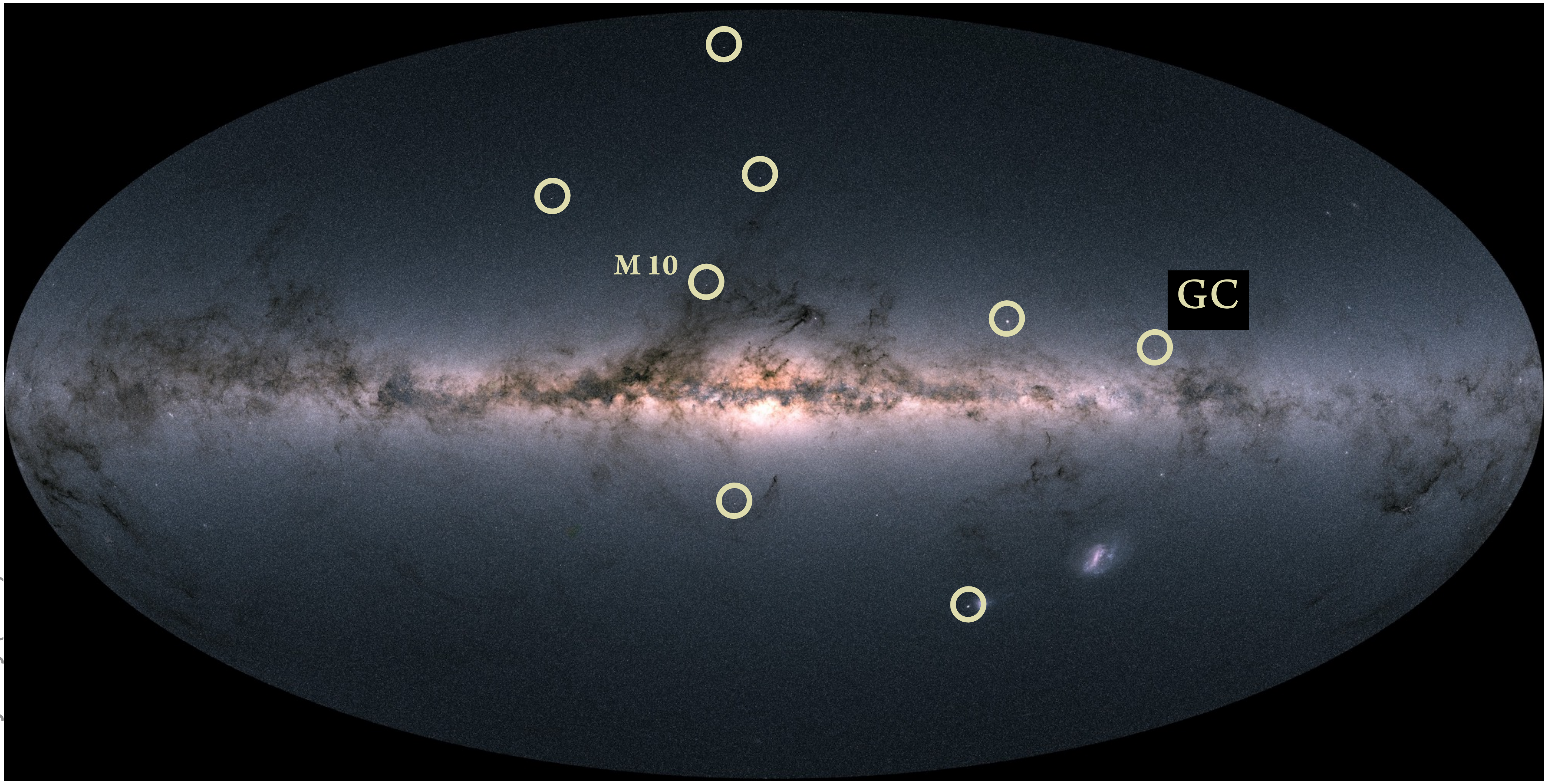
10 kpc



The Milky Way - Globular Clusters



More than 150 GCs detected today



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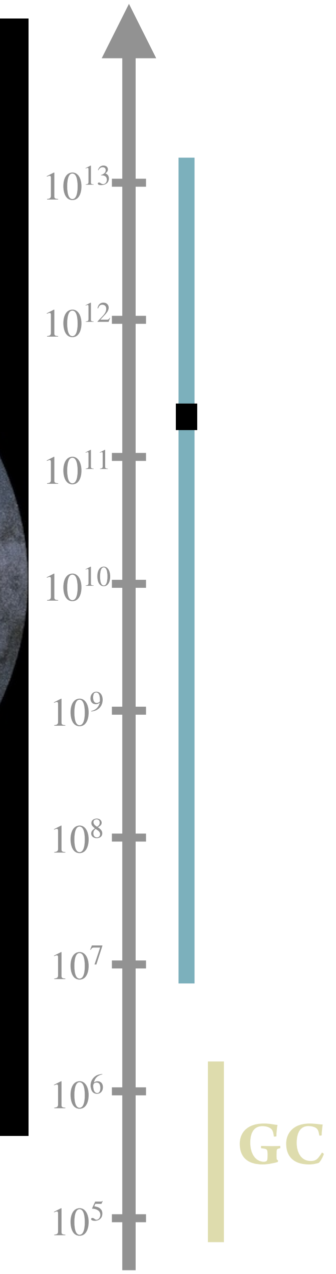
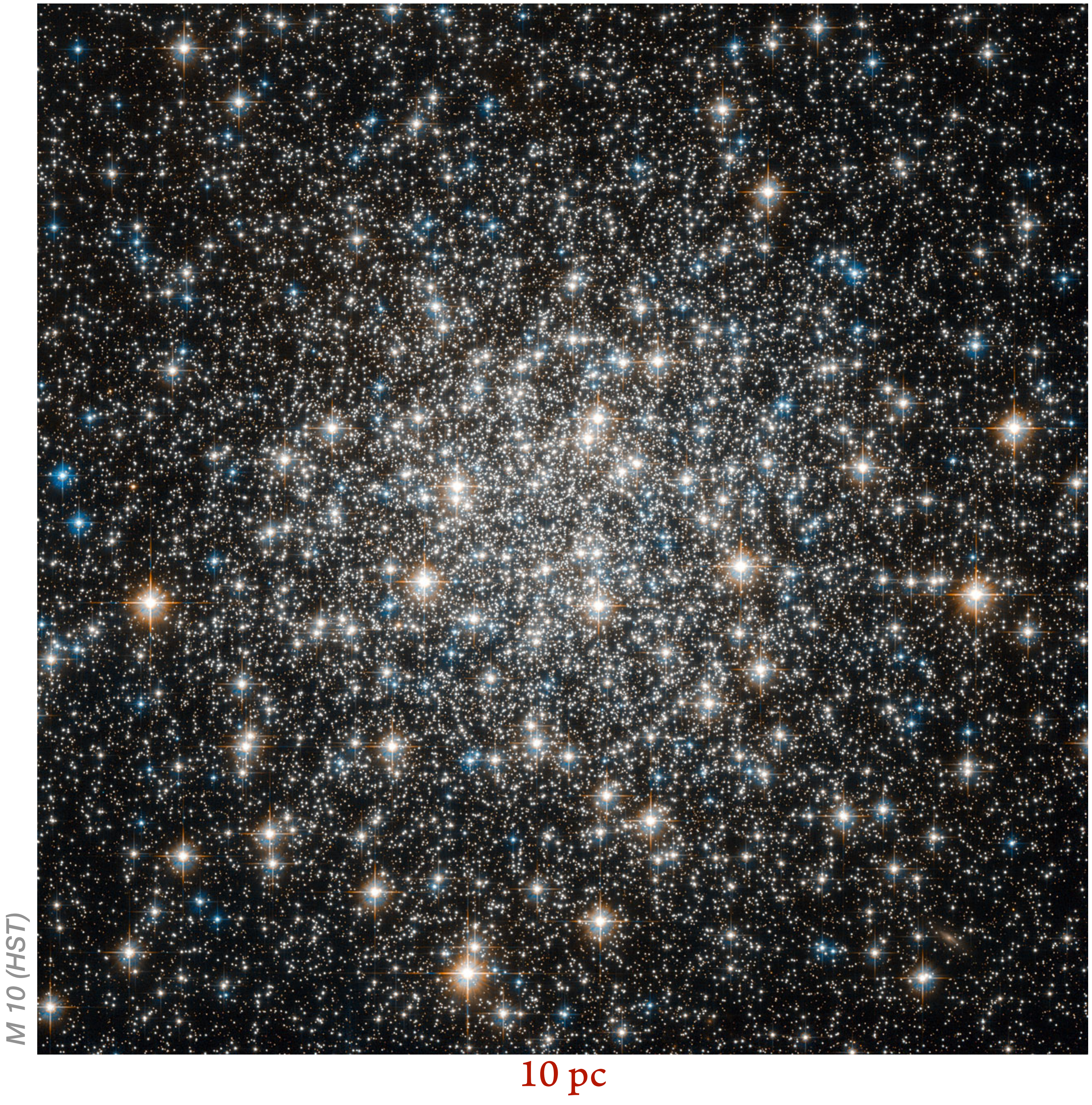
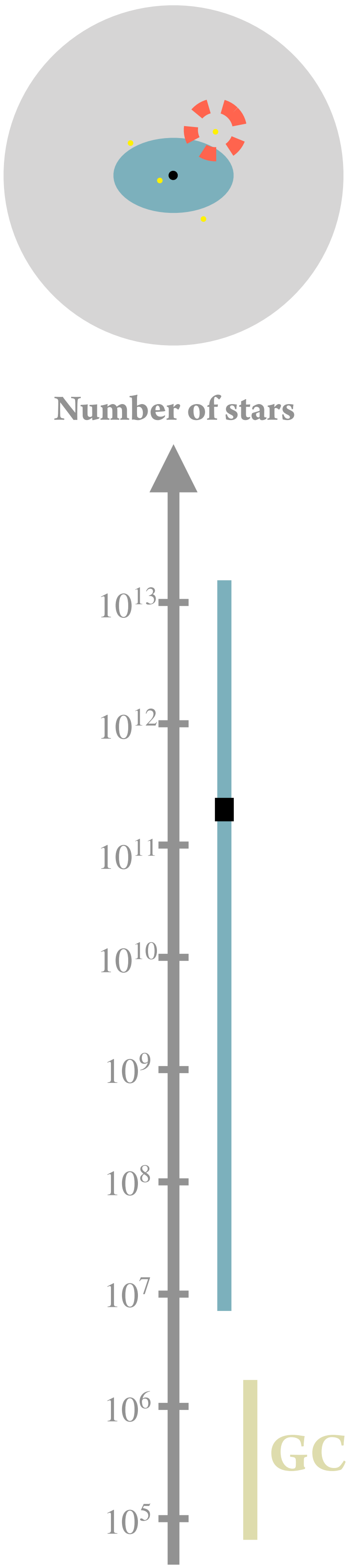
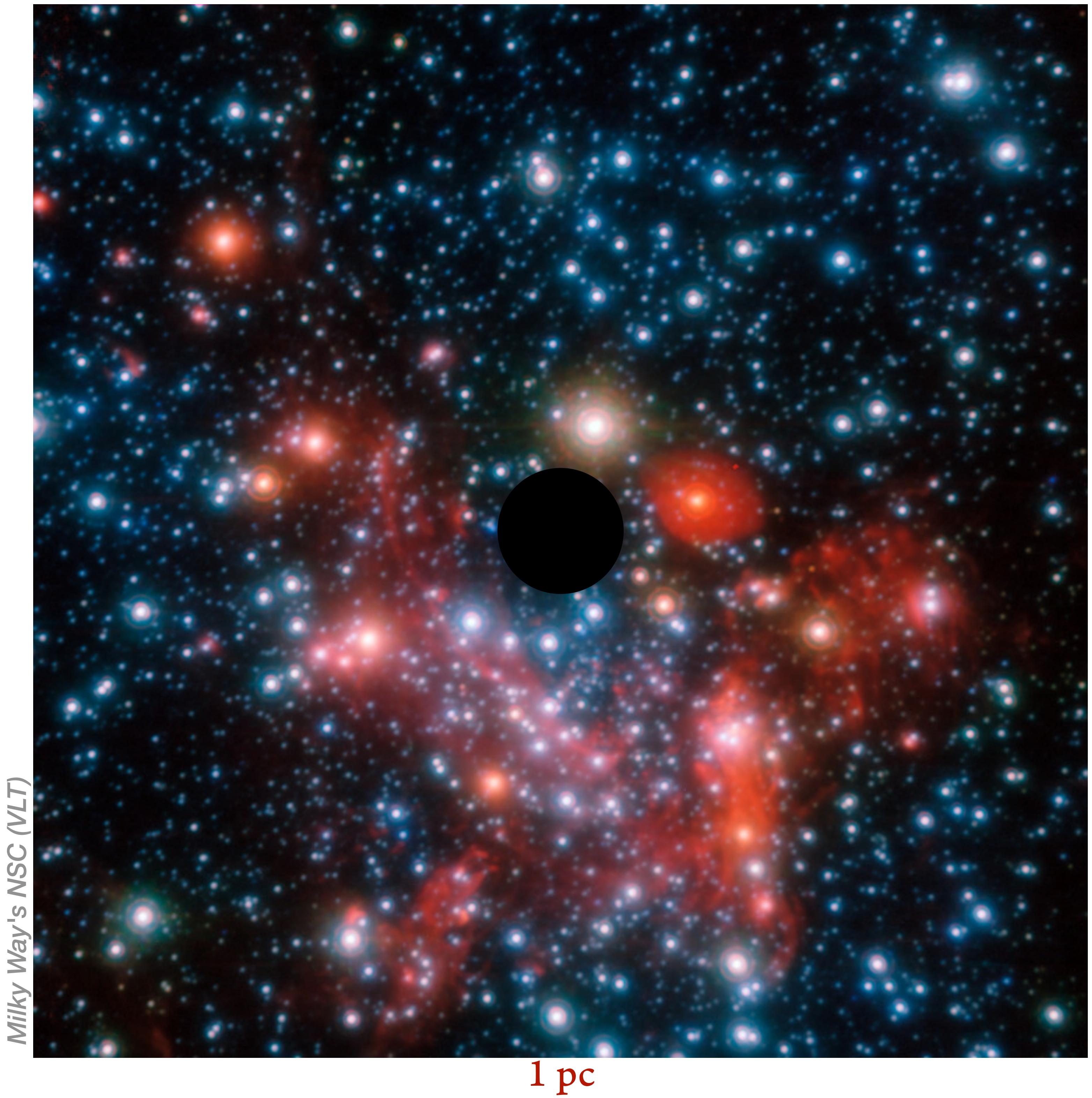


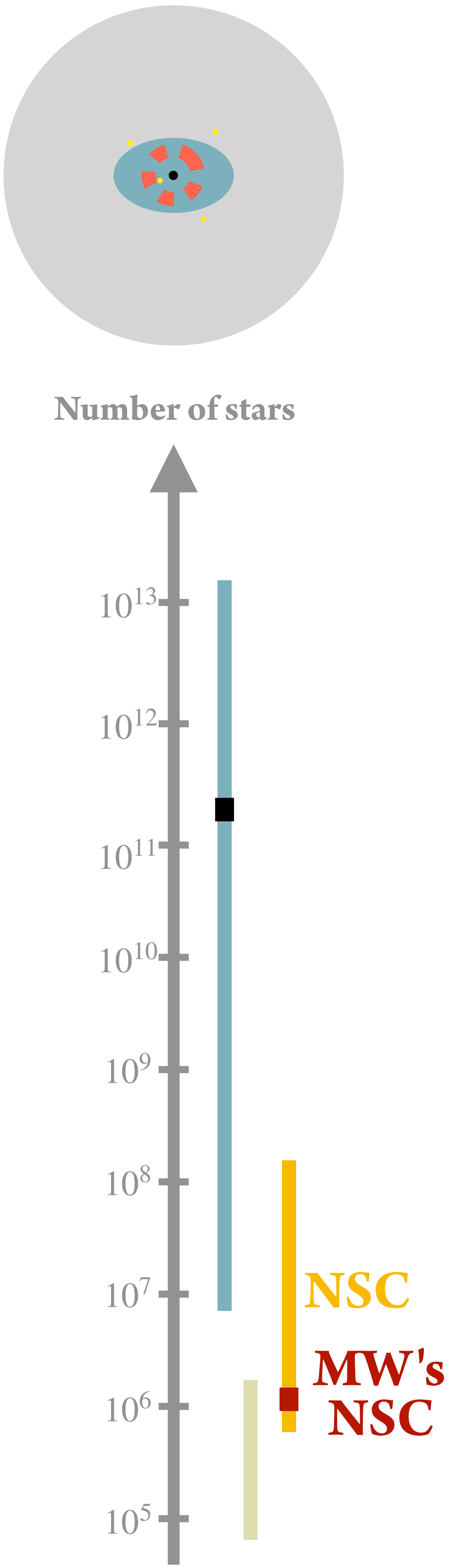
Globular Clusters



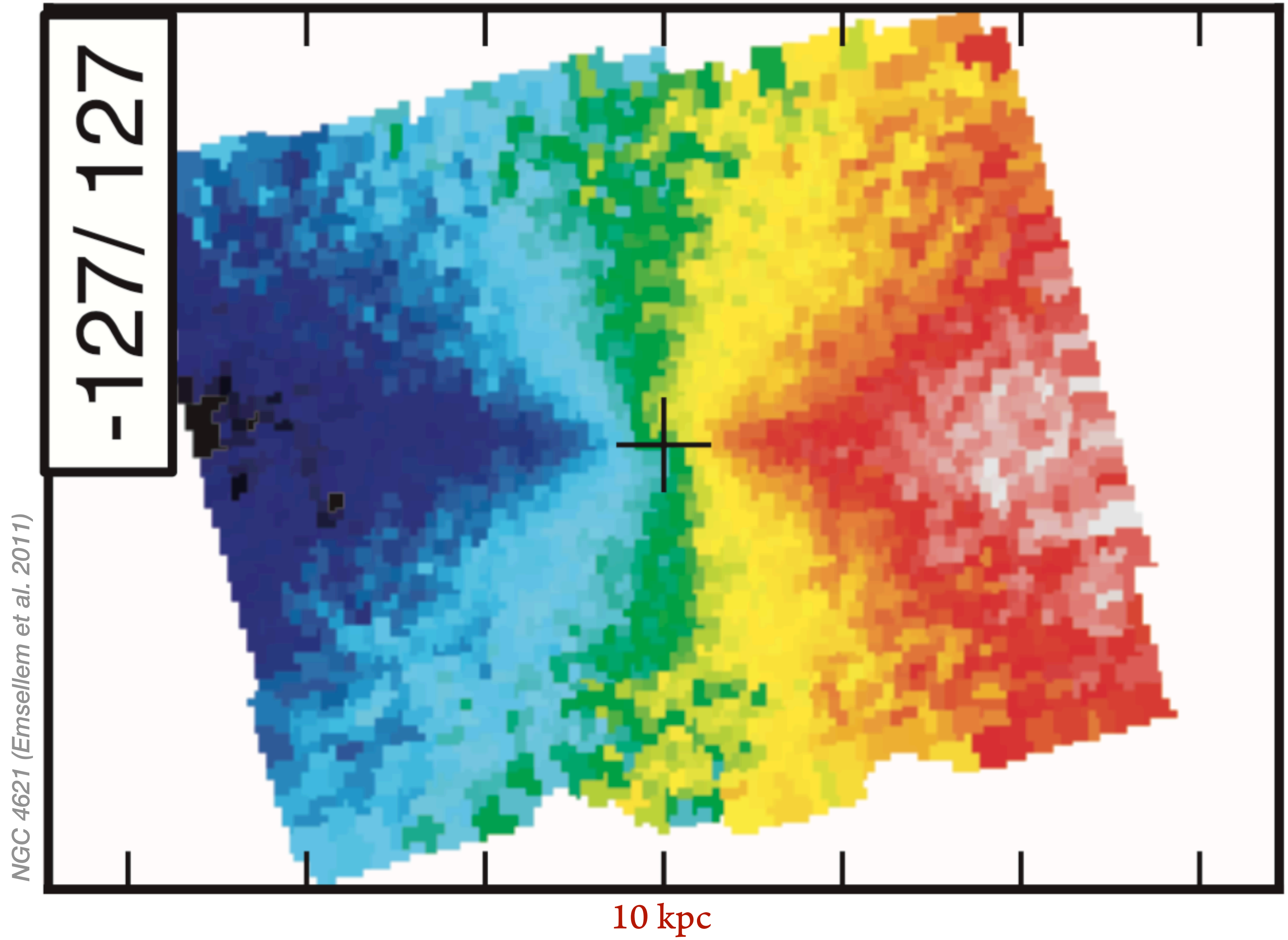


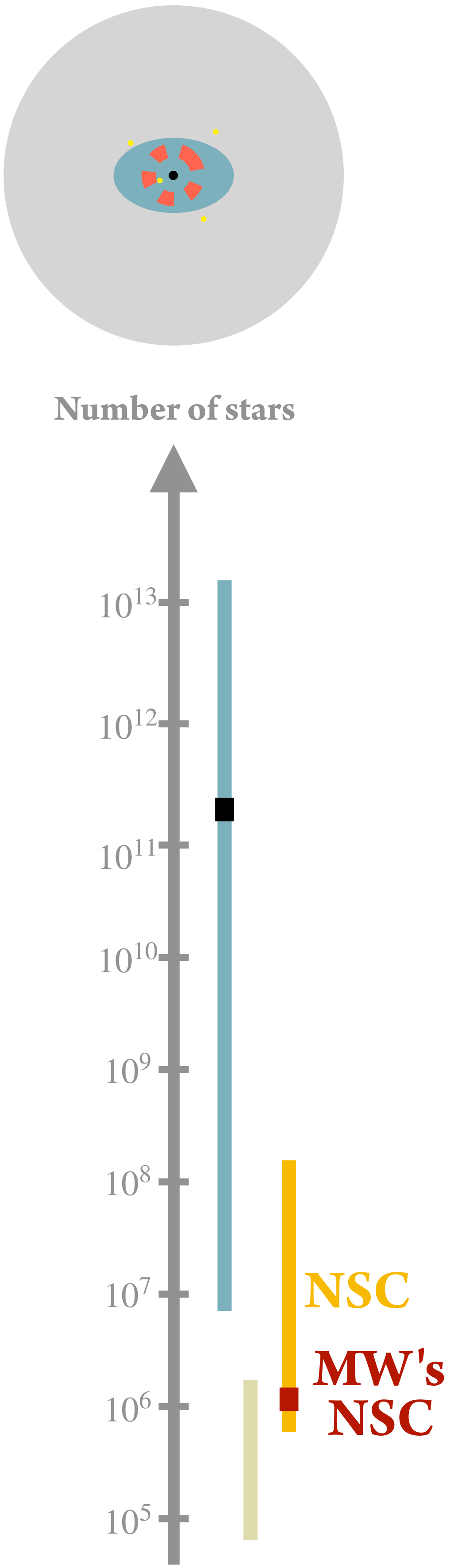
Nuclear Star Clusters



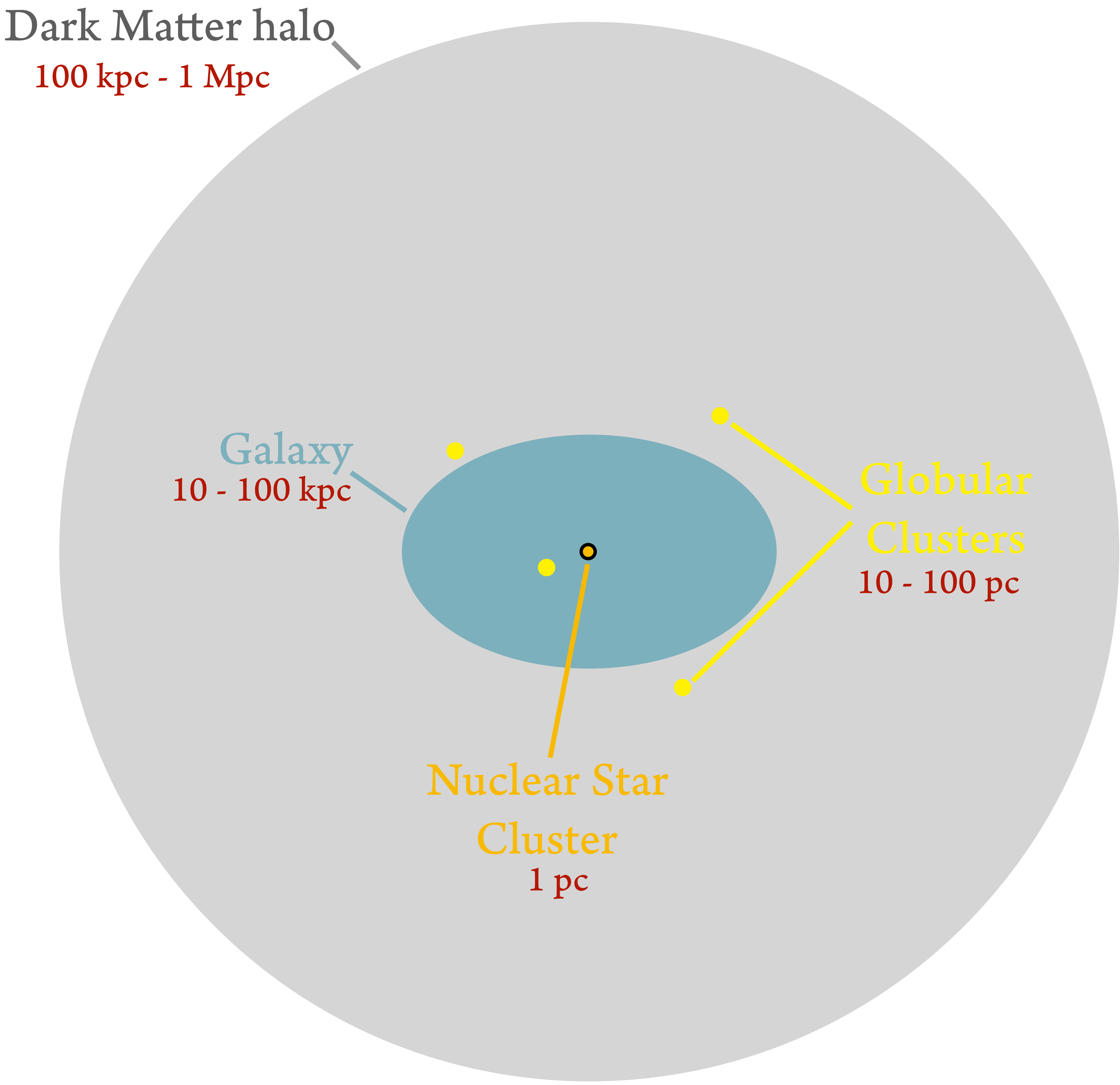


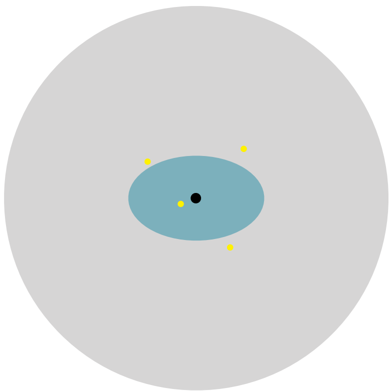
Kinematic diversity

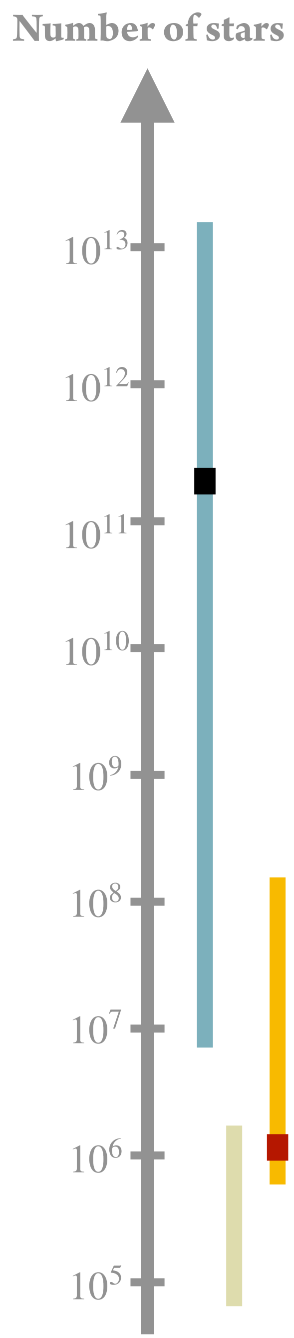




Stellar Clusters







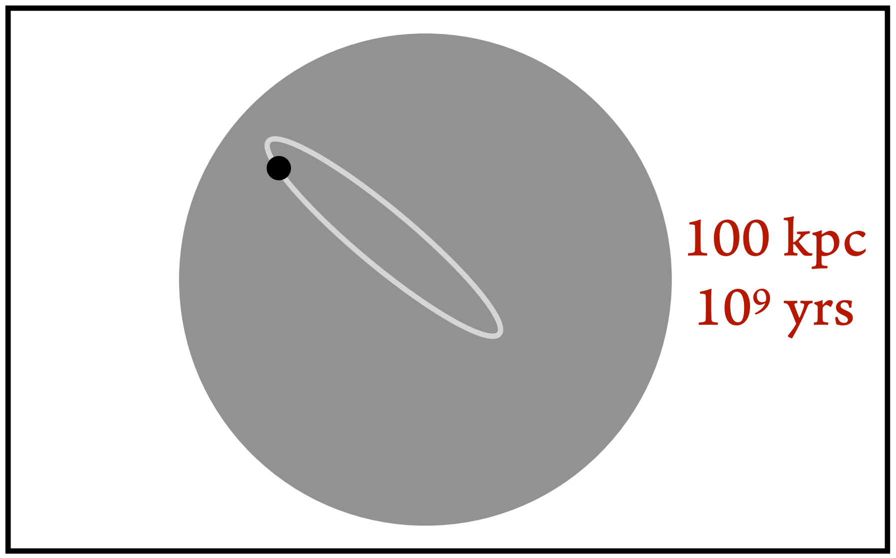
Scaling relation

$$\boxed{t_{\mathrm{dyn}} \propto \frac{1}{\sqrt{\rho}}}$$

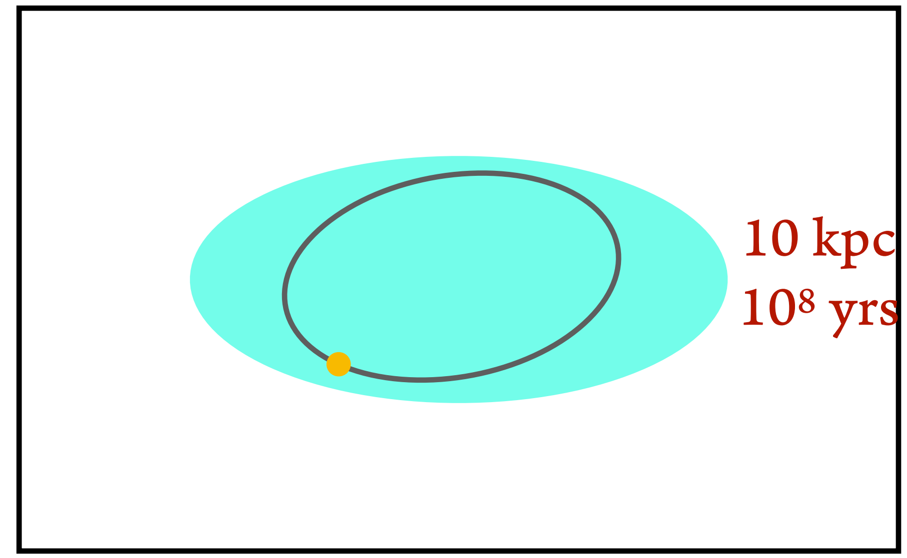
Changing scales amounts to **changing the clock**

Smaller system $\longrightarrow$ denser $\longrightarrow$ faster evolution

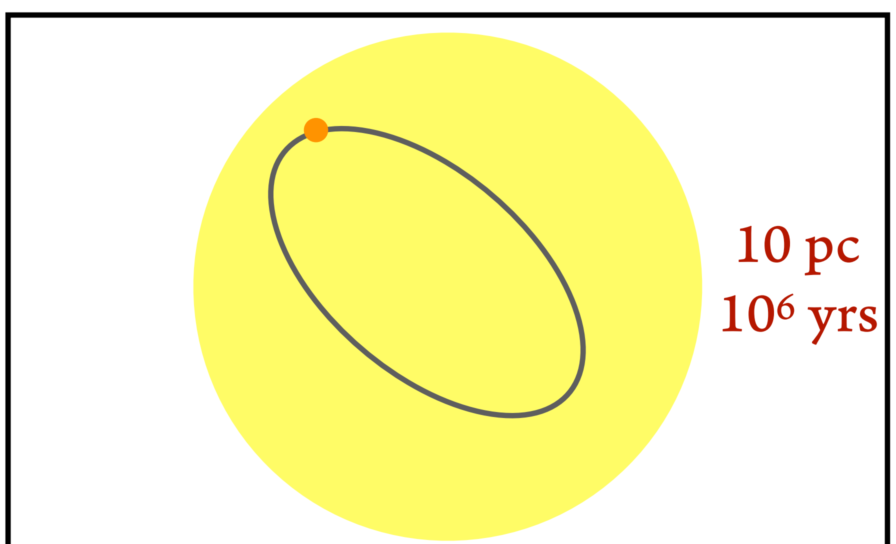
DM particle - DM halo



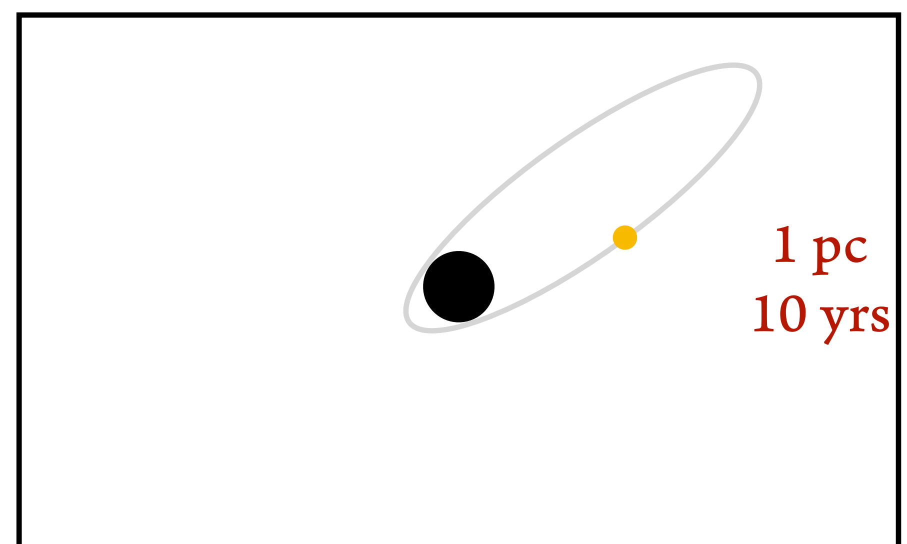
Star - Galaxy



Star - Globular cluster



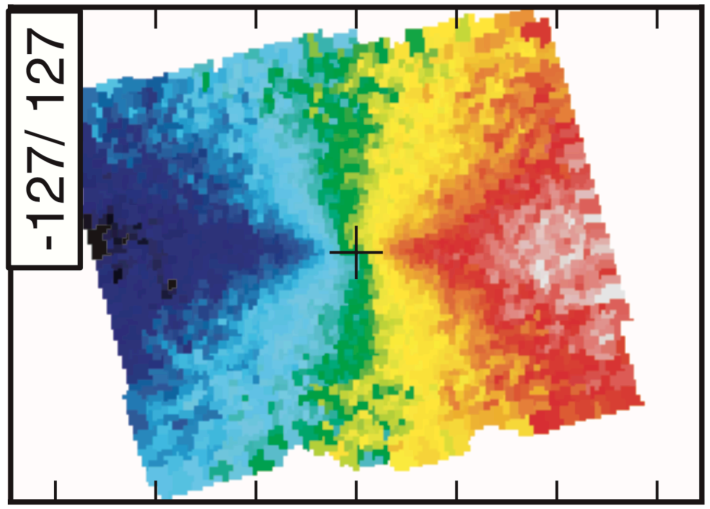
Star - Black hole (NSC)



Critical concepts

Gas brings order
⟶ **kinematic diversity**

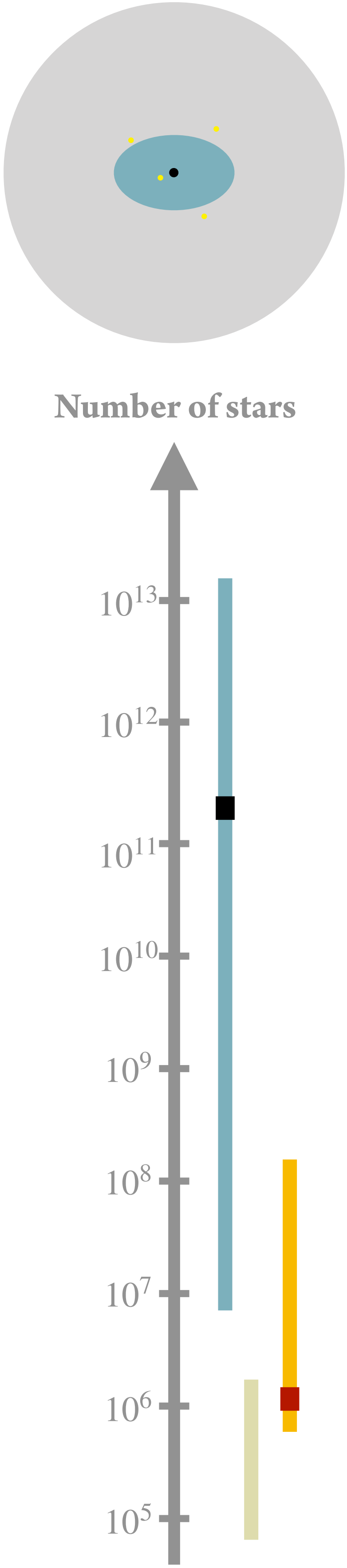
Steady state
⟶ dynamically **locked**



Interactions
⟶ **perturbations**

Neighbouring scales
⟶ **resonance**

$$t_{\rm dyn} \propto \frac{1}{\sqrt{\rho}}$$



Questions

Orbital reshuffling

Loose free energy

Spontaneous evolution

Instabilities

Effect of perturbations

Linear response

Processes

Long-range couplings
Resonances

New Horizon

10 kpc

Stellar Clusters

Orbital structure of stellar clusters

Linear Response Theory

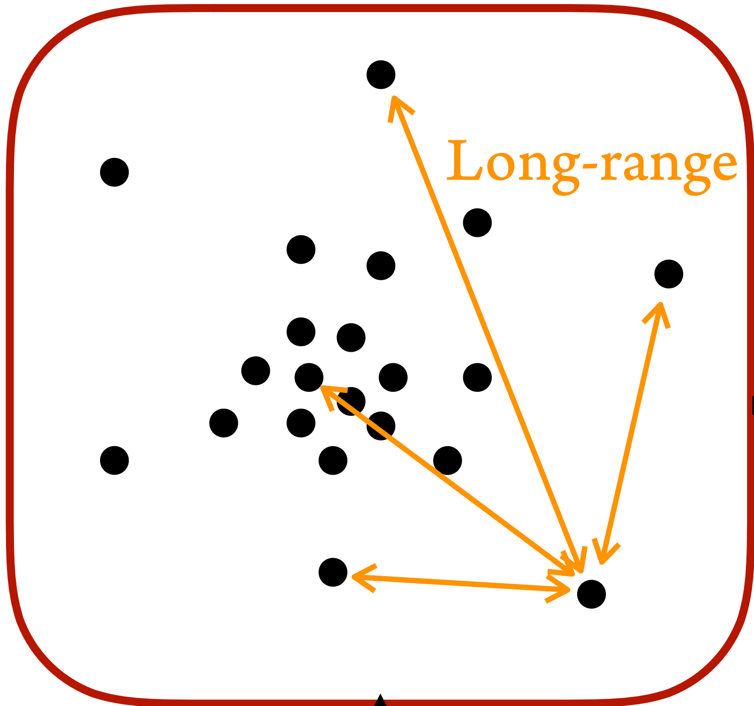
Identifying unstable modes

Anisotropic, rotating stellar clusters

Destabilisation processes

Conclusions and prospects

Mean field dynamics



ρ_0 : mass density

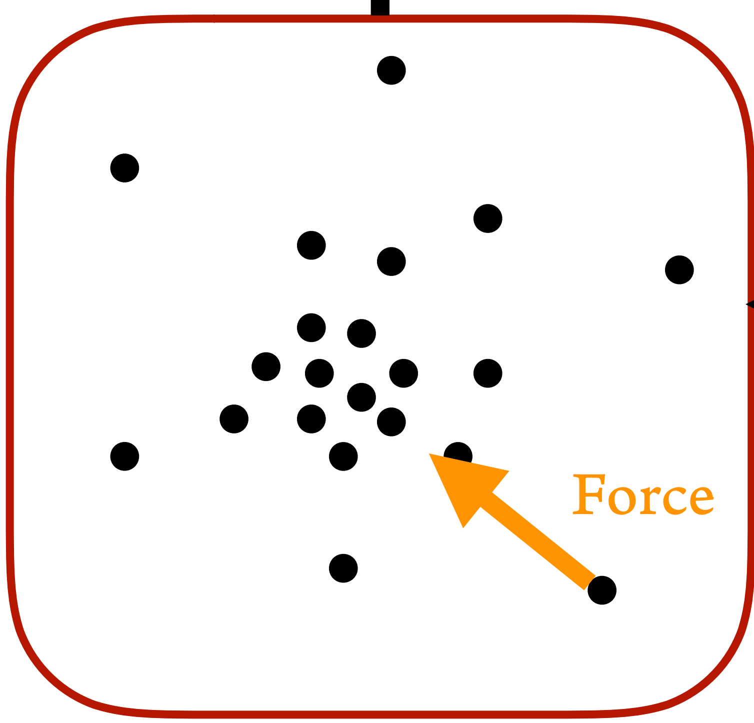
$$\rho_0(\mathbf{x}) = \int \mathrm{d}\mathbf{v}\, F(\mathbf{x}, \mathbf{v})$$

ψ_0: gravitational potential

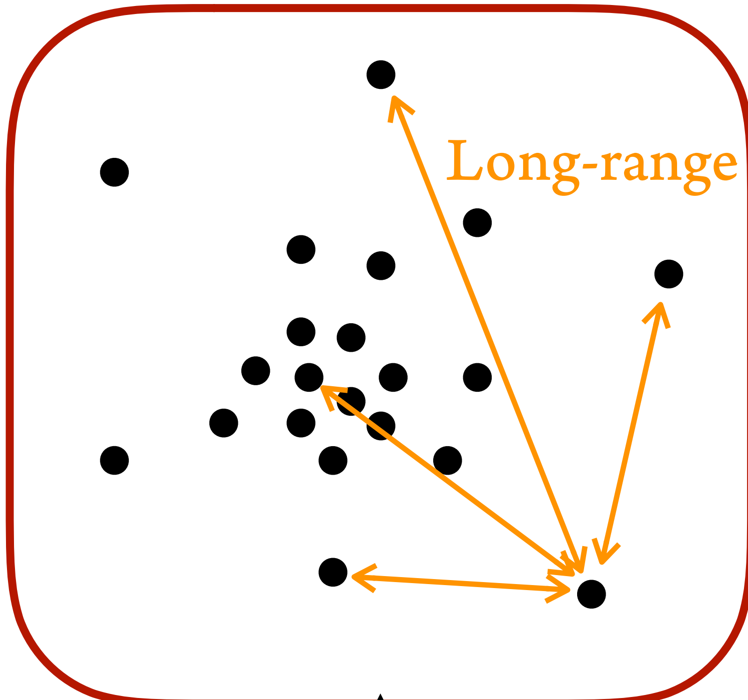
$$\psi_0(\mathbf{x}) = -G \int \mathrm{d}\mathbf{x}' \frac{\rho_0(\mathbf{x}')}{|\mathbf{x}' - \mathbf{x}|}$$

Poisson equation

$$\boxed{\Delta\psi_0 = 4\pi G \rho_0}$$



Mean field dynamics



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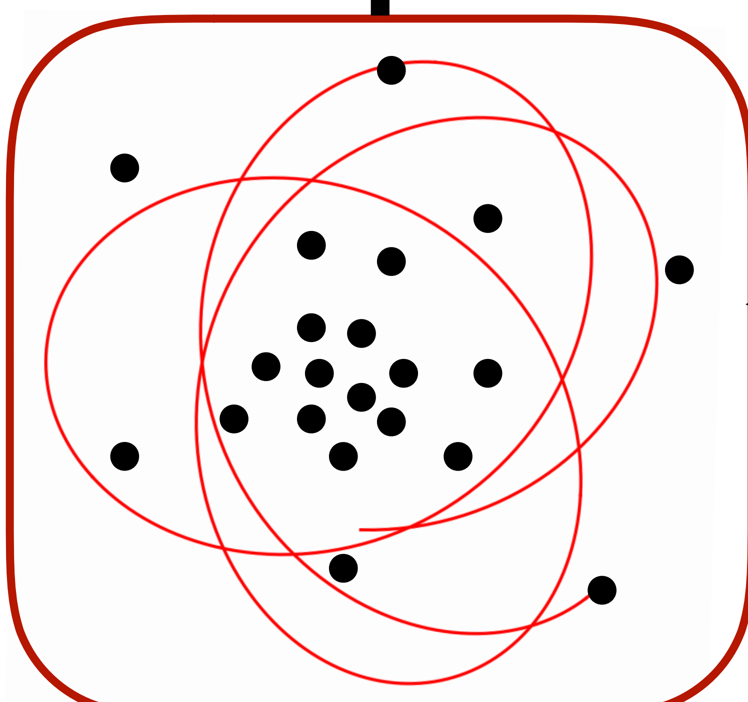
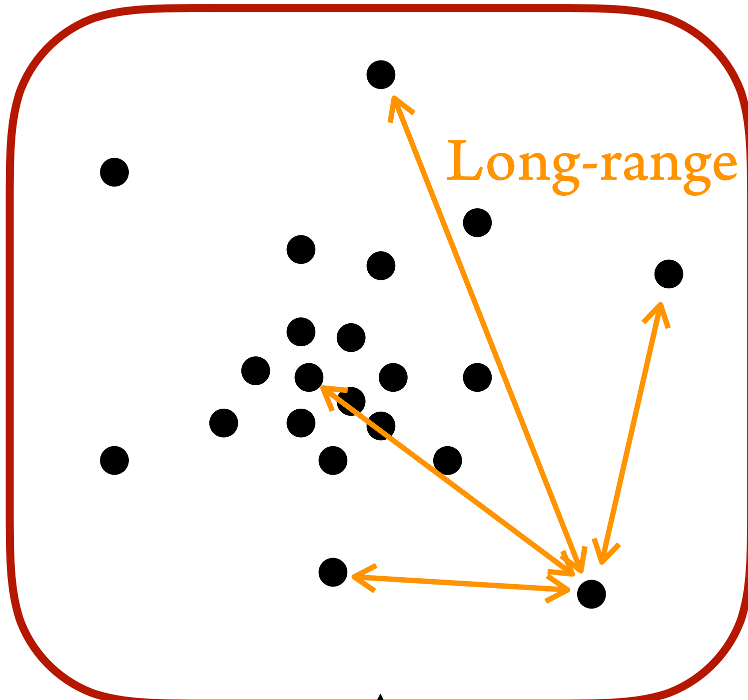


Mean-field dynamics

Mean field dynamics



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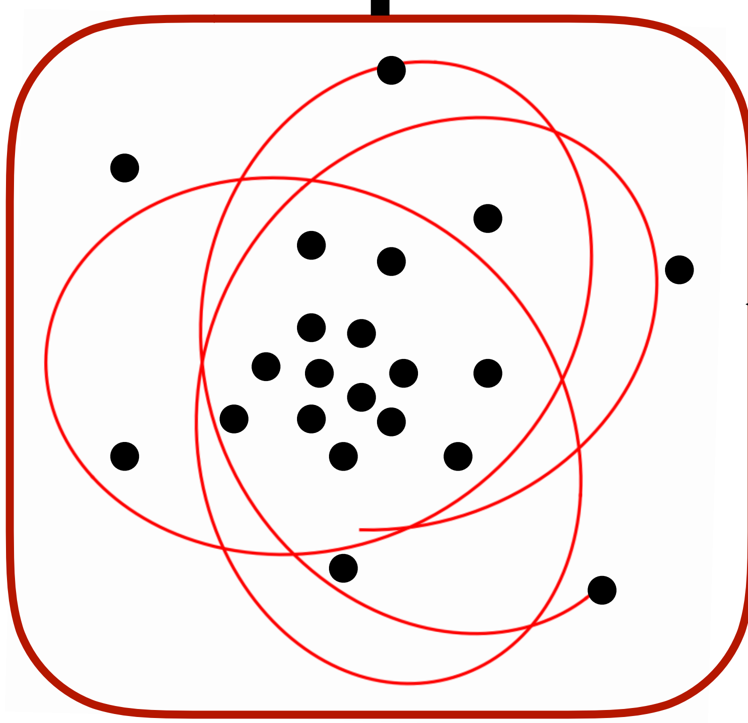
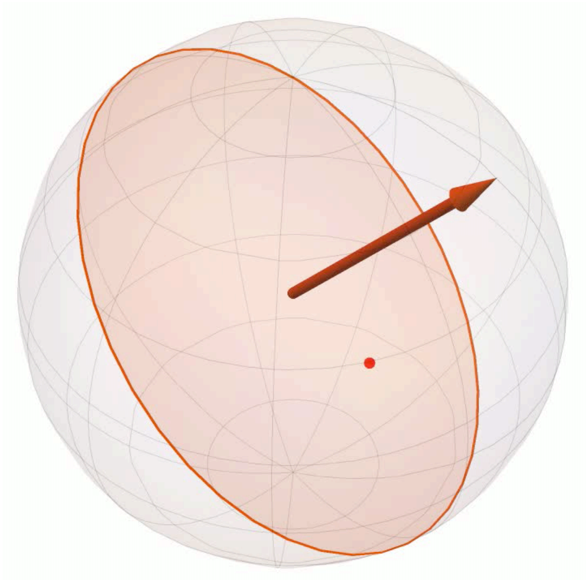
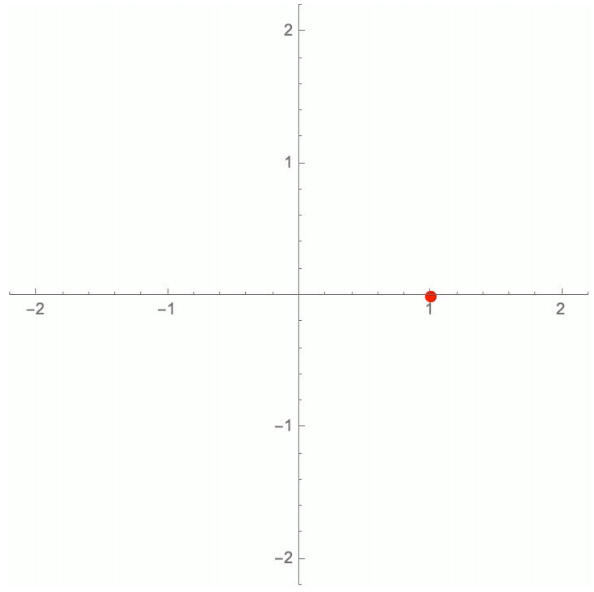


Mean-field dynamics

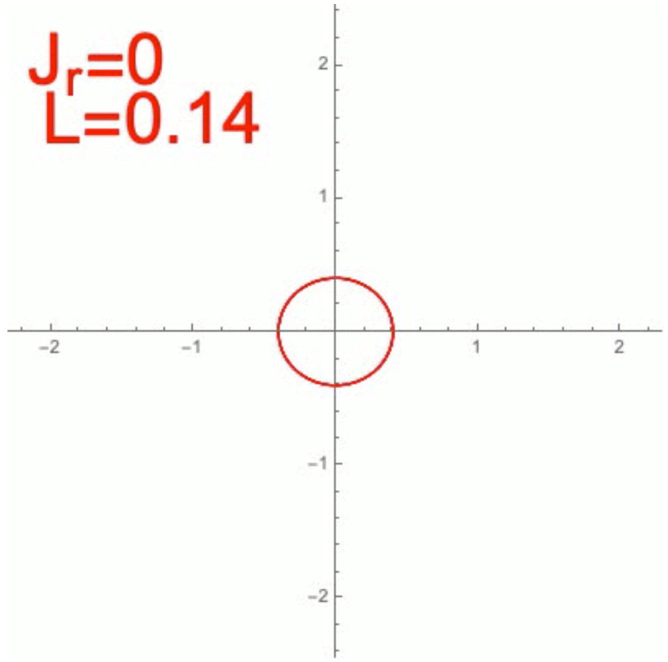
Spherical symmetry $\psi_0(\mathbf{x}) = \psi_0(r)$

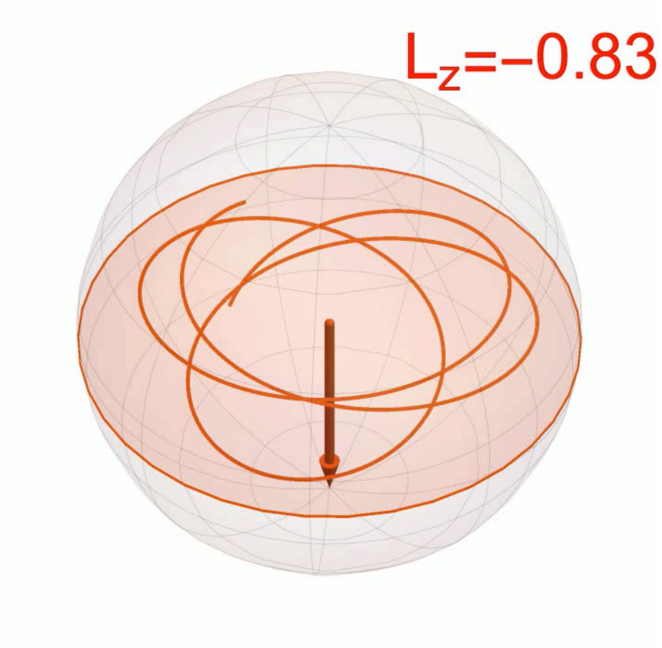
Orbits in a spherical potential





Spherical potential $\longrightarrow$ 3 conserved quantities $\mathbf{J} = (J_r, L, L_z)$
Actions used to label the orbits

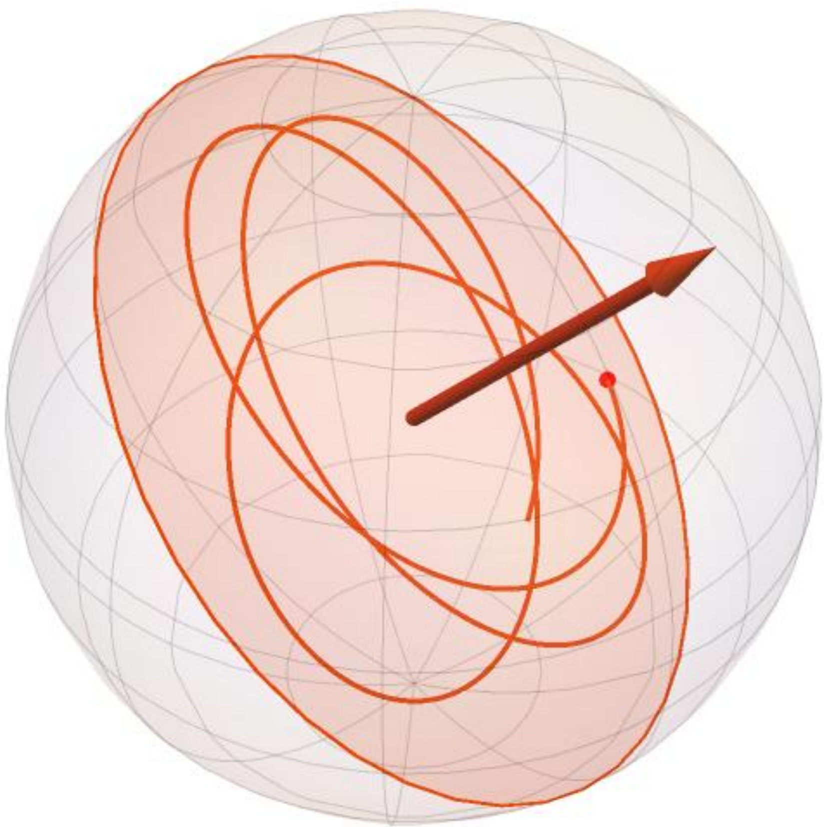


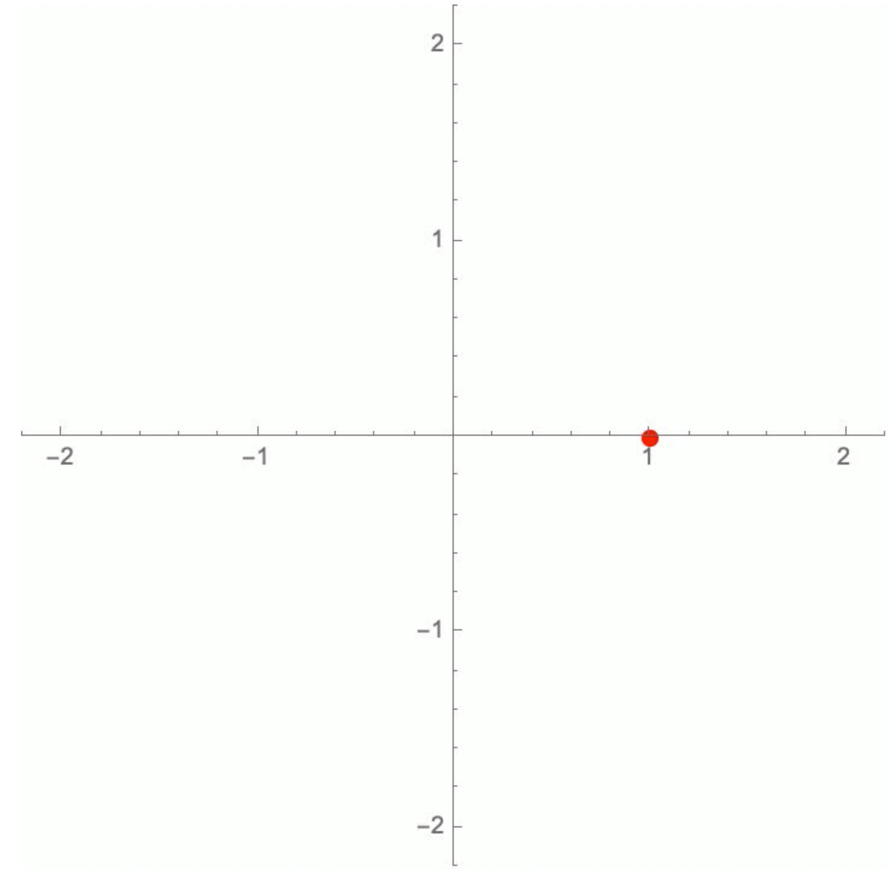


Angular momentum $L \sim$ average radius
Radial action $J_r \sim$ eccentricity

z component of the angular momentum
$L_z \sim$ inclination of the orbit

Orbital frequencies





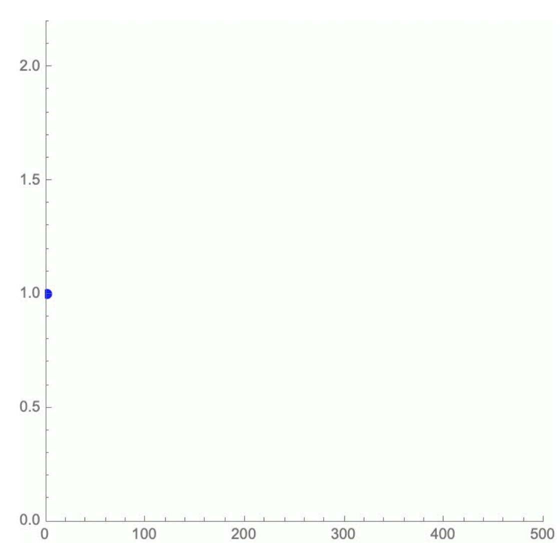
Quasi-periodic orbits

Two frequencies $\mathbf{\Omega} = \mathbf{\Omega}(\mathbf{J})$

Radial $\Omega_1 = \dfrac{2\pi}{T_r}$

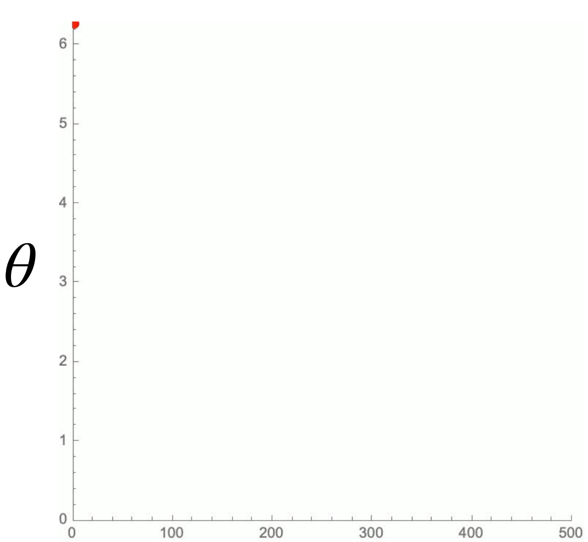
Azimuthal $\Omega_2 = \dfrac{2\pi}{T_\theta}$

r



T_r t

θ

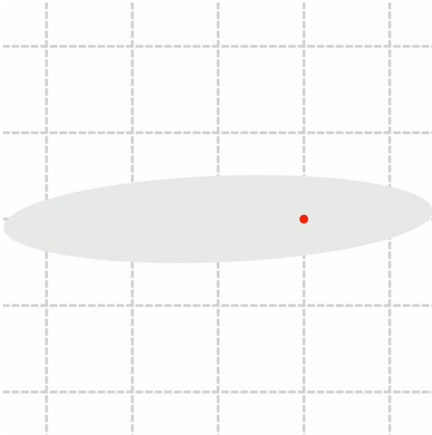
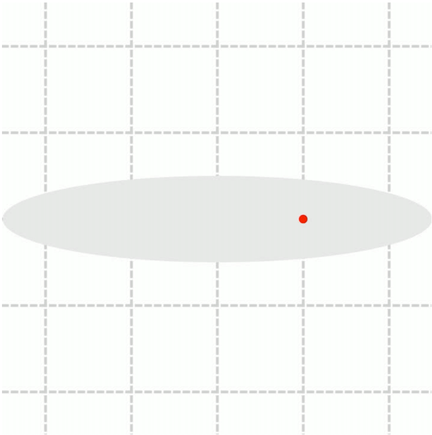


T_θ t

Orbital resonances

Frequency of the perturbation: $$\omega_0 = \mathbf{n} \cdot \boldsymbol{\Omega}(\mathbf{J})$$

Resonance $\longrightarrow$ **torque build-up** $\longrightarrow$ perturbation **modifies** the orbits

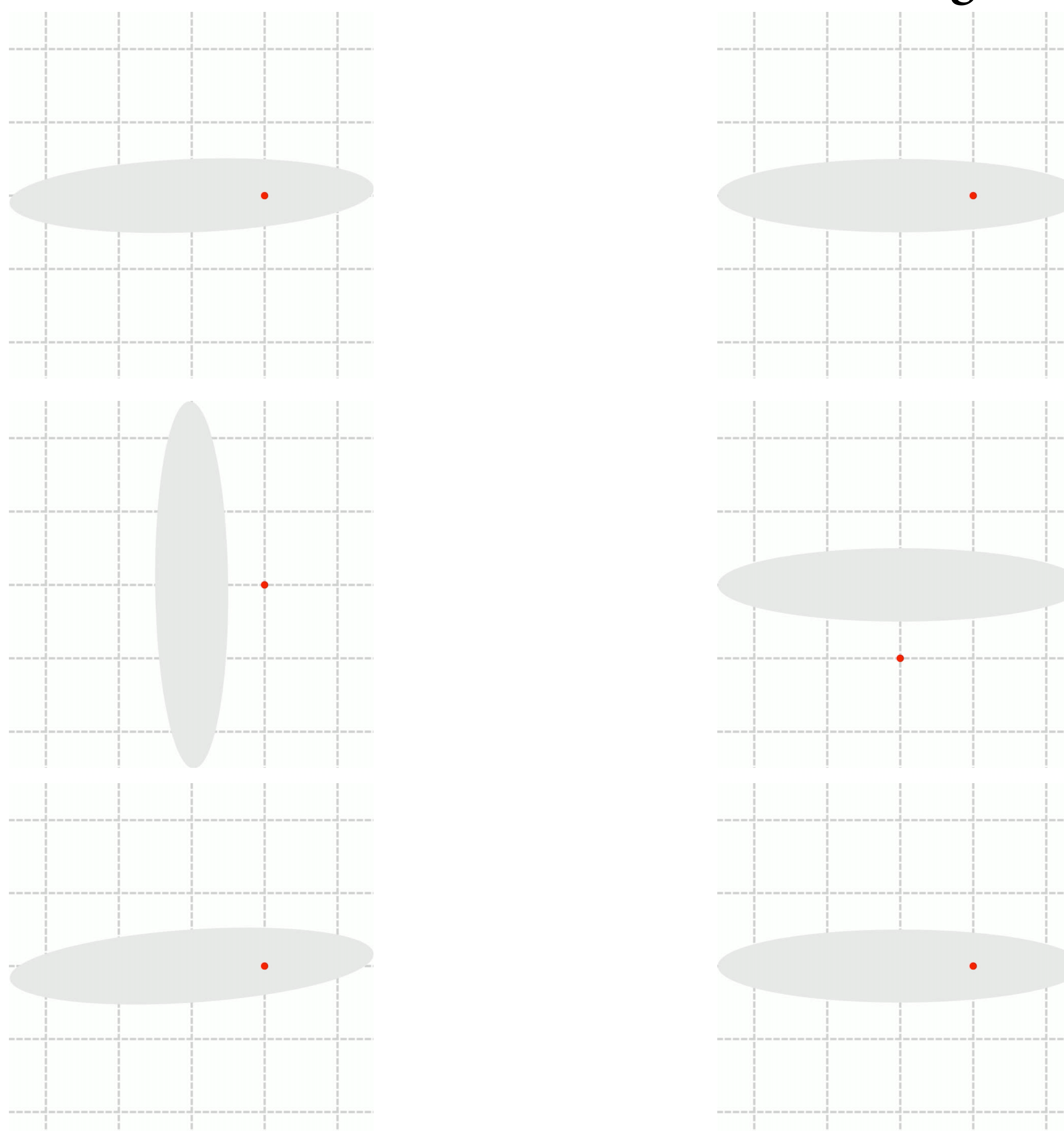
	Fixed	**Rotating**	
Corotation			$\omega_0 = 2\Omega_2$

Lindblad resonances

Frequency of the perturbation: $\omega_0 = \mathbf{n} \cdot \mathbf{\Omega}(\mathbf{J})$

Resonance $\longrightarrow$ **torque build-up** $\longrightarrow$ perturbation **modifies** the orbits

Fixed | **Rotating**



Corotation — $\omega_0 = 2\Omega_2$

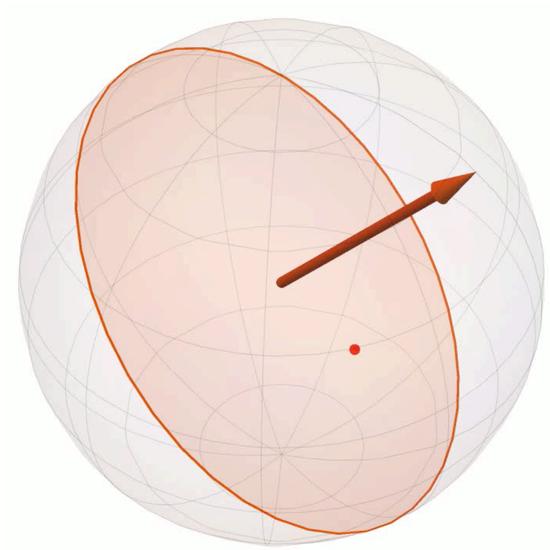
Inner Lindblad resonance (ILR) — $\omega_0 = 2\Omega_2 - \Omega_1$

Outer Lindblad resonance (OLR) — $\omega_0 = 2\Omega_2 + \Omega_1$

Orbital structure

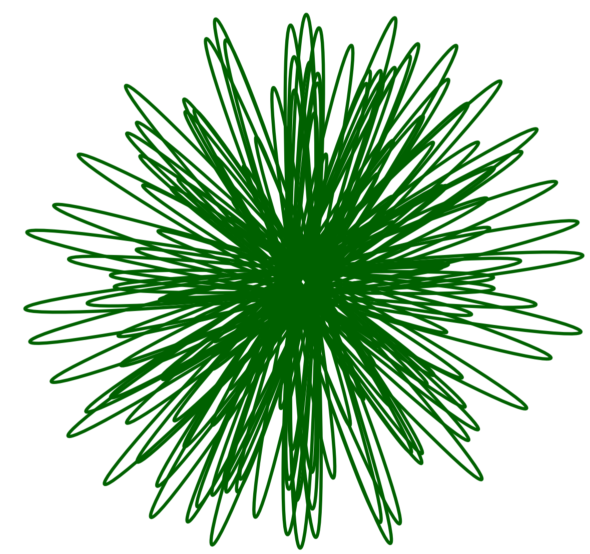
Spherical symmetry

$\psi_0(r)$



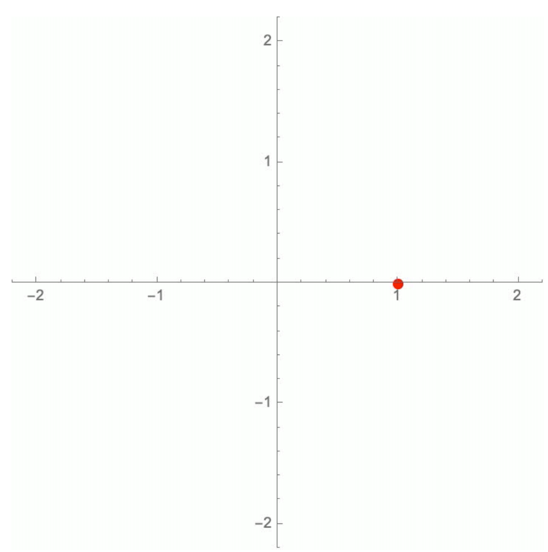
Distribution of orbits

$F(\mathbf{J})$



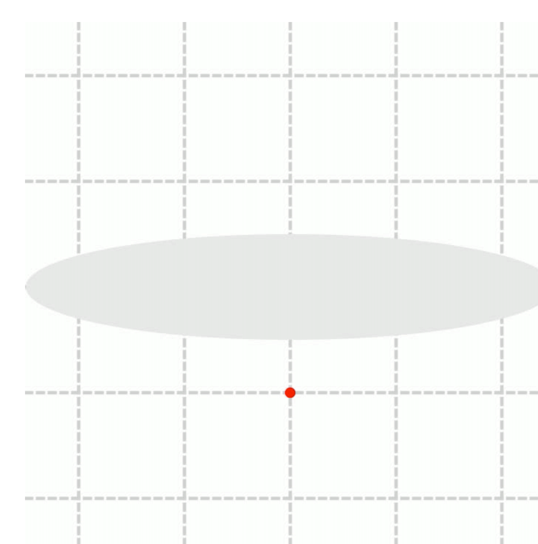
Quasi-periodic motion

$$\boldsymbol{\theta} = \boldsymbol{\theta}_0 + \boldsymbol{\Omega}(\mathbf{J})\, t$$



Resonances

$$\omega_0 = \mathbf{n} \cdot \boldsymbol{\Omega}(\mathbf{J})$$



Stellar Clusters

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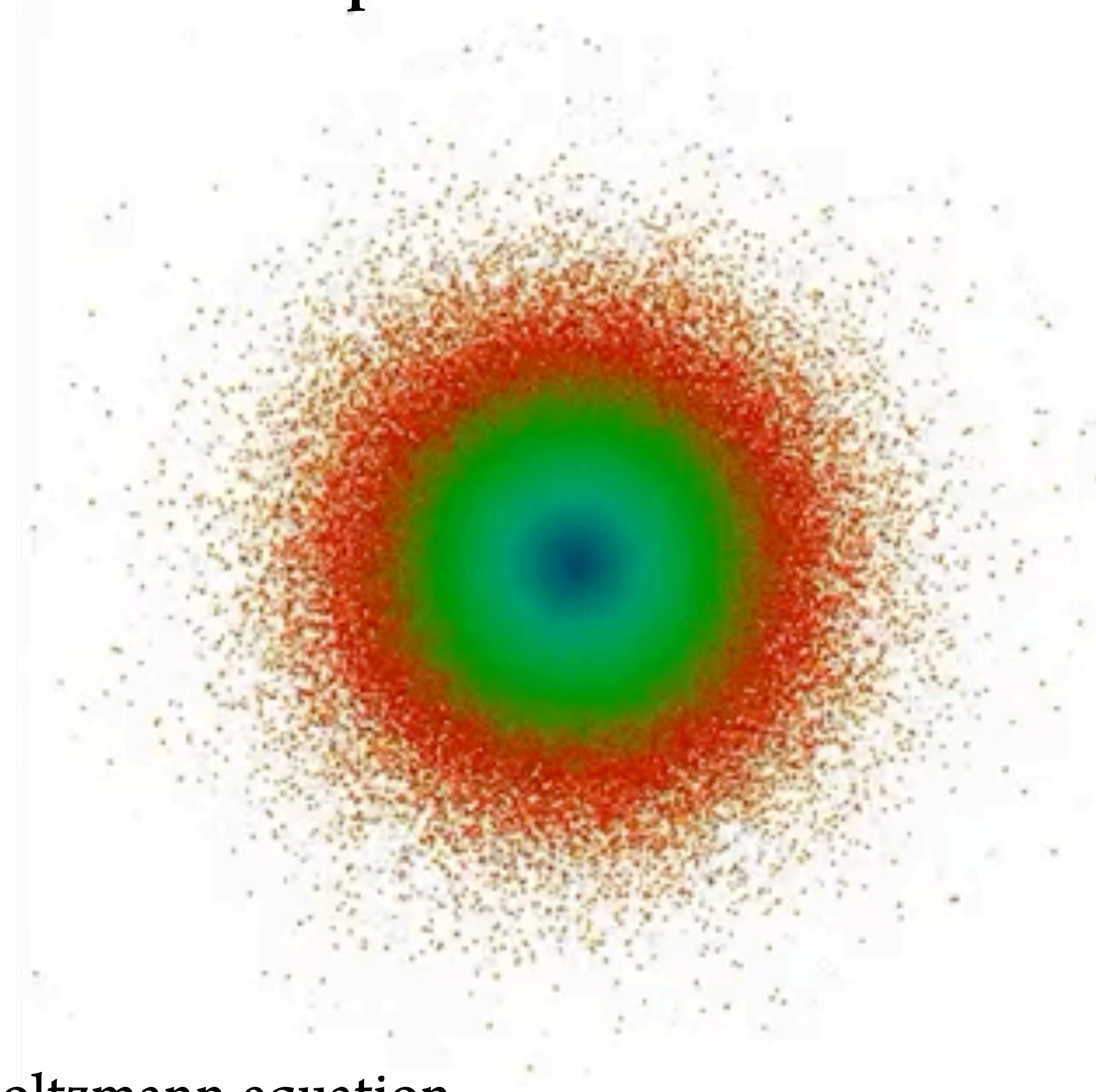
Linear Response Theory

How does a stellar system **respond** to an **external perturbation**?

External perturber $\psi_0 \to \psi_0 + \psi^{\mathrm{e}}$

Perturbed DF $F \to F + f$

System's response $\psi_0 \to \psi_0 + \psi^{\mathrm{e}} + \psi^{\mathrm{s}}[f]$



S. Chakrabarti

Linearised collisionless Boltzmann equation

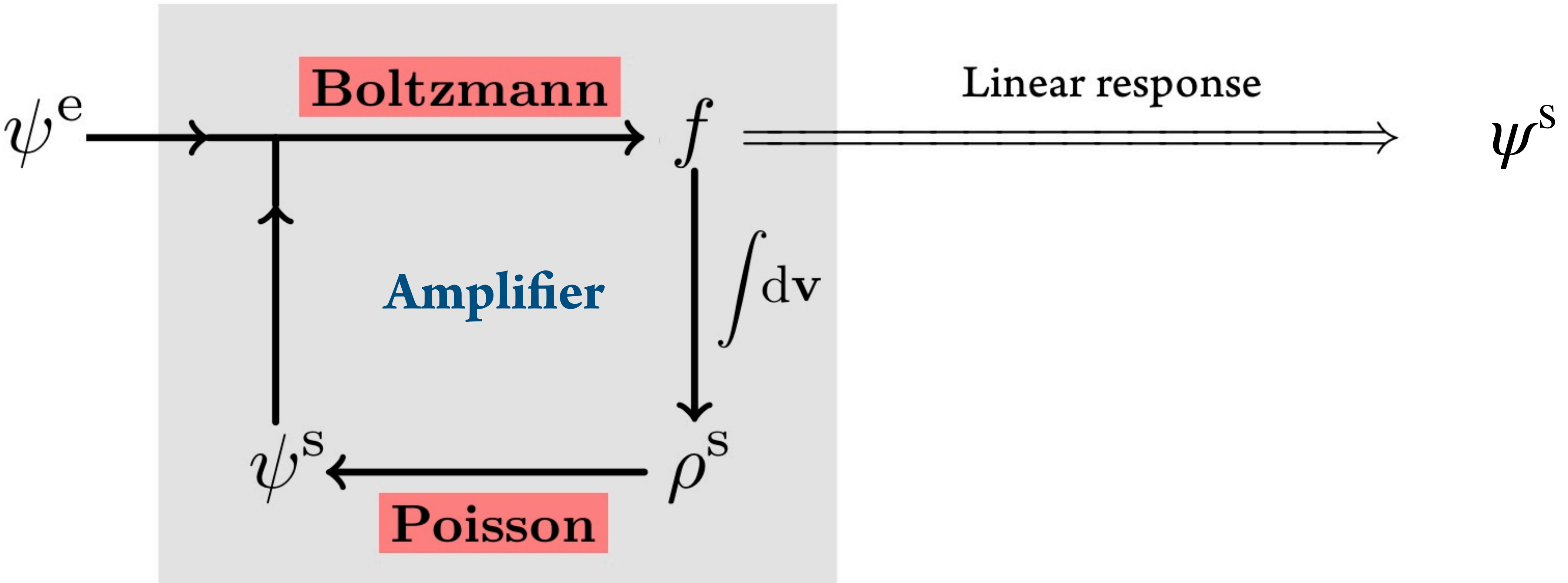
$$\boxed{\frac{\partial f}{\partial t} + \mathbf{\Omega}(\mathbf{J}) \cdot \frac{\partial f}{\partial \boldsymbol{\theta}} - \frac{\partial F}{\partial \mathbf{J}} \cdot \frac{\partial (\psi^{\mathrm{e}} + \psi^{\mathrm{s}})}{\partial \boldsymbol{\theta}} = 0}$$

Linear Response Theory

How does a stellar system **respond** to an **external perturbation**?

Linearised CBE

$$\boxed{\frac{\partial f}{\partial t} + \boldsymbol{\Omega}(\mathbf{J}) \cdot \frac{\partial f}{\partial \boldsymbol{\theta}} - \frac{\partial F}{\partial \mathbf{J}} \cdot \frac{\partial (\psi^{\mathrm{e}} + \psi^{\mathrm{s}})}{\partial \boldsymbol{\theta}} = 0}$$



Poisson

$$\boxed{\Delta \psi^{\mathrm{s}} = 4\pi G \rho^{\mathrm{s}}}$$

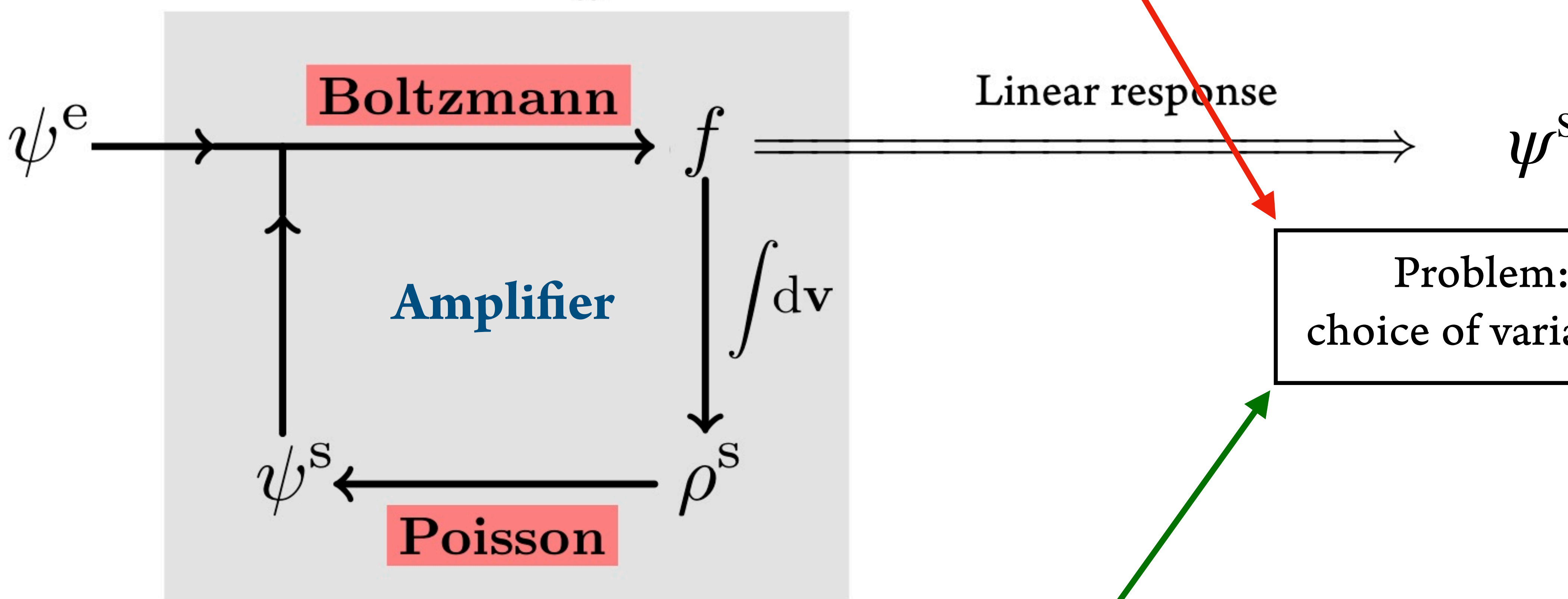
Linear Response Theory

How does a stellar system **respond** to an **external perturbation**?

Linearised CBE

$$\boxed{\frac{\partial f}{\partial t} + \mathbf{\Omega}(\mathbf{J}) \cdot \frac{\partial f}{\partial \boldsymbol{\theta}} - \frac{\partial F}{\partial \mathbf{J}} \cdot \frac{\partial (\psi^{\mathrm{e}} + \psi^{\mathrm{s}})}{\partial \boldsymbol{\theta}} = 0}$$

Easier in $(\boldsymbol{\theta}, \mathbf{J})$



Problem:
choice of variables

Poisson

$$\boxed{\Delta \psi^{\mathrm{s}} = 4\pi G \rho^{\mathrm{s}}}$$

Easier in $(\mathbf{x}, \mathbf{v})$

Projection on a basis *Kalnajs 1976*

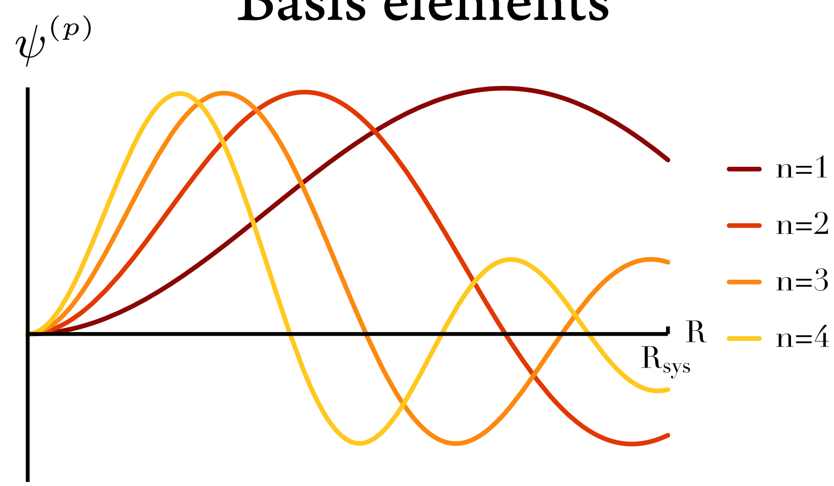
Time Laplace transform $\psi^{\mathrm{e}}(\mathbf{x},\omega)$

Complex frequency $\omega = \omega_0 + \mathrm{i}\,\eta$

Pattern speed ω_0

Growth rate η

Basis elements



The basis **solves the Poisson equation**

$$\psi^{\mathrm{e}}(\mathbf{x},\omega) \longrightarrow \mathbf{b}(\omega)$$

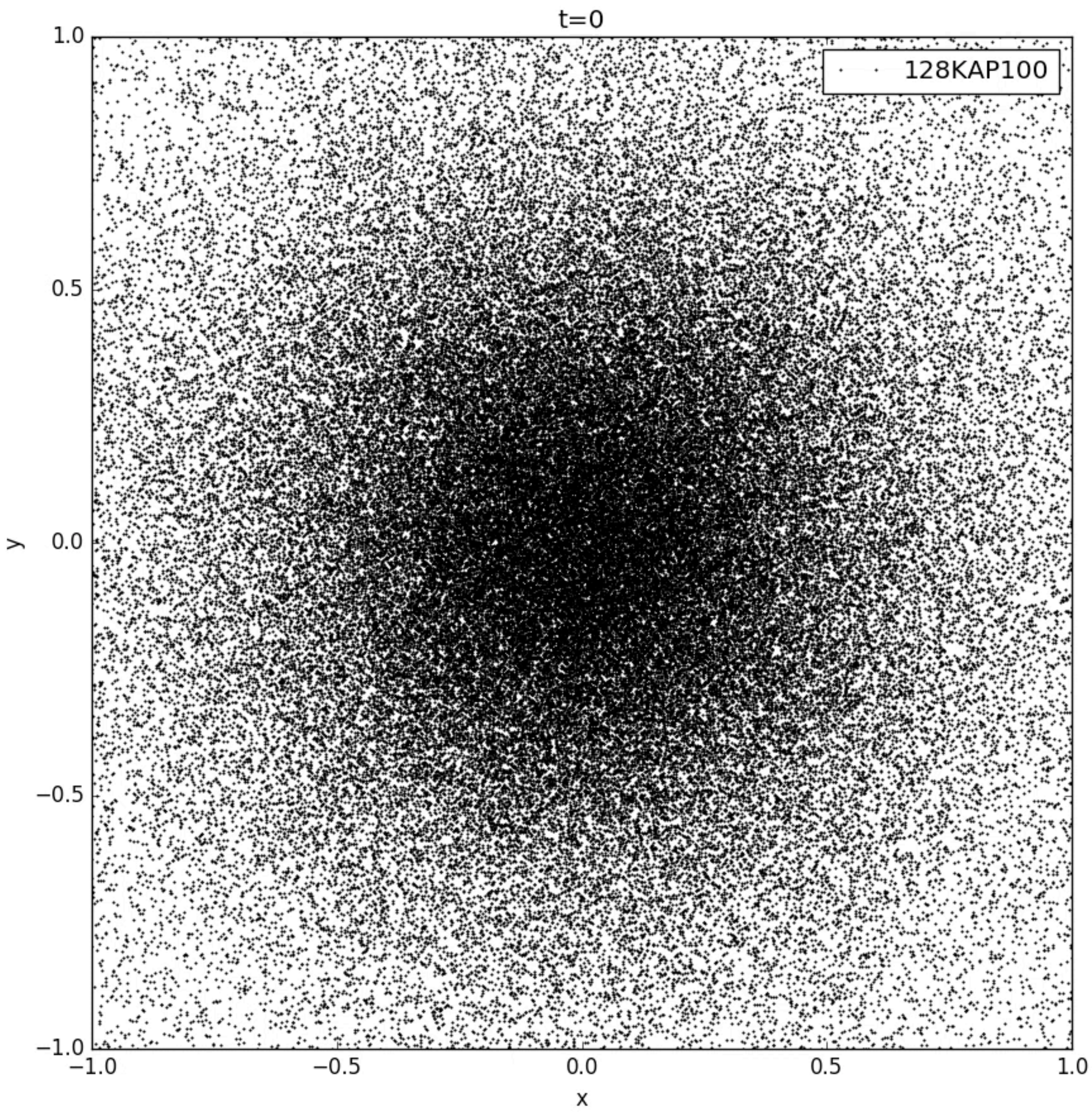
$$\psi^{\mathrm{s}}(\mathbf{x},\omega) \longrightarrow \mathbf{a}(\omega)$$

$\mathbf{M}(\omega)$ Response matrix

$$\mathbf{a}(\omega) = \mathbf{M}(\omega)\cdot[\mathbf{I}-\mathbf{M}(\omega)]^{-1}\cdot\mathbf{b}(\omega) \longrightarrow \text{Linear Response}$$

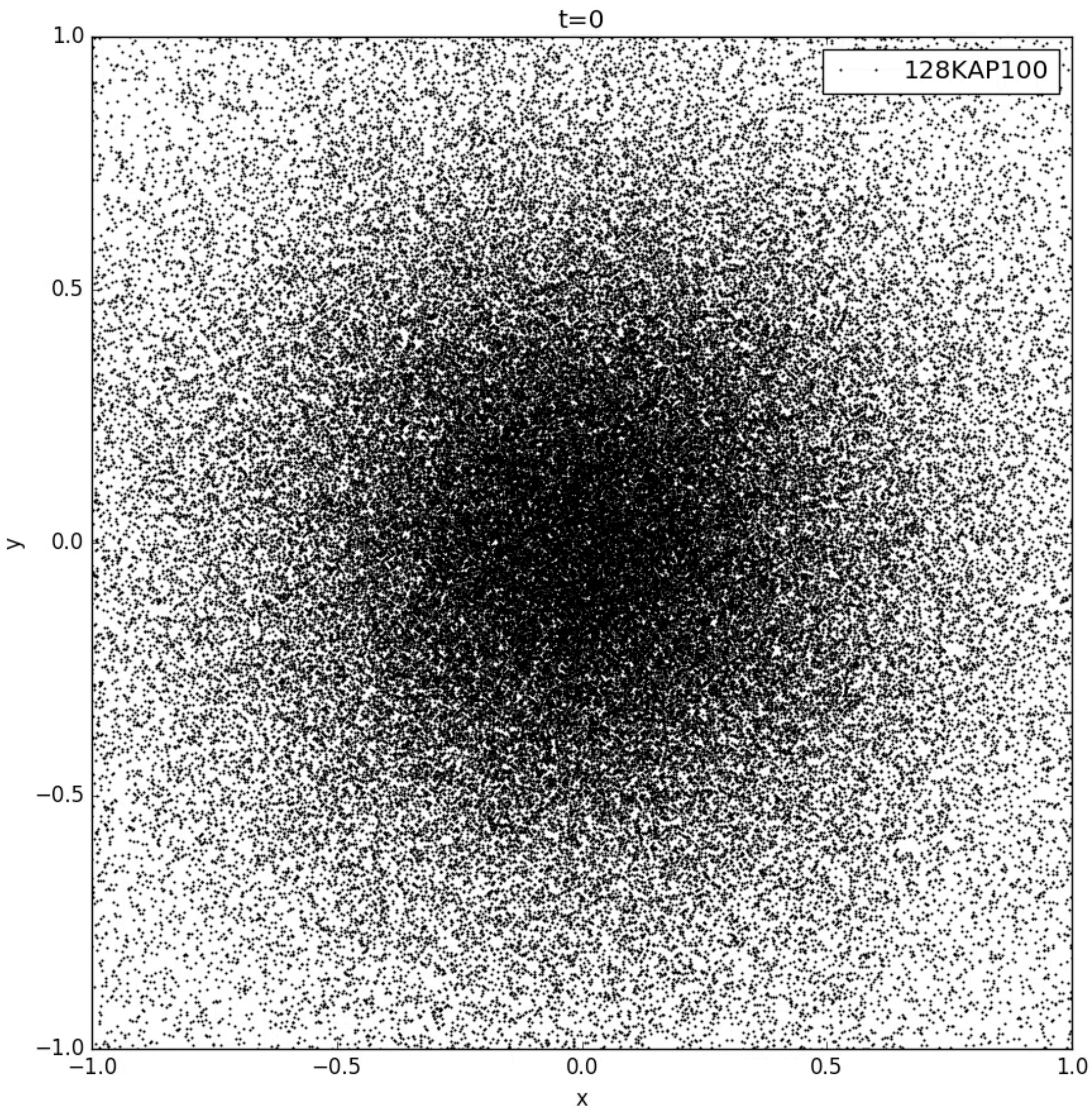
$$\mathbf{a}(\omega) = \mathbf{M}(\omega)\cdot[\mathbf{I}-\mathbf{M}(\omega)]^{-1}\cdot 0^{+} \longrightarrow \textbf{Linear Instabilities}$$

Unstable equilibrium

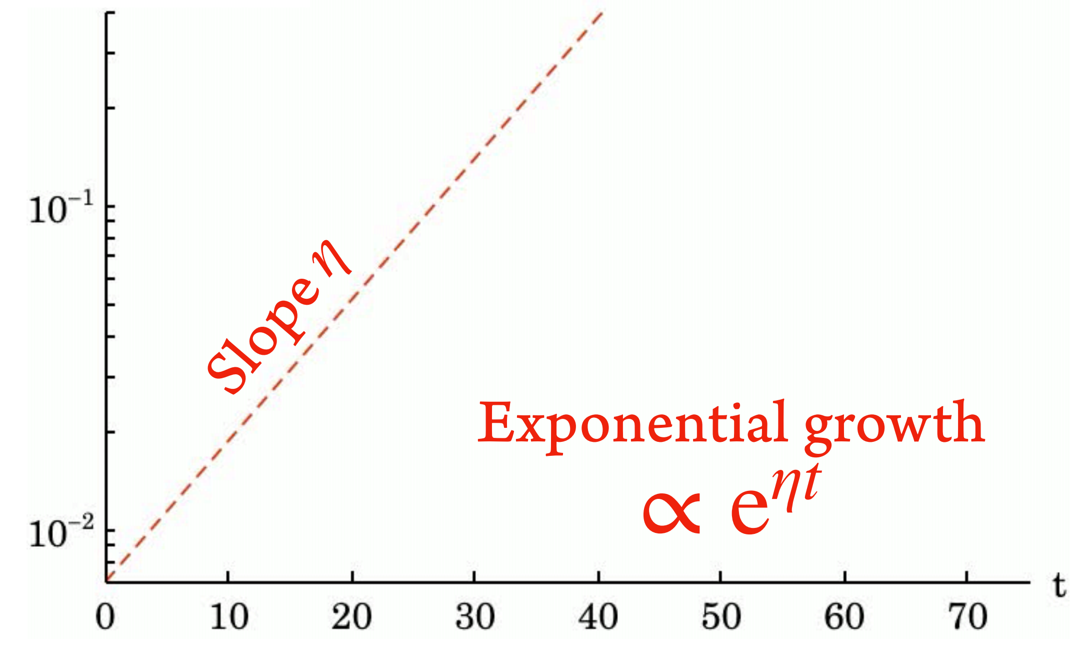


Rozier *et al. 2019*

Unstable equilibrium

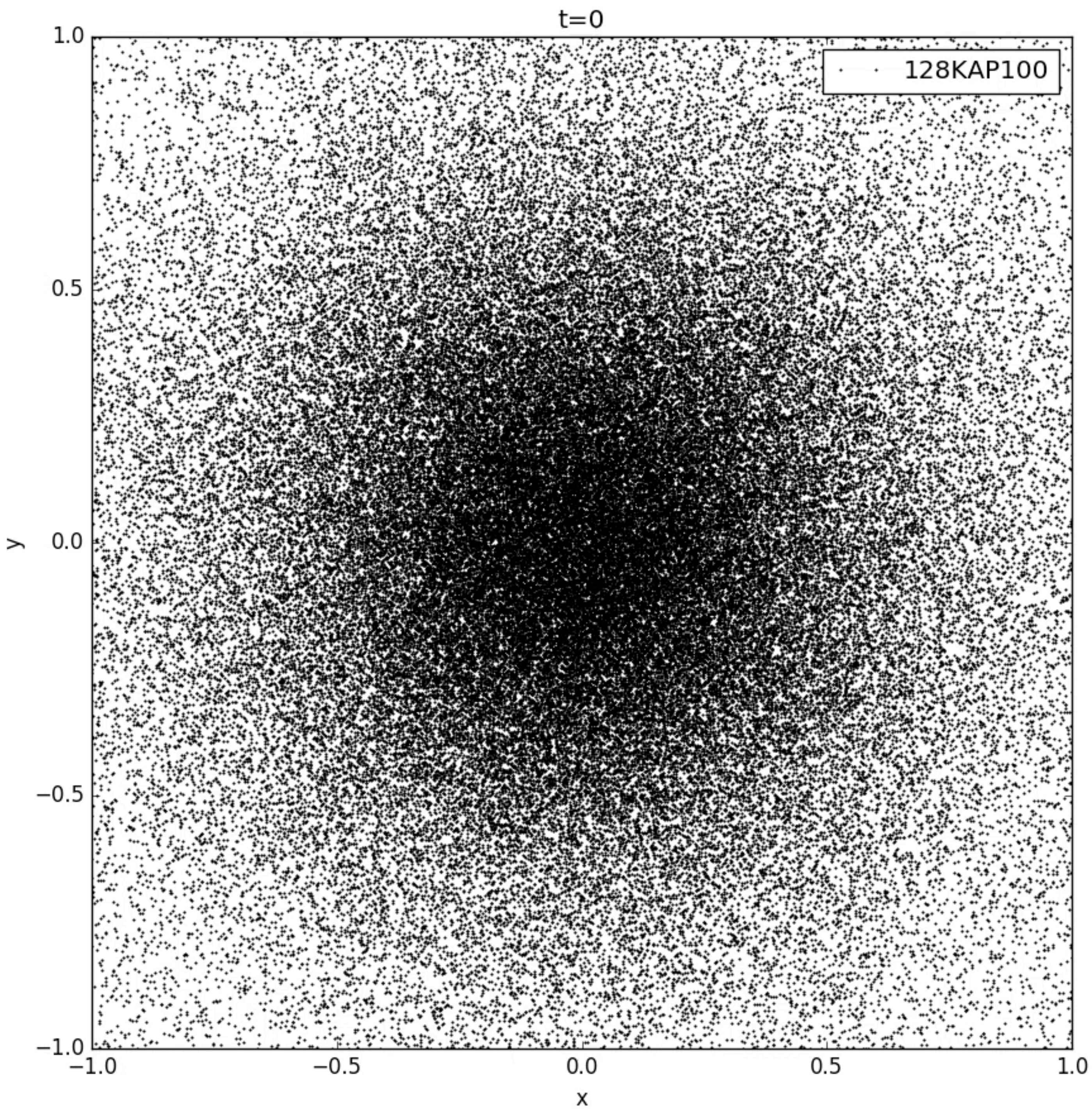


bar **amplitude**

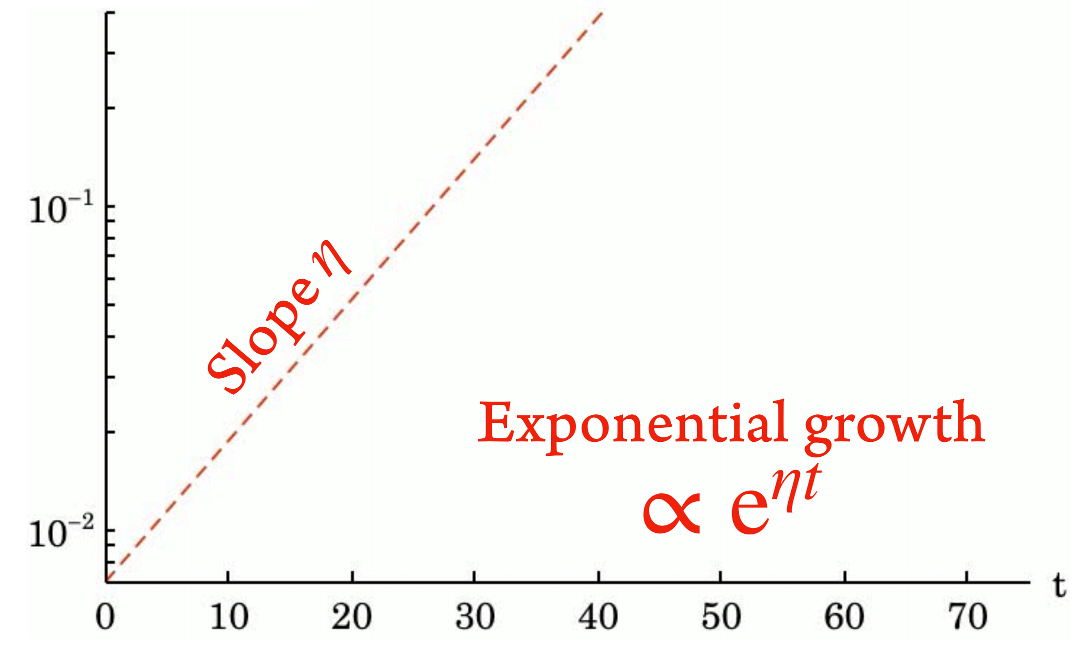


***Rozier** et al. 2019*

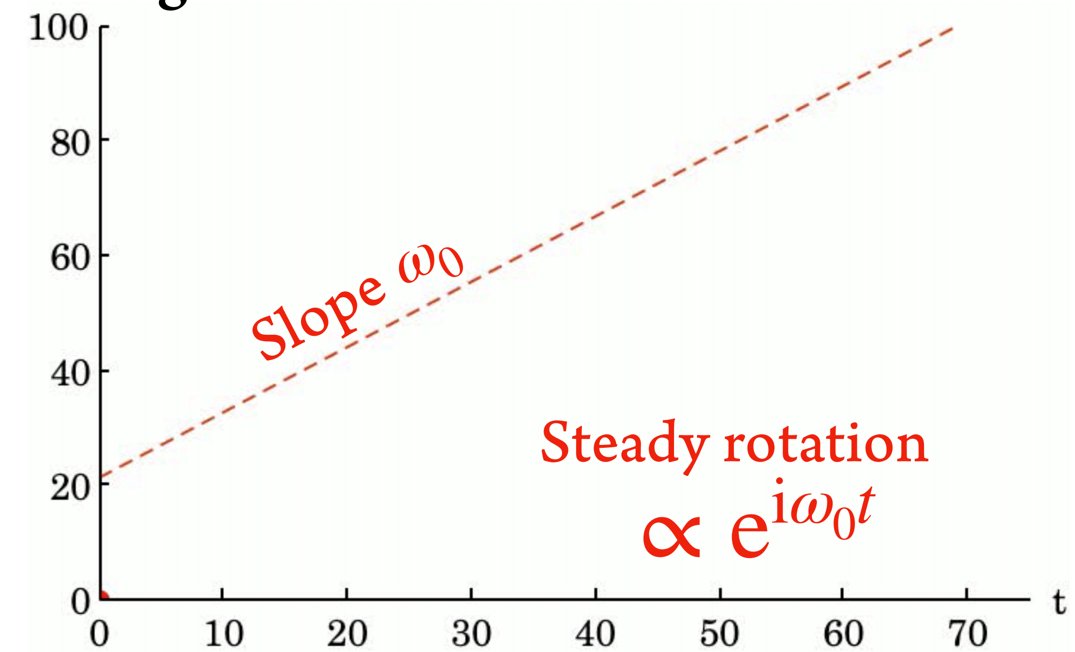
Unstable equilibrium



bar **amplitude**



bar **angle**



Rozier et al. 2019

Modes

Mode = **Self-sustained** response via spontaneous orbital distortions

Condition for a **mode:** $\det[\mathbf{I} - \mathbf{M}(\omega)] = 0$ $\qquad \omega = \omega_0 + \mathrm{i}\,\eta$

Eigenvector $\longrightarrow$ **mode shape** $\qquad \psi^{\mathrm{s}} \propto \mathrm{e}^{\eta t}\, \mathrm{e}^{\mathrm{i}\omega_0 t}$

$\eta < 0$ The mode is **damped**	$\eta = 0$ The mode is **neutral**	$\eta > 0$ The mode is **growing**

INSTABILITY

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Computation of $\mathbf{M}(\omega)$

$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \sum_{\mathbf{n}} \int \mathrm{d}\mathbf{J}\, \frac{\mathbf{n} \cdot \partial F / \partial \mathbf{J}}{\omega - \mathbf{n} \cdot \mathbf{\Omega}}\, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J})\, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

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Sum over **resonance vectors (3D)**

Computation of $\mathbf{M}(\omega)$

$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \boxed{\sum_{\mathbf{n}}} \boxed{\int \mathrm{d}\mathbf{J}} \, \frac{\mathbf{n} \cdot \partial F / \partial \mathbf{J}}{\omega - \mathbf{n} \cdot \mathbf{\Omega}} \, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J}) \, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

Sum over **resonance vectors** (3D)

Integral over **action space** (3D)

Computation of $\mathbf{M}(\omega)$

$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \boxed{\sum_{\mathbf{n}}} \boxed{\int \mathrm{d}\mathbf{J}} \frac{\mathbf{n}\cdot\partial F/\partial \mathbf{J}}{\boxed{\omega - \mathbf{n}\cdot\mathbf{\Omega}}} \psi_{\mathbf{n}}^{(p)*}(\mathbf{J})\, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

Sum over **resonance vectors** (3D)

Integral over **action space** (3D)

Resonant denominator
at the intrinsic frequencies

Computation of $\mathbf{M}(\omega)$

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Sum over **resonance vectors** (3D)

Integral over **action space** (3D)

Resonant denominator
at the intrinsic frequencies

Potential **basis functions**

Computation of $\mathbf{M}(\omega)$

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Sum over **resonance vectors** (3D)

Integral over **action space** (3D)

Resonant denominator
at the intrinsic frequencies

Potential **basis functions**

Gradient of the **distribution function**

Computation of $\mathbf{M}(\omega)$

$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \sum_{\mathbf{n}} \int \mathrm{d}\mathbf{J} \, \frac{\mathbf{n} \cdot \partial F / \partial \mathbf{J}}{\omega - \mathbf{n} \cdot \mathbf{\Omega}} \, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J}) \, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

Sum over **resonance vectors** (3D)	**Truncation** of the sum **Low-order resonances** matter the most
Integral over **action space** (3D) **Resonant denominator** at the intrinsic frequencies	
Potential **basis functions**	
Gradient of the **distribution function**	

Computation of $\mathbf{M}(\omega)$

$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \sum_{\mathbf{n}} \int \mathrm{d}\mathbf{J} \, \frac{\mathbf{n} \cdot \partial F / \partial \mathbf{J}}{\omega - \mathbf{n} \cdot \mathbf{\Omega}} \, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J}) \, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

Sum over **resonance vectors (3D)**	**Truncation** of the sum **Low-order resonances** matter the most
Integral over **action space (3D)** **Resonant denominator** at the intrinsic frequencies	**Computing the integral** **Tailor made approximations** to account for the **resonant denominator**
Potential **basis functions**	
Gradient of the **distribution function**	

Computation of $\mathbf{M}(\omega)$

$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \sum_{\mathbf{n}} \int \mathrm{d}\mathbf{J} \, \frac{\mathbf{n} \cdot \partial F / \partial \mathbf{J}}{\omega - \mathbf{n} \cdot \mathbf{\Omega}} \, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J}) \, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

Sum over **resonance vectors** (3D)	**Truncation** of the sum **Low-order resonances** matter the most
Integral over **action space** (3D) **Resonant denominator** at the intrinsic frequencies	**Computing the integral** **Tailor made approximations** to account for the **resonant denominator**
Potential **basis functions**	**Computing the harmonic transform of the basis** $\psi^{(p)}(\mathbf{x}) \to \psi^{(p)}(\mathbf{J}, \boldsymbol{\theta}) \to \psi_{\mathbf{n}}^{(p)}(\mathbf{J})$ **Runge-Kutta scheme** to compute the **nested integrals** Gain in performance: about 100
Gradient of the **distribution function**	

Computation of $\mathbf{M}(\omega)$

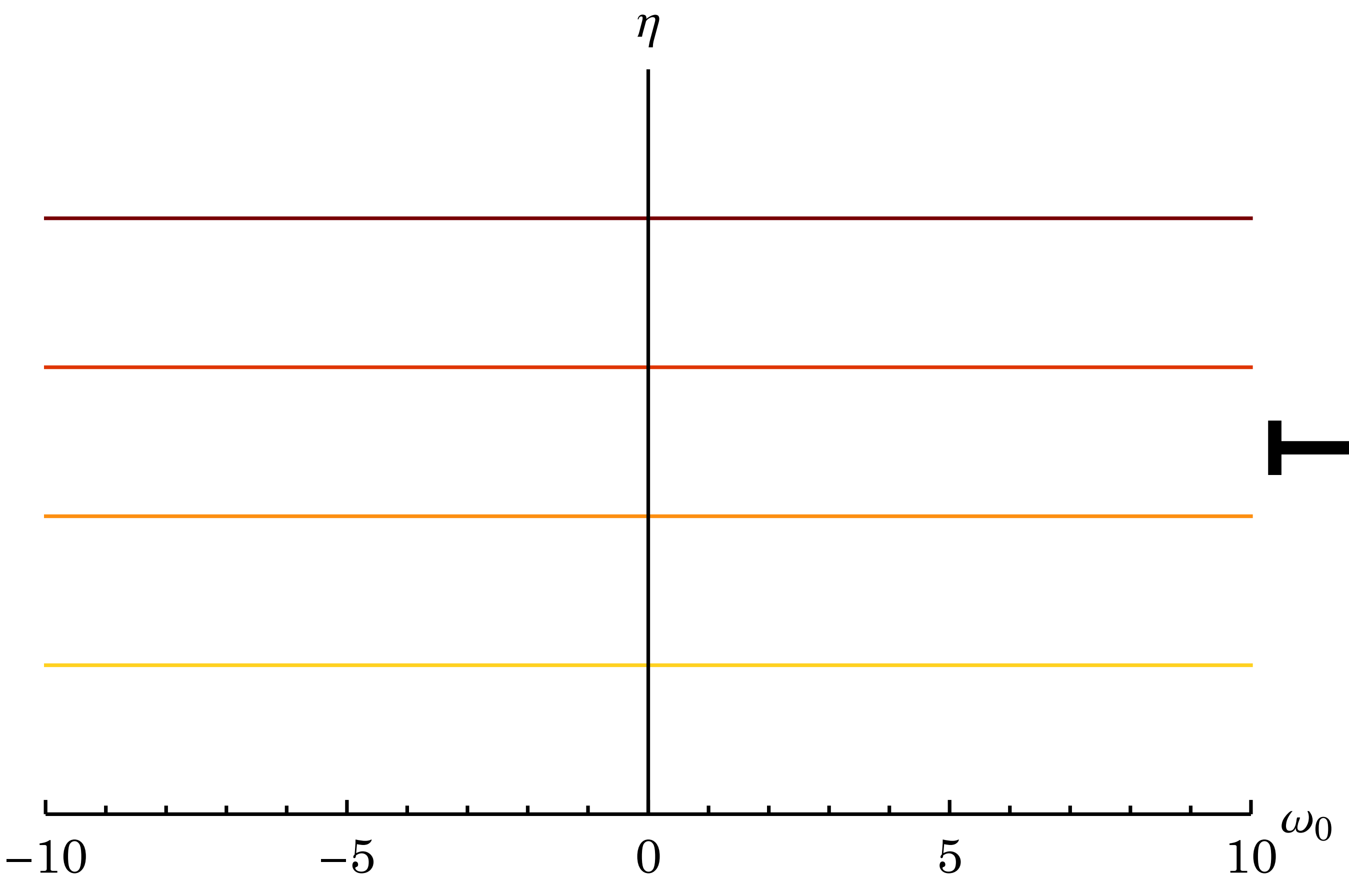
$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \sum_{\mathbf{n}} \int \mathrm{d}\mathbf{J} \, \frac{\mathbf{n}\cdot \partial F/\partial \mathbf{J}}{\omega - \mathbf{n}\cdot\mathbf{\Omega}} \, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J})\, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

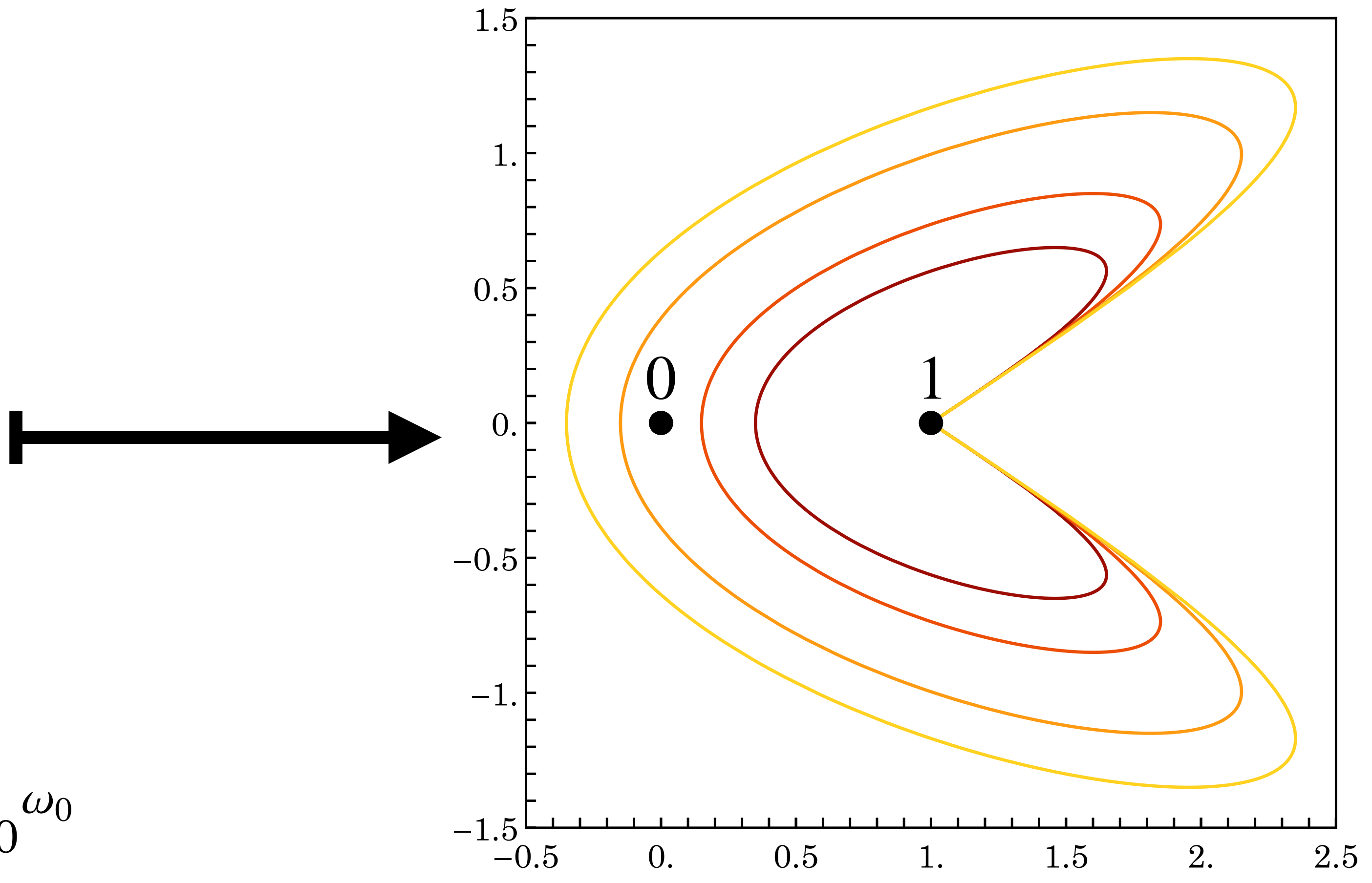
Sum over **resonance vectors** (3D)	**Truncation** of the sum **Low-order resonances** matter the most
Integral over **action space** (3D) **Resonant denominator** at the intrinsic frequencies	**Computing the integral** **Tailor made approximations** to account for the **resonant denominator**
Potential **basis functions**	**Computing the harmonic transform of the basis** $\psi^{(p)}(\mathbf{x}) \to \psi^{(p)}(\mathbf{J},\boldsymbol{\theta}) \to \psi_{\mathbf{n}}^{(p)}(\mathbf{J})$ **Runge-Kutta scheme** to compute the **nested integrals** Gain in performance: about 100
Gradient of the **distribution function**	**Rotating** cluster No rotation: $F(J_r, L)$ With rotation: $F(J_r, L, \mathbf{L_z})$

Nyquist contours

$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \sum_{\mathbf{n}} \int \mathrm{d}\mathbf{J} \, \frac{\mathbf{n}\cdot\partial F/\partial \mathbf{J}}{\omega - \mathbf{n}\cdot\mathbf{\Omega}} \, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J}) \, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

Nyquist diagrams $\qquad \omega_0 \mapsto \det[\mathbf{I} - \mathbf{M}(\omega_0 + \mathrm{i}\,\eta)]$

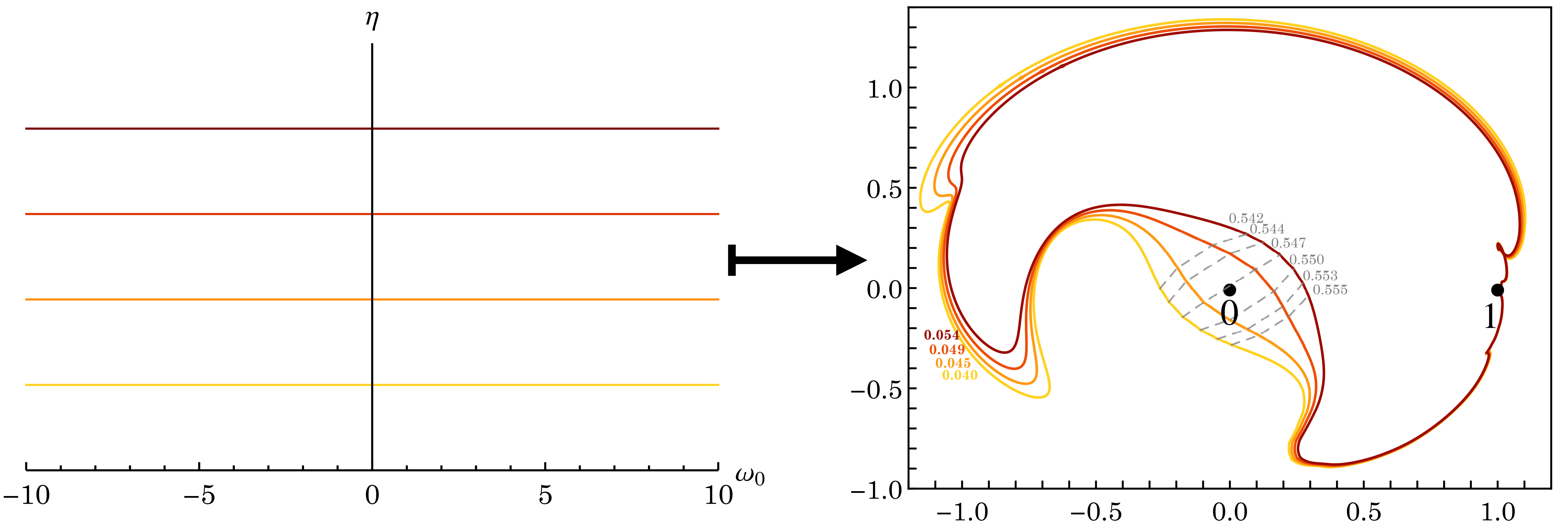




Nyquist contours

$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \sum_{\mathbf{n}} \int \mathrm{d}\mathbf{J}\, \frac{\mathbf{n}\cdot\partial F/\partial\mathbf{J}}{\omega - \mathbf{n}\cdot\mathbf{\Omega}}\, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J})\, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

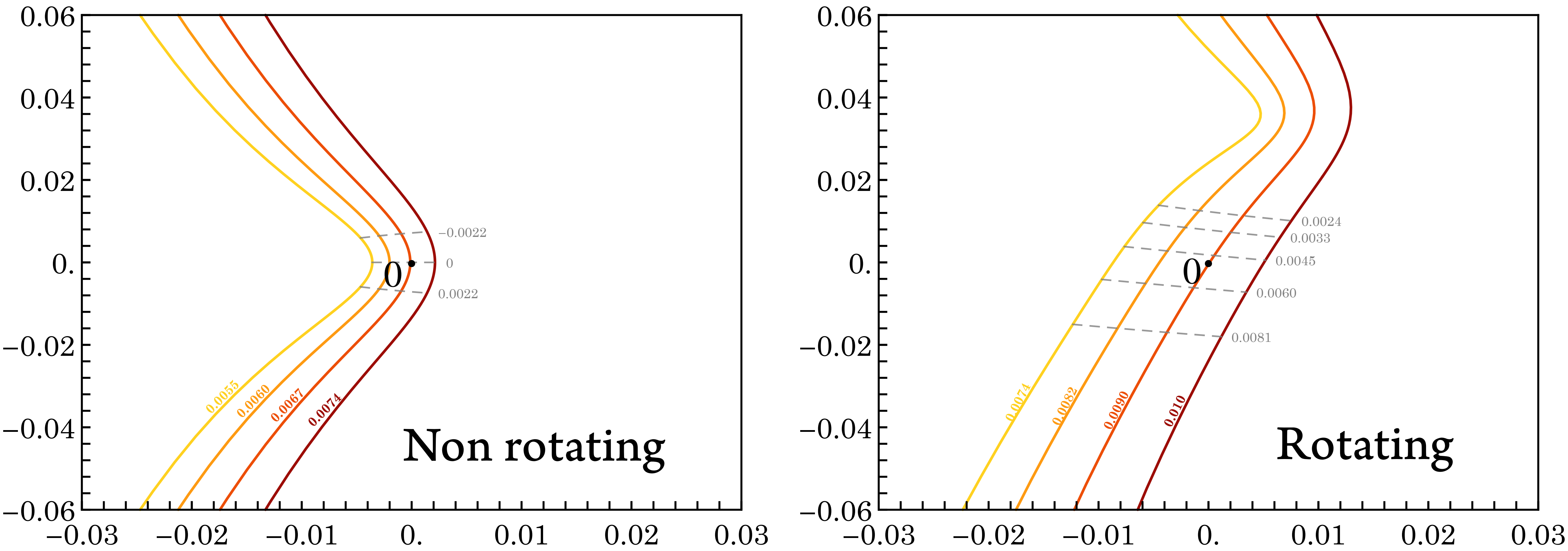
Nyquist diagrams $\quad \omega_0 \mapsto \det[\mathbf{I} - \mathbf{M}(\omega_0 + \mathrm{i}\,\eta)]$



Influence of rotation

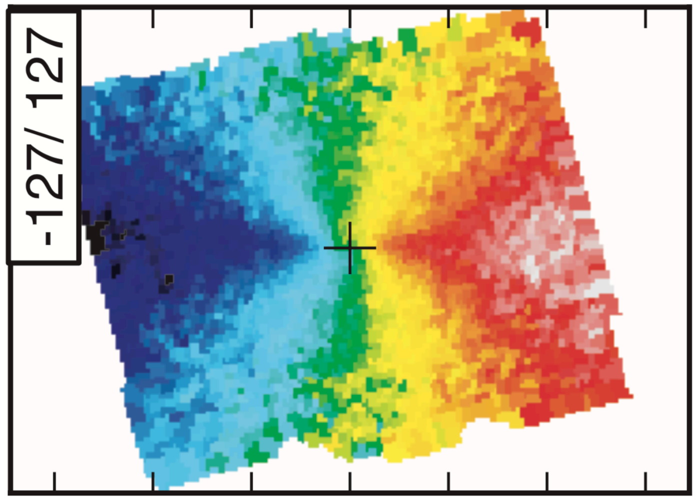
$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \sum_{\mathbf{n}} \int \mathrm{d}\mathbf{J}\, \frac{\mathbf{n}\cdot \partial F/\partial \mathbf{J}}{\omega - \mathbf{n}\cdot\mathbf{\Omega}}\, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J})\, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

Nyquist diagrams $\quad \omega_0 \mapsto \det[\mathbf{I} - \mathbf{M}(\omega_0 + \mathrm{i}\,\eta)]$



Outline of the method

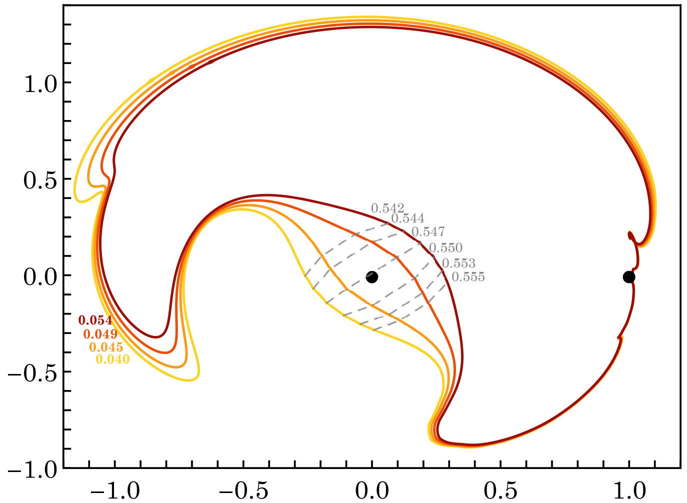
System $F(\mathbf{J})$



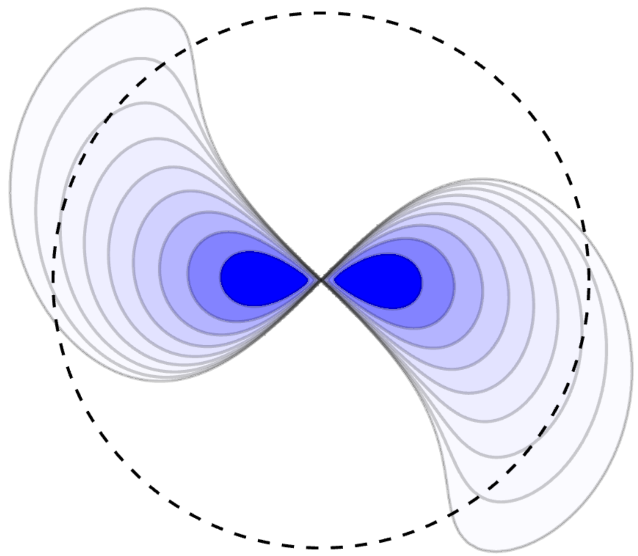
Response matrix

$$\mathbf{M}(\omega)$$

Nyquist contours



Measurements



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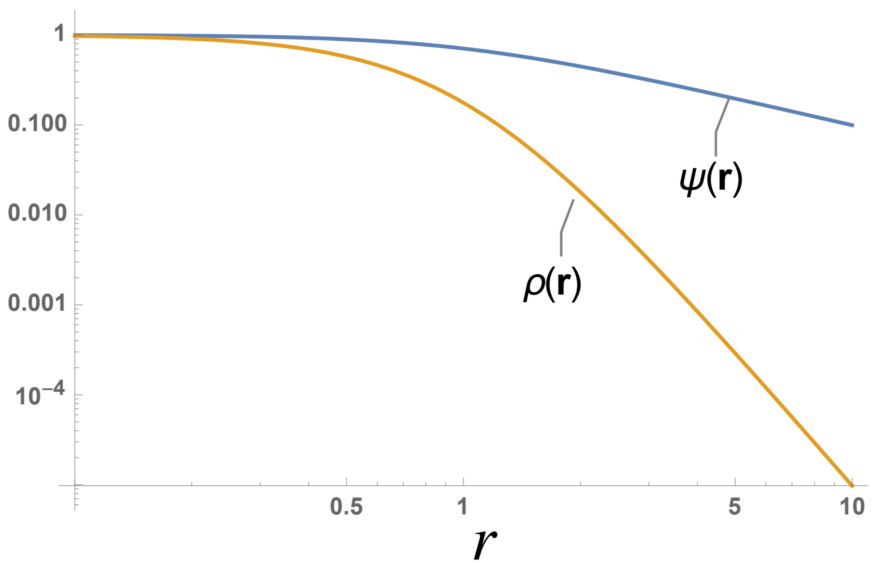
A series of equilibria

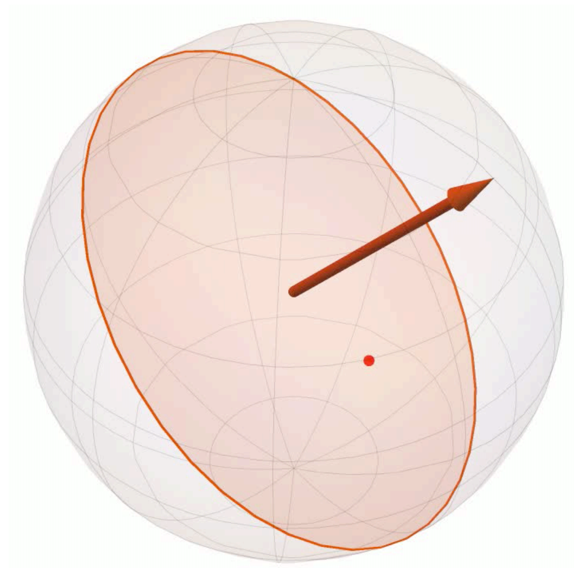
Plummer potential

Plummer 1911



Rotation: "Lynden-Bell's demon"

Lynden-Bell 1962

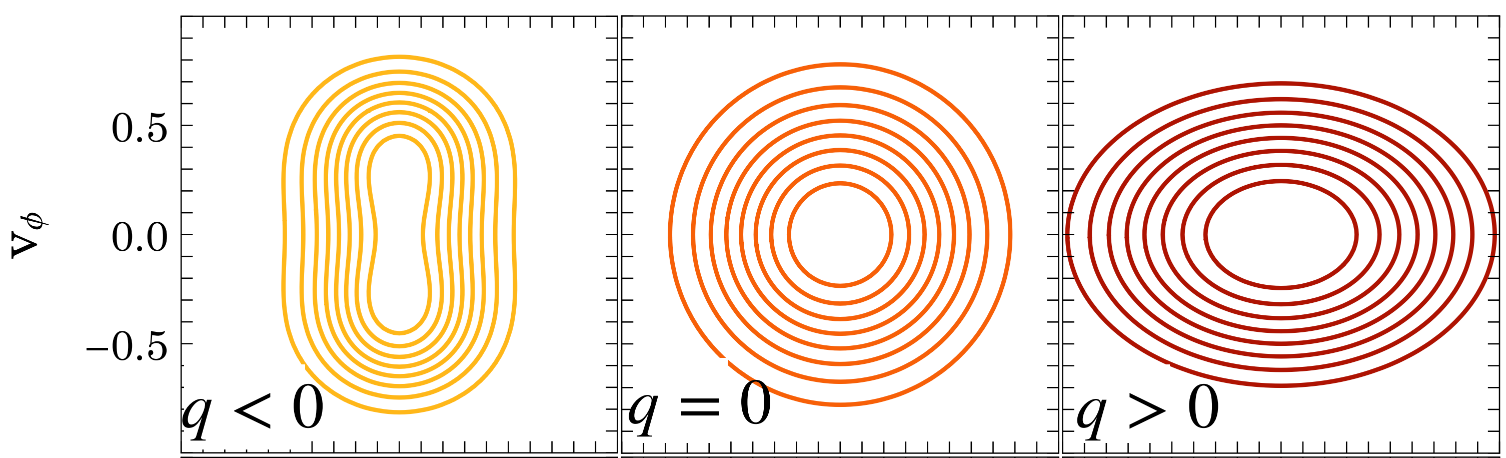


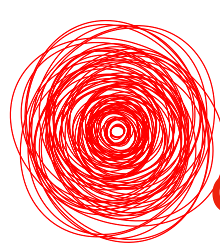
$\alpha = 0$: no rotation

$\alpha = 1$: maximum rotation

Anisotropic family of DFs

Dejonghe 1987



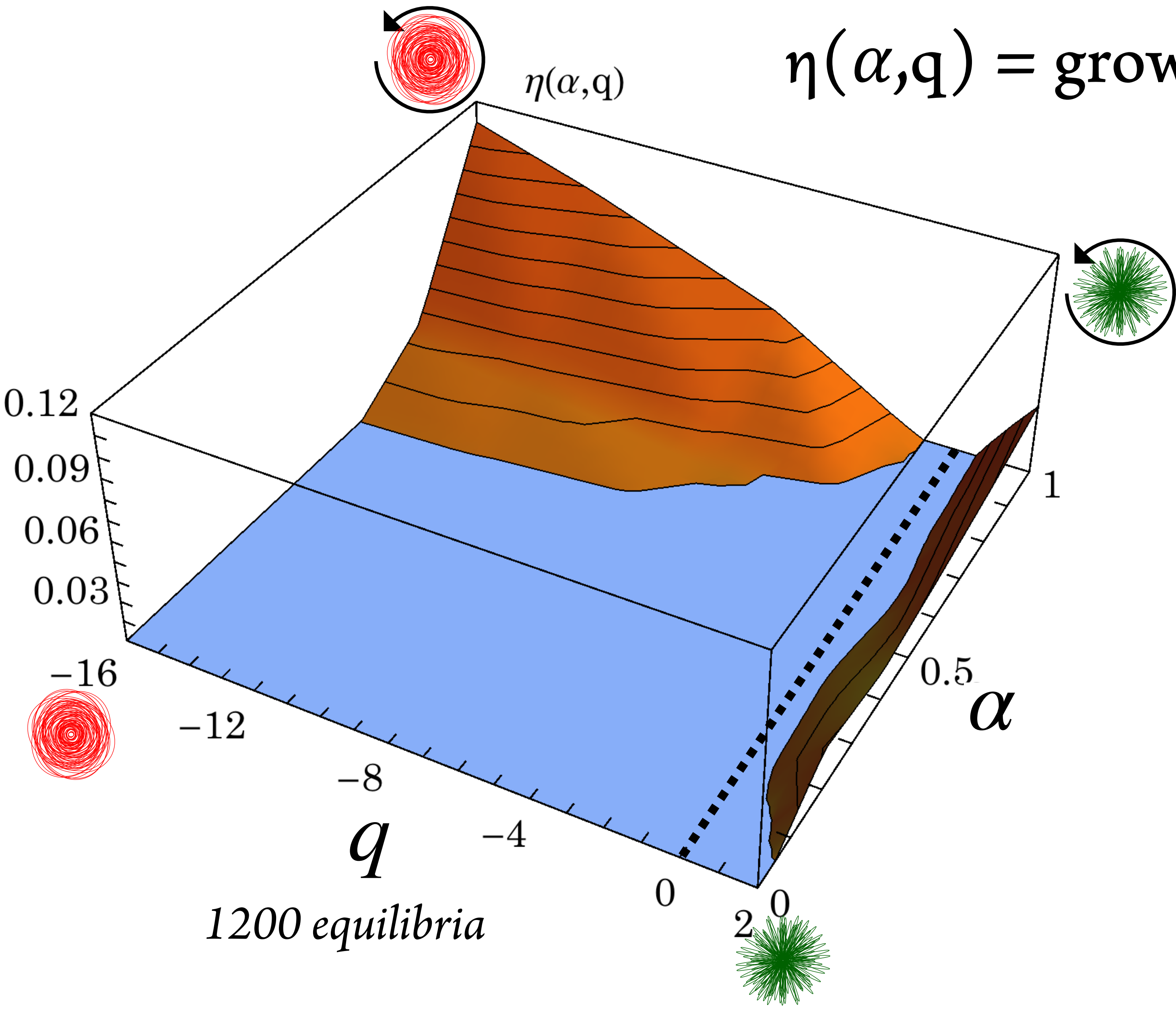


Circular

Radial

Mapping the stability landscape

$\eta(\alpha, q)$ = growth rate



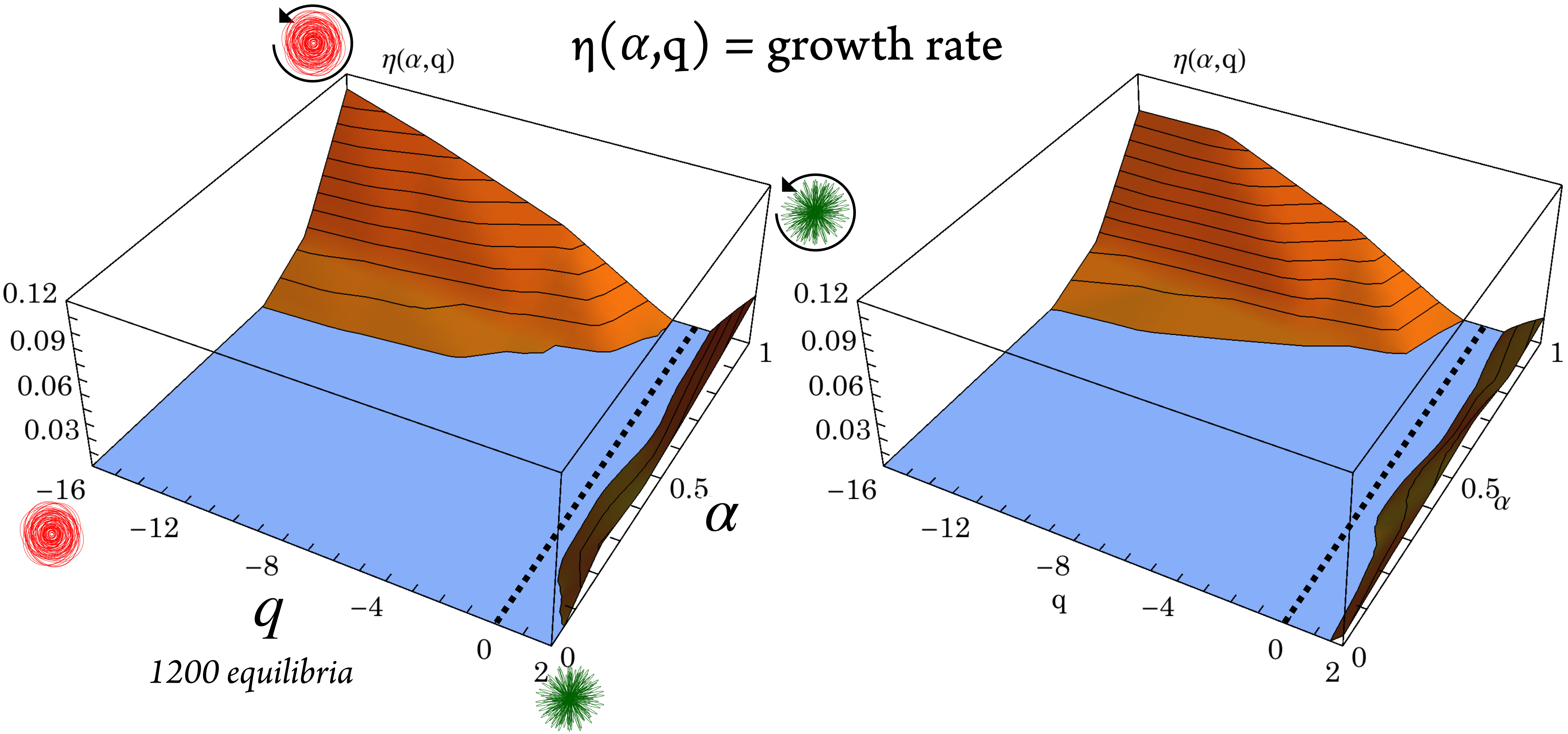
1200 equilibria

Linear Response Theory

q: anisotropy $\qquad$ α: rotation

Mapping the stability landscape

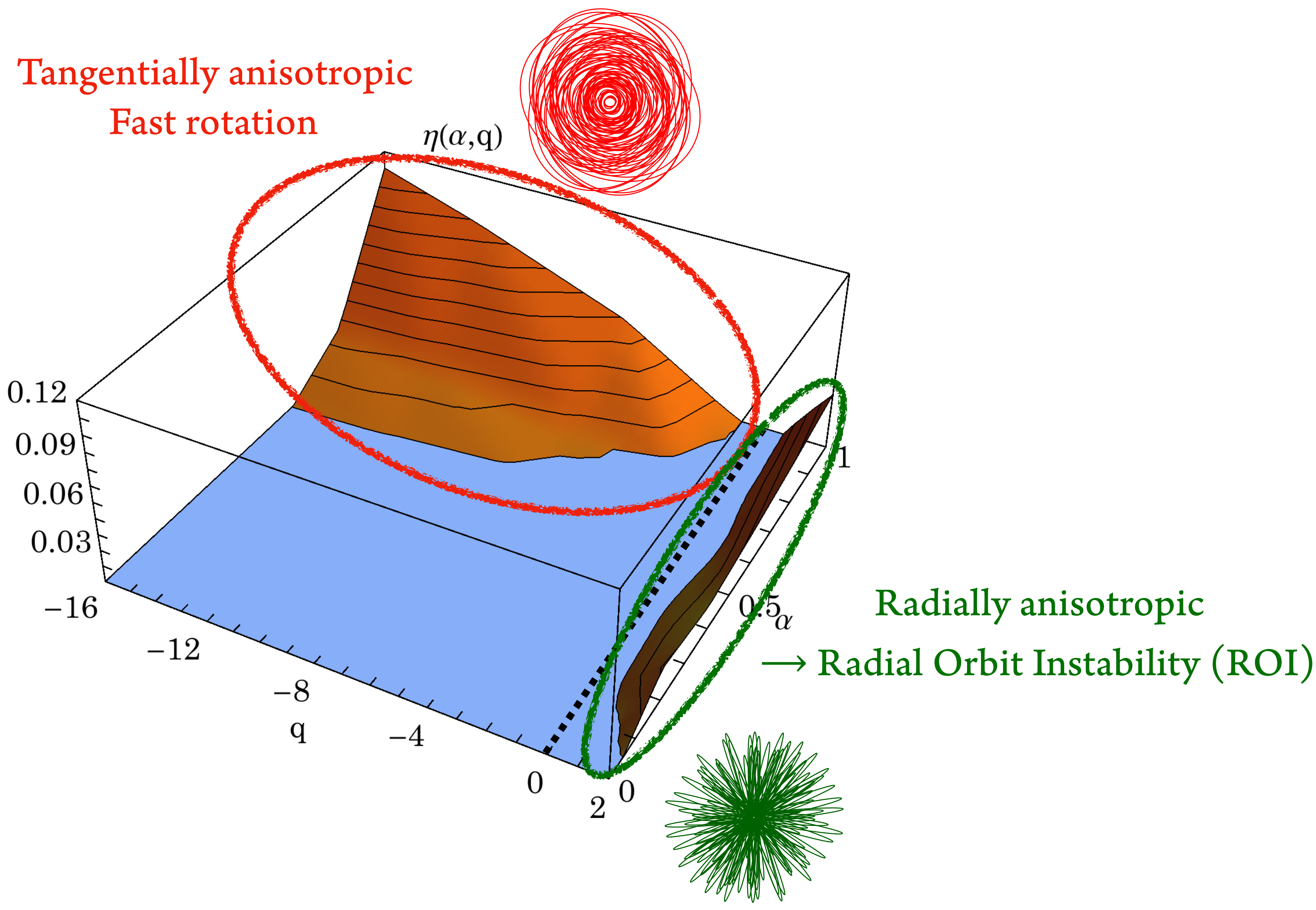
$\eta(\alpha,q)$ = growth rate



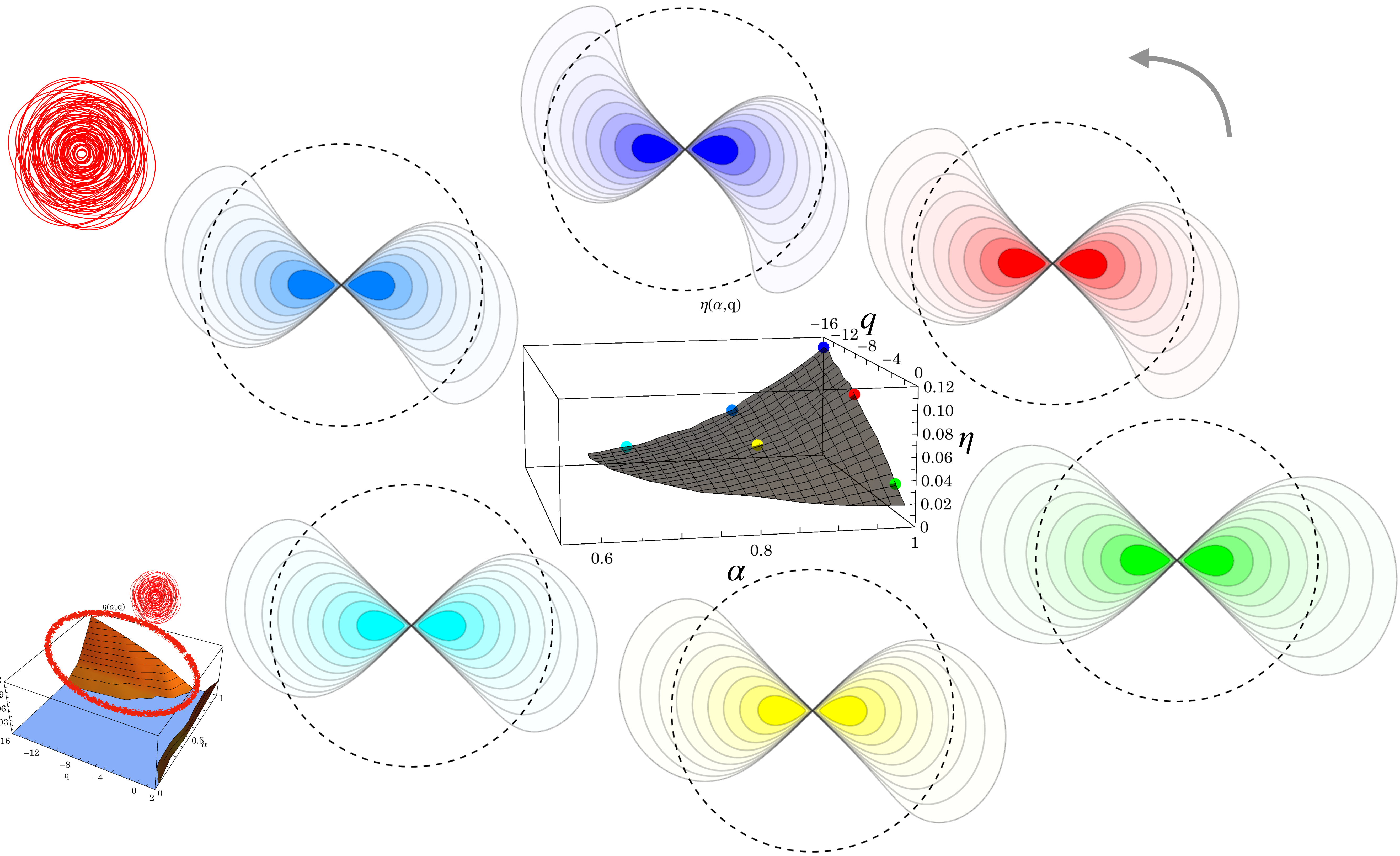
Linear Response Theory **N-body simulations**

q: anisotropy α: rotation

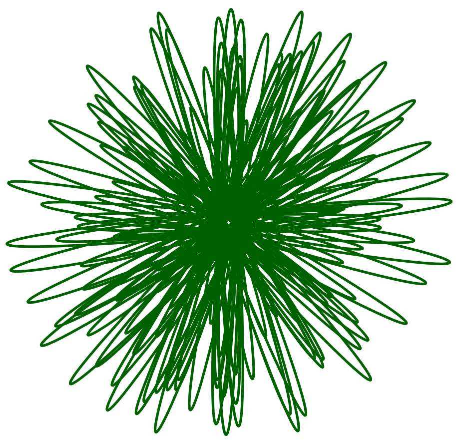
Results: Stability mapping

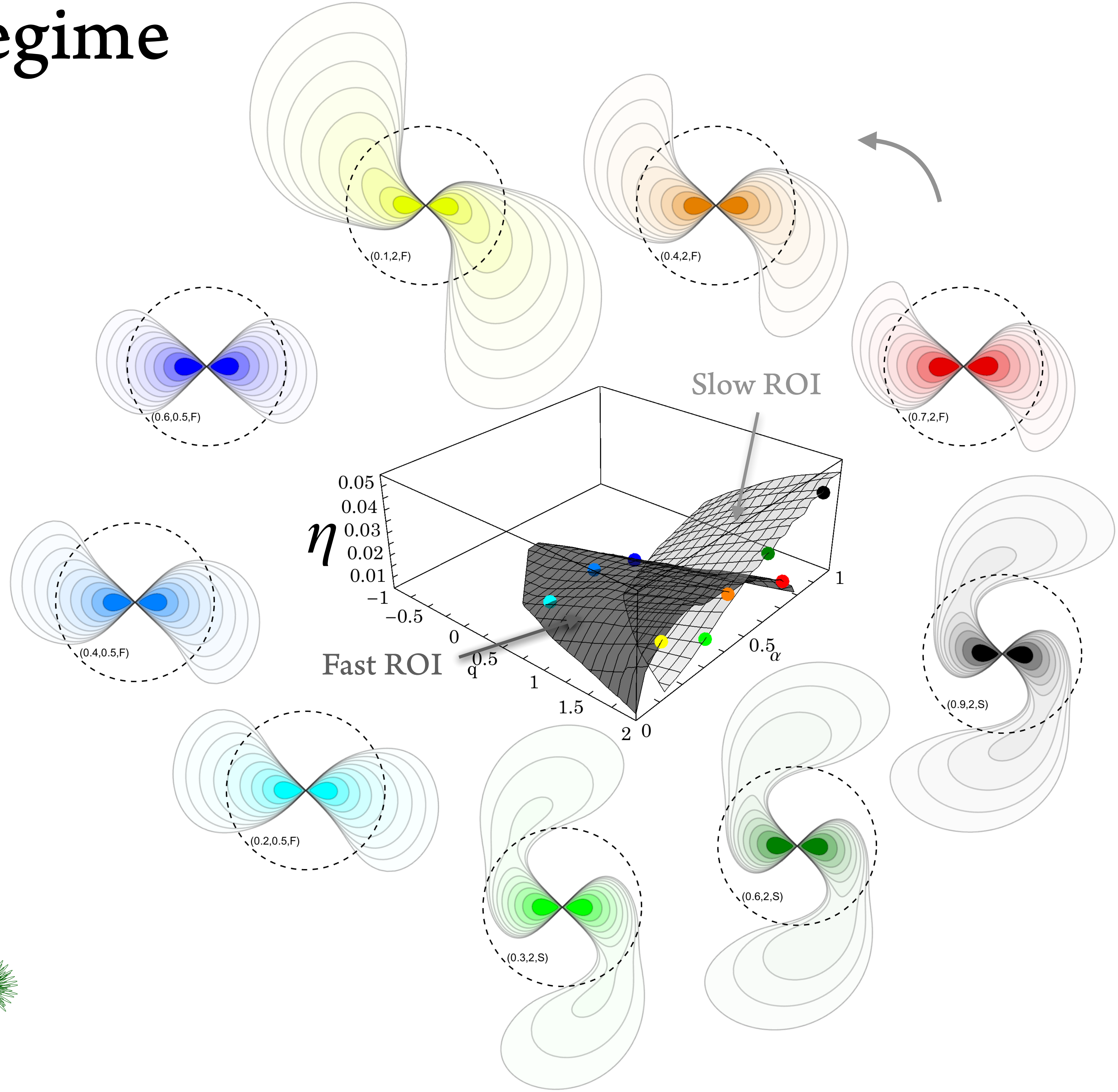


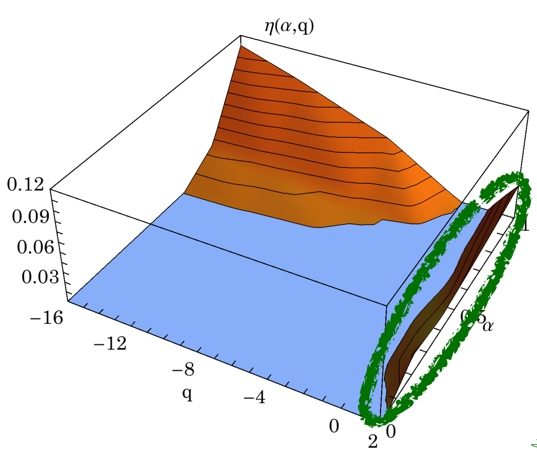
Tangential regime

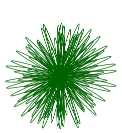


Radial regime









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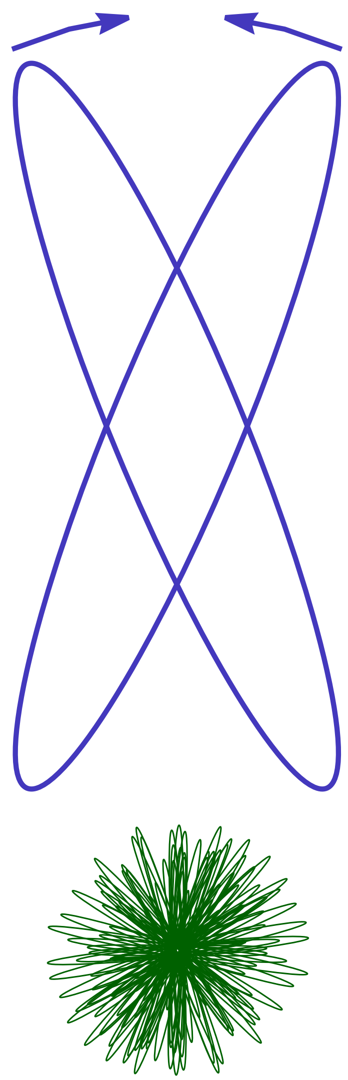
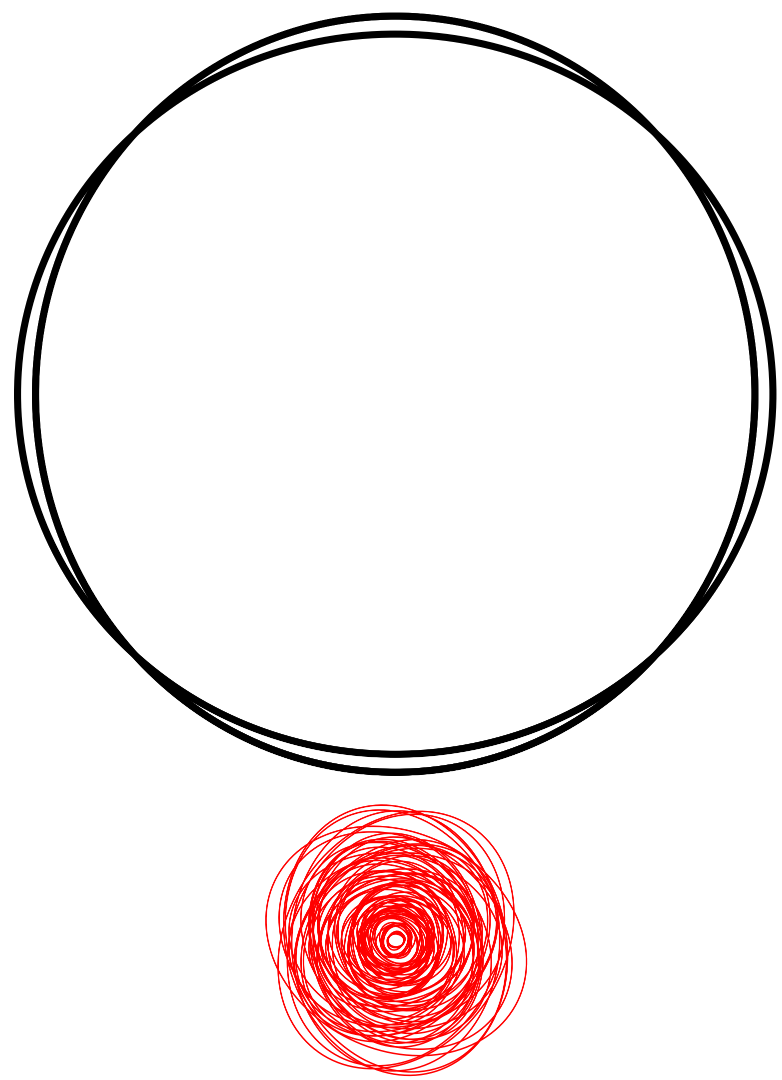
3 resonant processes

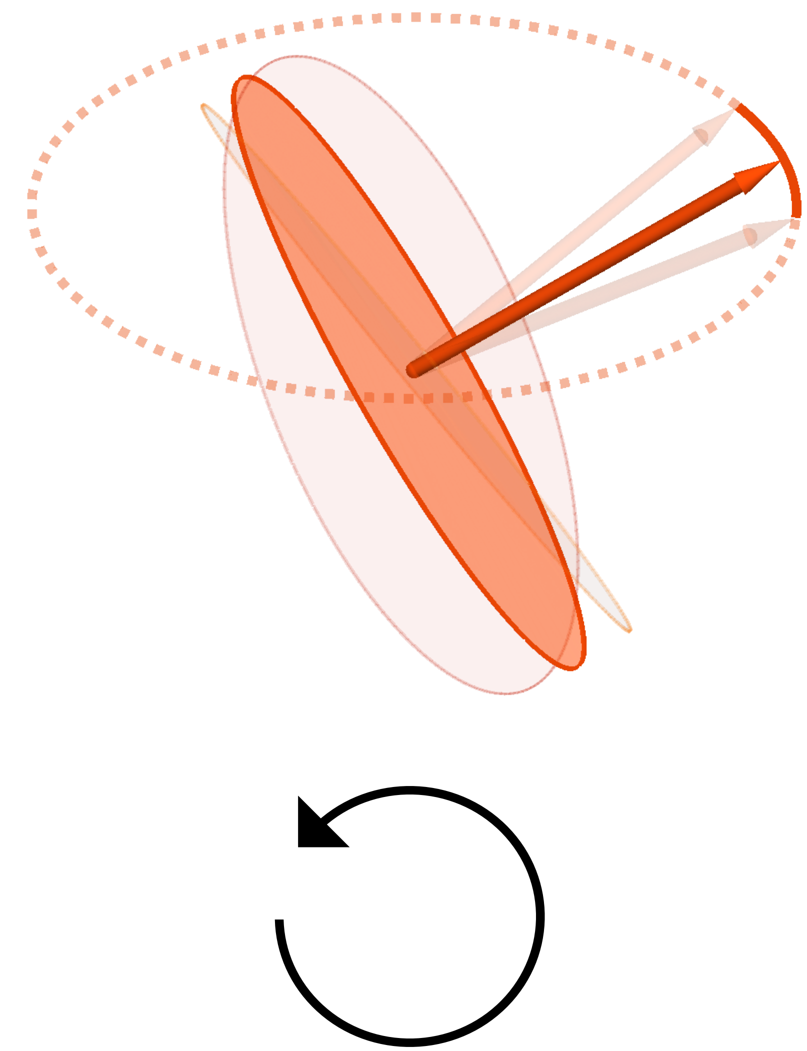
Radial orbit
instability



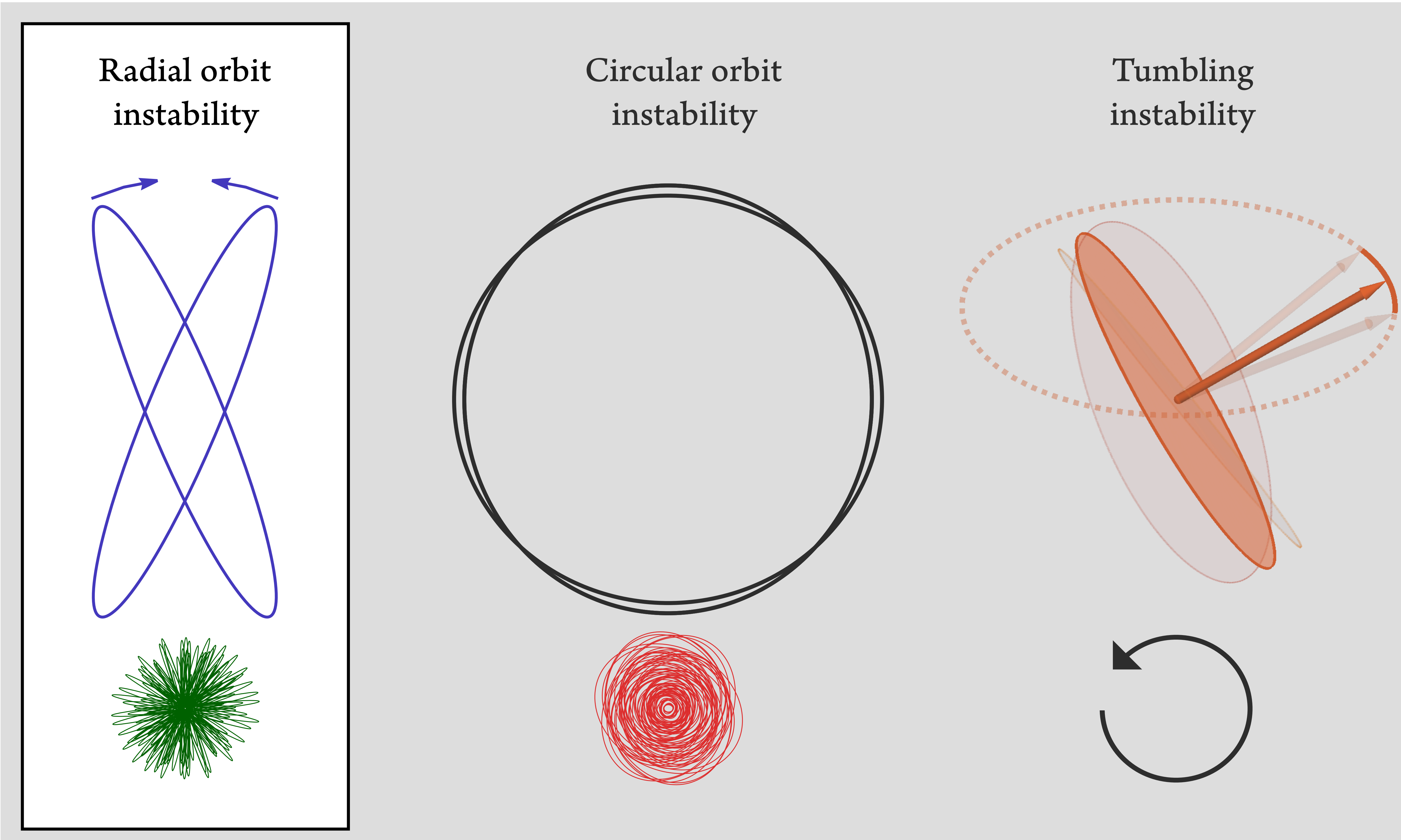
Circular orbit
instability



Tumbling
instability



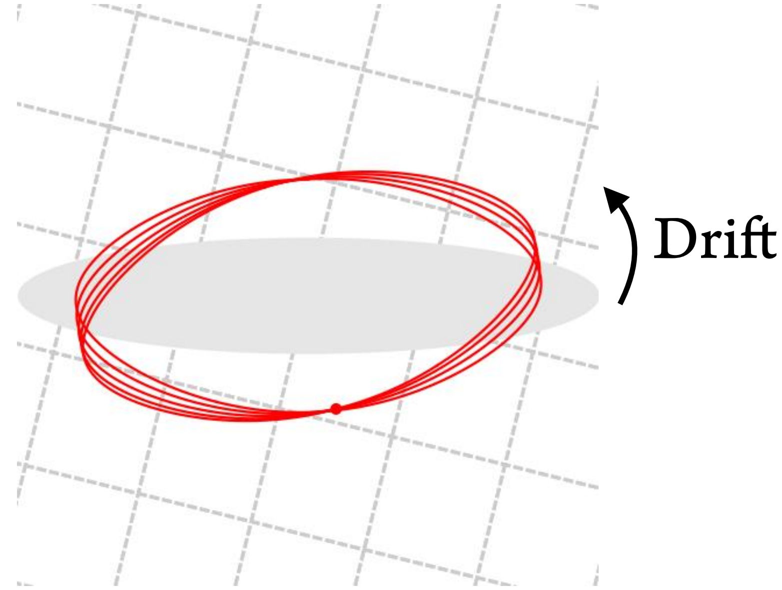
3 resonant processes



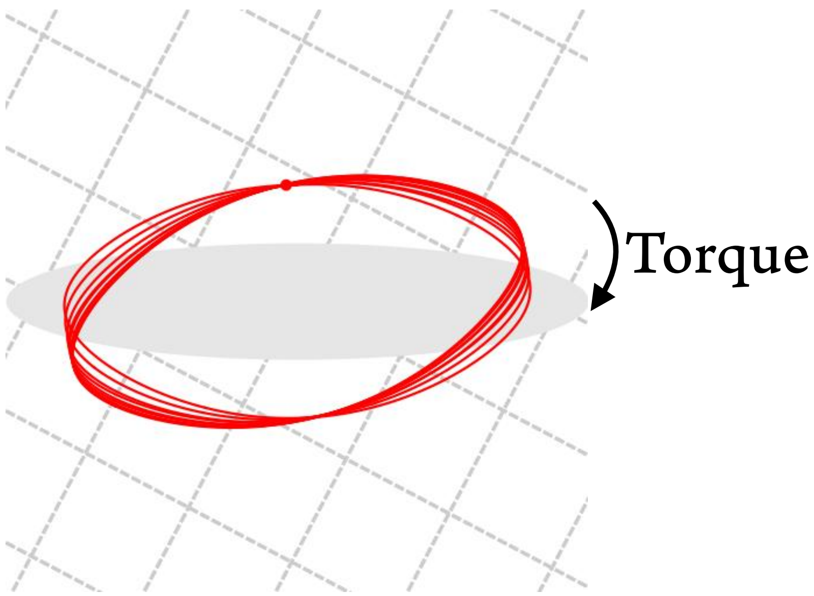
Radial orbit instability

Antonov 1973, Hénon 1973

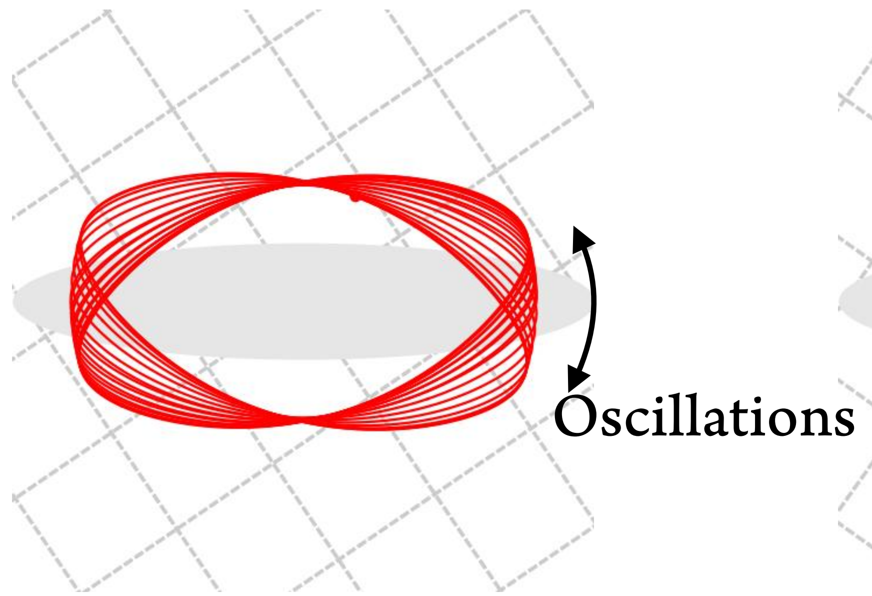
Quasi-resonant orbit at the inner Lindblad resonance



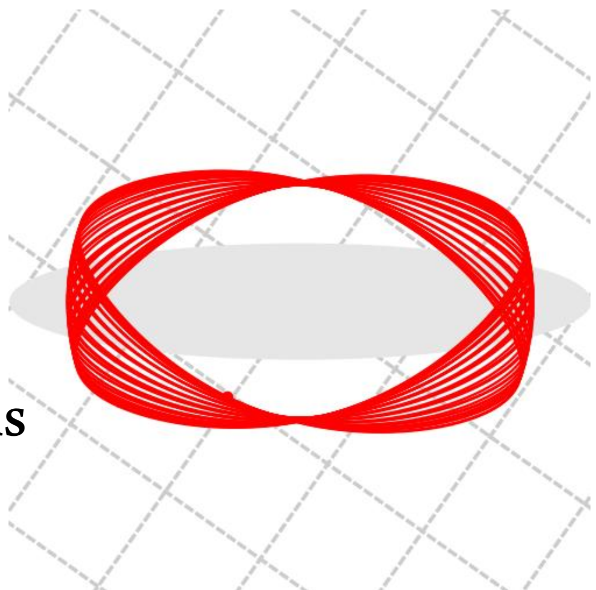
Torque bar ⟶ orbit



Oscillation



Trapped orbits ⟶ bar enhanced



gravitational torque VS **azimuthal pressure**

Restricted matrix method

ROI

Relies on the ILR

$(n_1, n_2, n_3) = (-1,2,2)$

Occurs in **radial** systems

COI

Relies on the ILR

$(n_1, n_2, n_3) = (-1,2,2)$

Occurs in **tangential** systems

TI

Relies on the TR

$(n_1, n_2, n_3) = (0,0,2)$

Occurs in **rotating** systems

Method to identify the **resonant processes** sourcing **instabilities**

$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \boxed{\sum_{(n_1,n_2,n_3)}} \int \mathrm{d}\mathbf{J} \, \frac{\mathbf{n} \cdot \partial F / \partial \mathbf{J}}{\omega - \mathbf{n} \cdot \boldsymbol{\Omega}} \, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J}) \, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

Isolate the contribution from **each resonance**

ILR
$(n_1, n_2, n_3) = (-1,2,2)$

TR
$(n_1, n_2, n_3) = (0,0,2)$

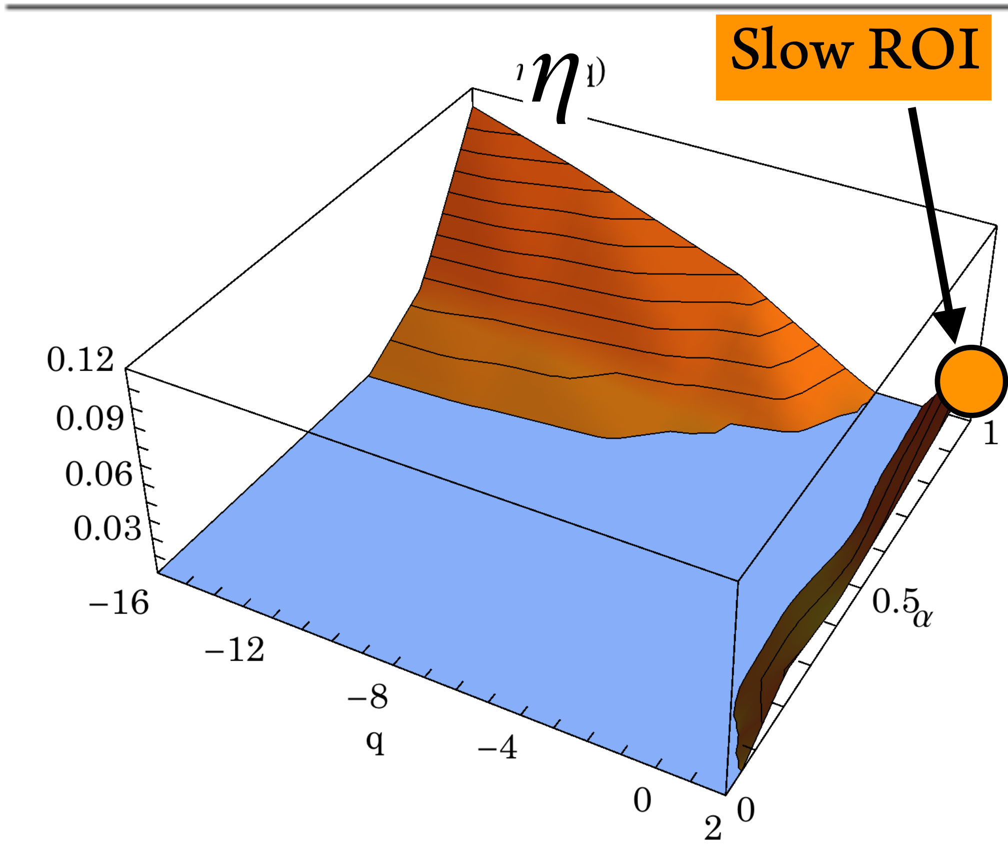
Other resonances
(n_1, n_2, n_3)

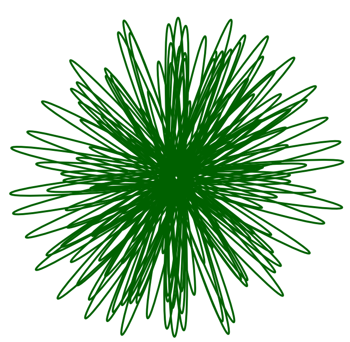
$\longrightarrow$ Perform the **full mode search**, but selecting only **a fraction of the resonant terms**

Application to the rotating ROI

$\mathcal{I}$	$\eta(\alpha = 1)$
Reference	0.040
$(-1, 2), (0, 0)$	0.040
$\mathcal{I}_0 \setminus (-1, 2)$	$< 10^{-4}$
$\mathcal{I}_0 \setminus (0, 0)$	0.015
$(-1, 2)$	0.008
$(0, 0)$	$< 10^{-4}$

*Breen, **Rozier** et al. 2020, submitted to MNRAS*





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$(-1, 2)$	0.008
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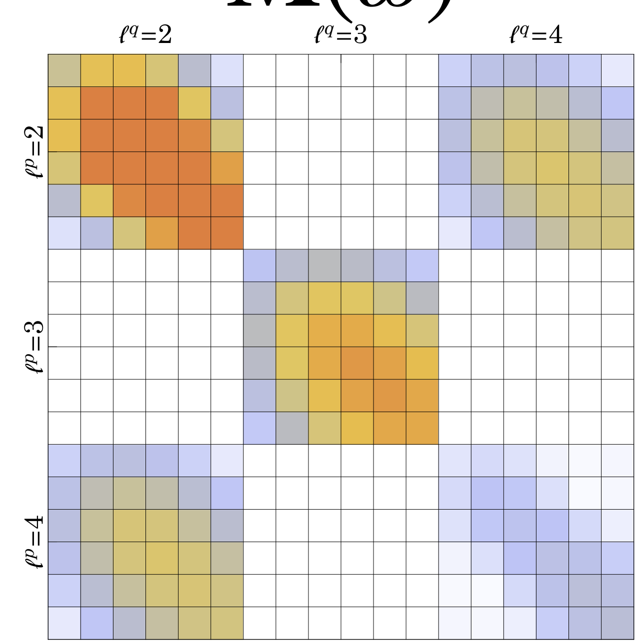
Reference = Full matrix

$\mathbf{M}(\omega)$



Application to the rotating ROI

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$(-1, 2)$	0.008
$(0, 0)$	$< 10^{-4}$

Reference = Full matrix

Remove the ILR $\longrightarrow$ **no more unstable**

Application to the rotating ROI

$\mathcal{I}$	$\eta(\alpha = 1)$
Reference	0.040
$(-1, 2), (0, 0)$	0.040
$\mathcal{I}_0 \setminus (-1, 2)$	$< 10^{-4}$
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$(-1, 2)$	0.008
$(0, 0)$	$< 10^{-4}$

Reference = Full matrix

Remove the ILR $\longrightarrow$ **no more unstable**

Remove the TR $\longrightarrow$ growth rate **strongly affected**

Application to the rotating ROI

$\mathcal{I}$	$\eta(\alpha = 1)$
Reference	0.040
$(-1, 2), (0, 0)$	0.040
$\mathcal{I}_0 \setminus (-1, 2)$	$< 10^{-4}$
$\mathcal{I}_0 \setminus (0, 0)$	0.015
$(-1, 2)$	0.008
$(0, 0)$	$< 10^{-4}$

Reference = Full matrix

Remove the ILR $\longrightarrow$ **no more unstable**

Remove the TR $\longrightarrow$ growth rate **strongly affected**

ILR or TR only $\longrightarrow$ instability **strongly affected**

Application to the rotating ROI

$\mathcal{I}$	$\eta(\alpha = 1)$
Reference	0.040
$(-1, 2), (0, 0)$	0.040
$\mathcal{I}_0 \setminus (-1, 2)$	$< 10^{-4}$
$\mathcal{I}_0 \setminus (0, 0)$	0.015
$(-1, 2)$	0.008
$(0, 0)$	$< 10^{-4}$

Reference = Full matrix

Remove the ILR $\longrightarrow$ **no more unstable**

Remove the TR $\longrightarrow$ growth rate **strongly affected**

ILR or TR only $\longrightarrow$ instability **strongly affected**

ILR + TR $\longrightarrow$ instability **barely changes**

$\longrightarrow$ **The ROI and the TI cooperate** for the instability

Stellar Clusters

Orbital structure of stellar clusters

Linear Response Theory

Identifying unstable modes

Anisotropic, rotating stellar clusters

Destabilisation processes

Conclusions and prospects

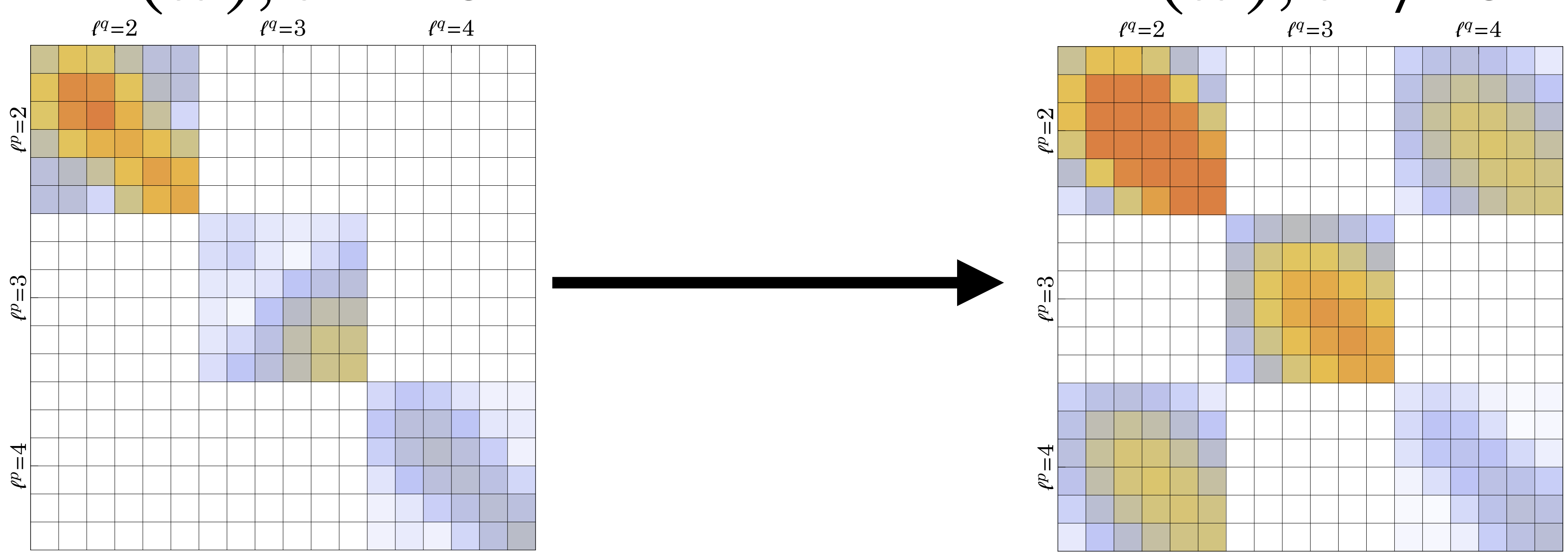
Conclusions

- **Algorithmic improvements** $\longrightarrow$ parameter space explorations

$$\boxed{\mathbf{M}_{pq}(\omega) = (2\pi)^3 \sum_{\mathbf{n}} \int \mathrm{d}\mathbf{J} \, \frac{\mathbf{n} \cdot \partial F / \partial \mathbf{J}}{\omega - \mathbf{n} \cdot \mathbf{\Omega}} \, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J}) \, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})}$$

- Computation of the matrix for **rotating spheres**

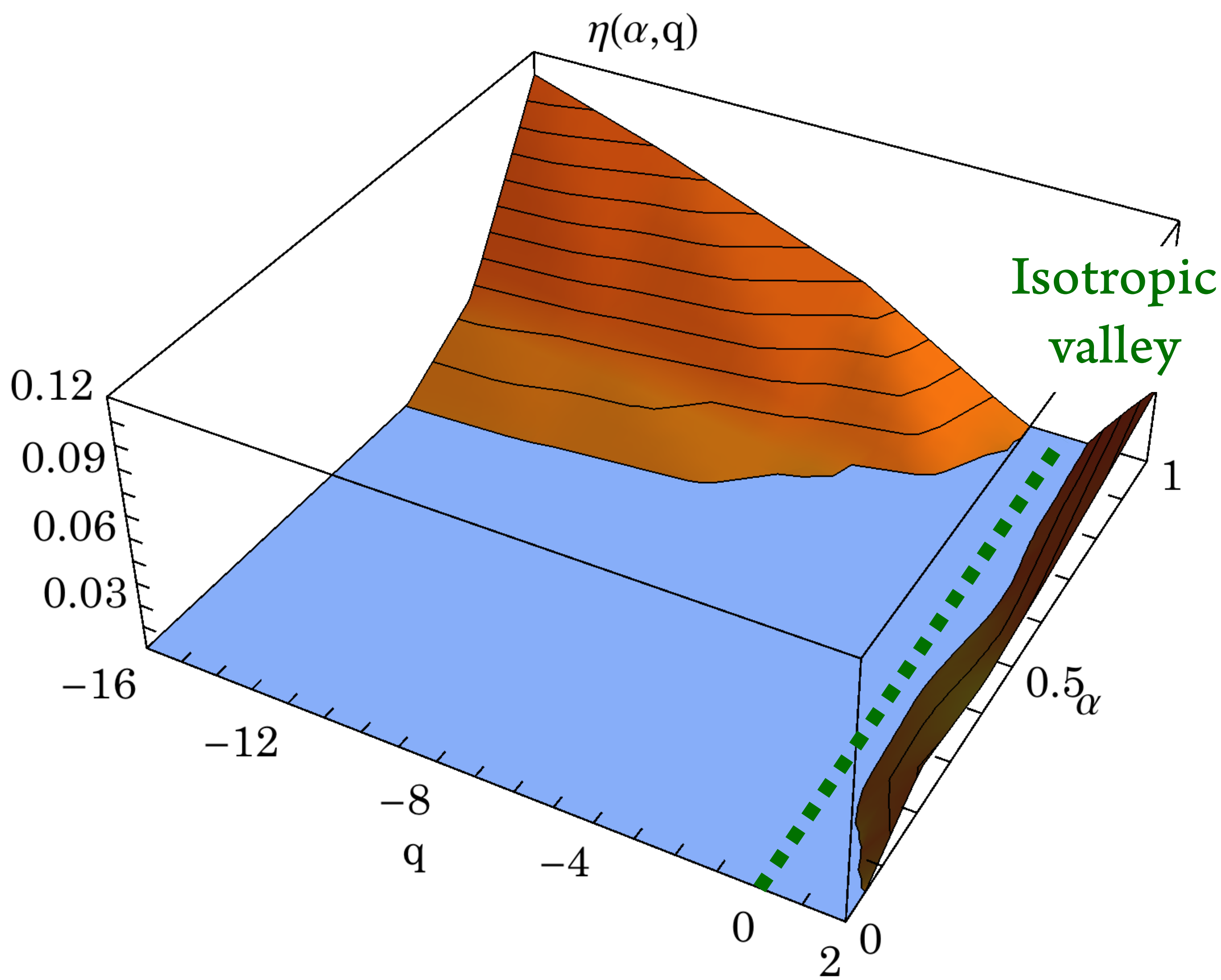
$\mathbf{M}(\omega), \alpha = 0$ $\qquad\qquad$ $\mathbf{M}(\omega), \alpha \neq 0$



Conclusions

- The **stability region** around **isotropic** clusters is narrow

⟶ **Dynamically cold** clusters tend to be **linearly unstable**

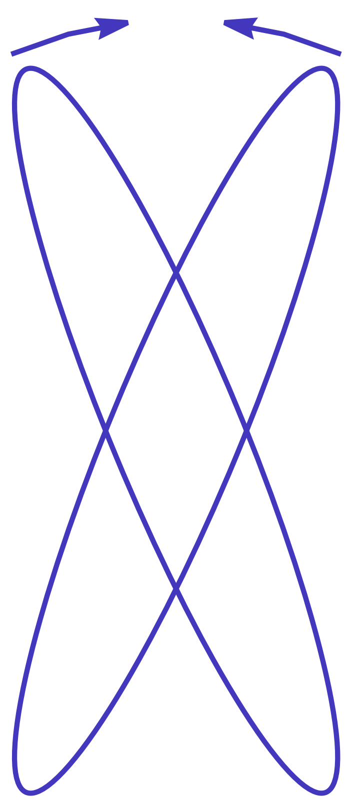


Conclusions

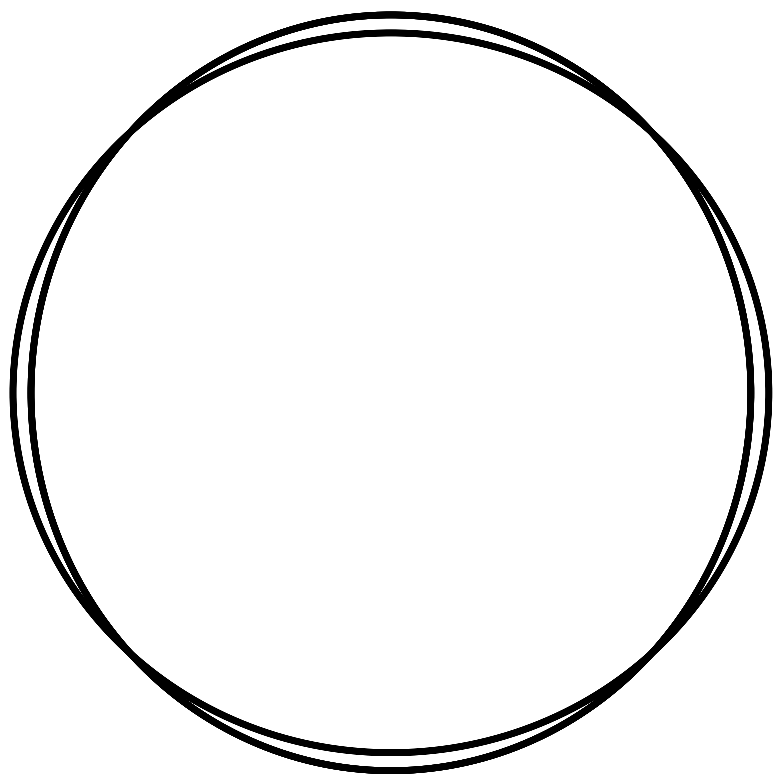
Restricted matrix method
$\longrightarrow$ **destabilisation processes**

TI enhances
pre-existing instabilities

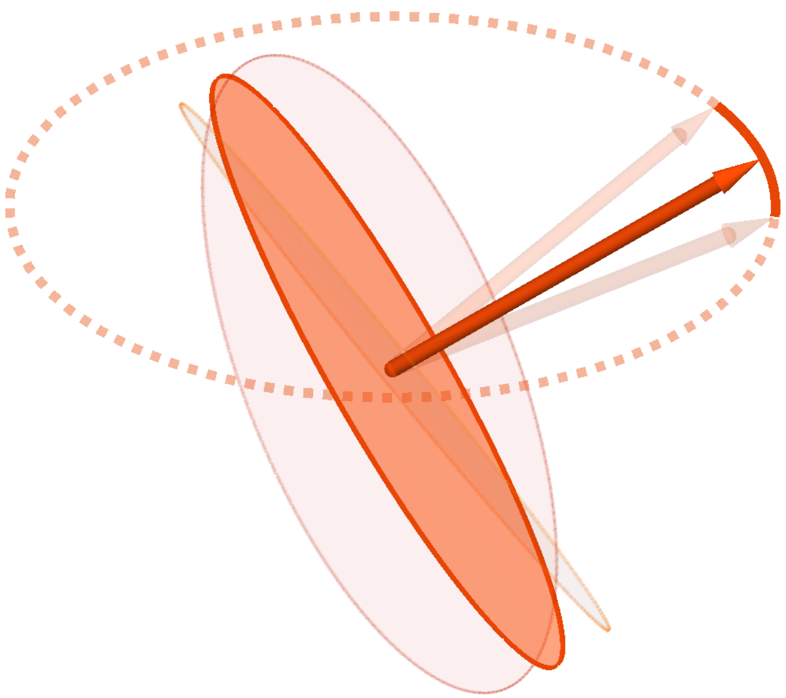
Resonant processes
$\longrightarrow$ **important drivers** of the linear response (ILR)



ROI



COI



TI

Questions

Orbital reshuffling

Yes, in highly ordered systems

Spontaneous evolution

Yes, many linear instabilities

Effect of perturbations

New analytical tools
+ relevant processes

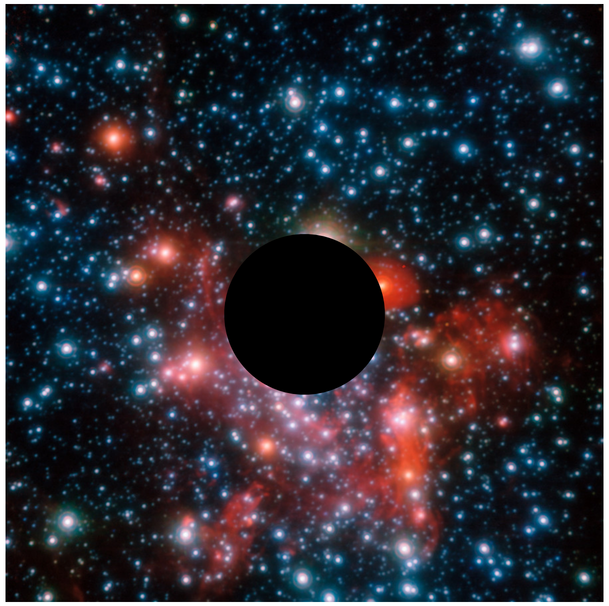
Processes

Resonances

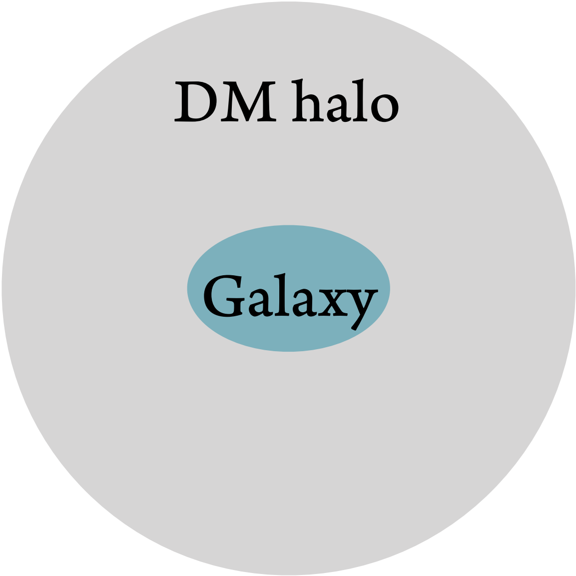
Prospects

- Use the same numerical methods in other contexts:

Cuspy systems

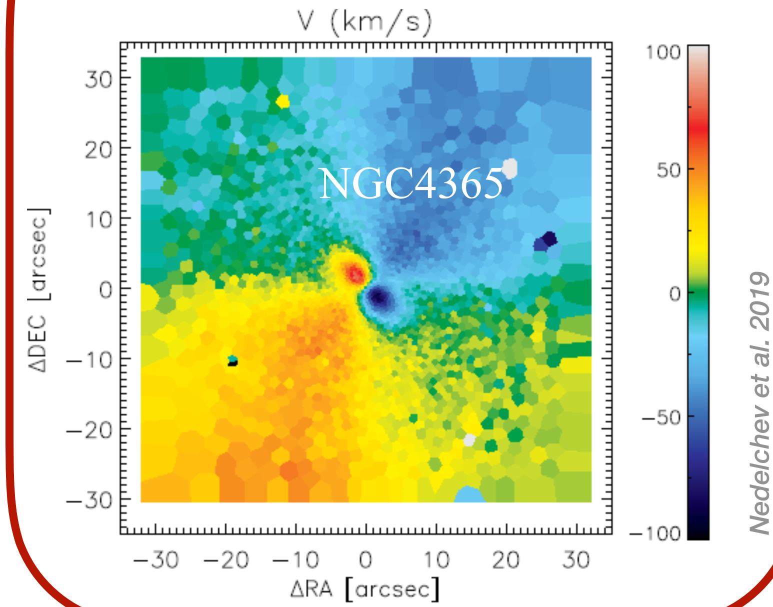


Multi-component systems



Axisymmetric systems

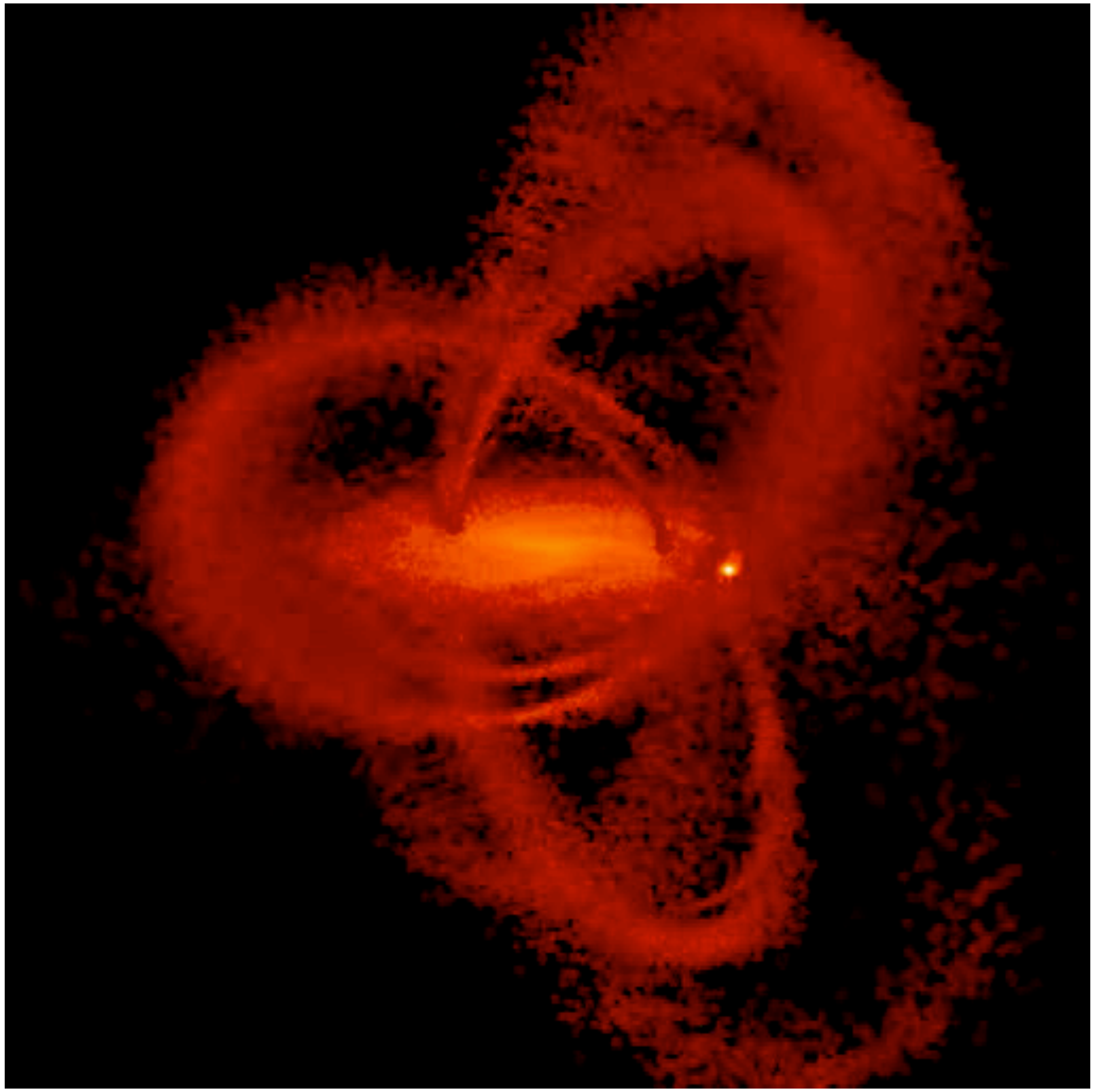
Less regular DF



Prospects

- Study **linear evolution**

$$\mathbf{a}(t) = \int_{-\infty}^{t} \mathrm{d}t'\, \mathbf{M}(t'-t)\cdot(\mathbf{a}(t')+\mathbf{b}(t'))$$



S. Colombi

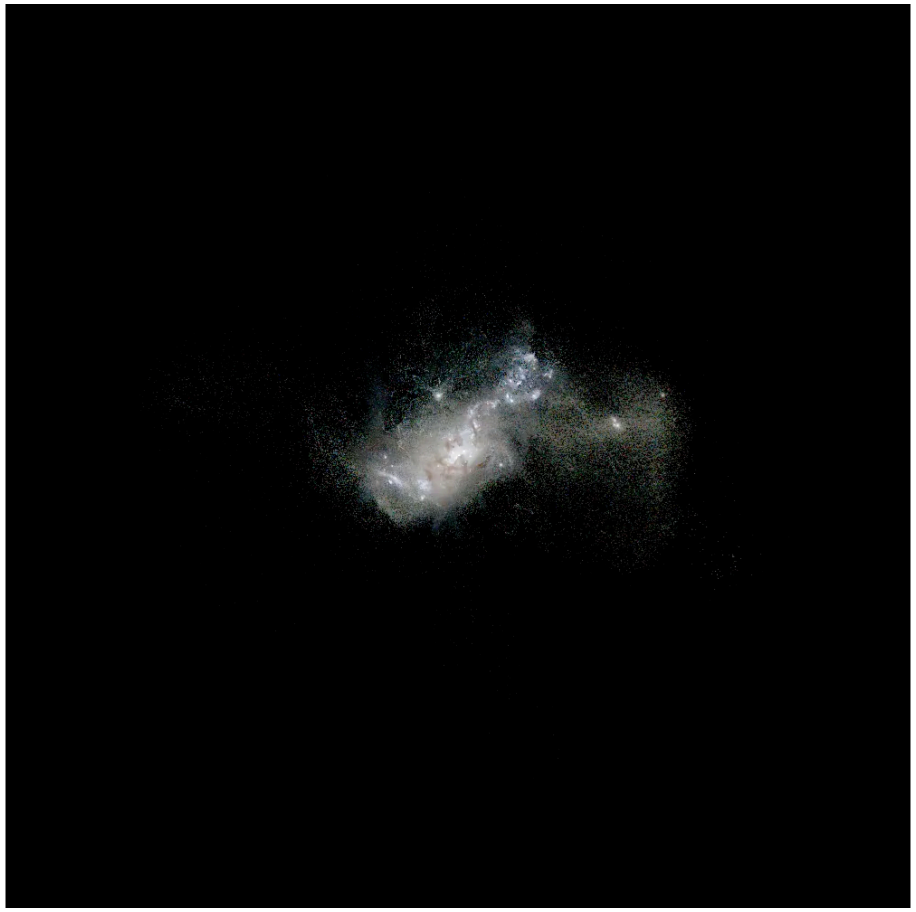
Prospects

- Study **secular evolution**

Balescu-Lenard

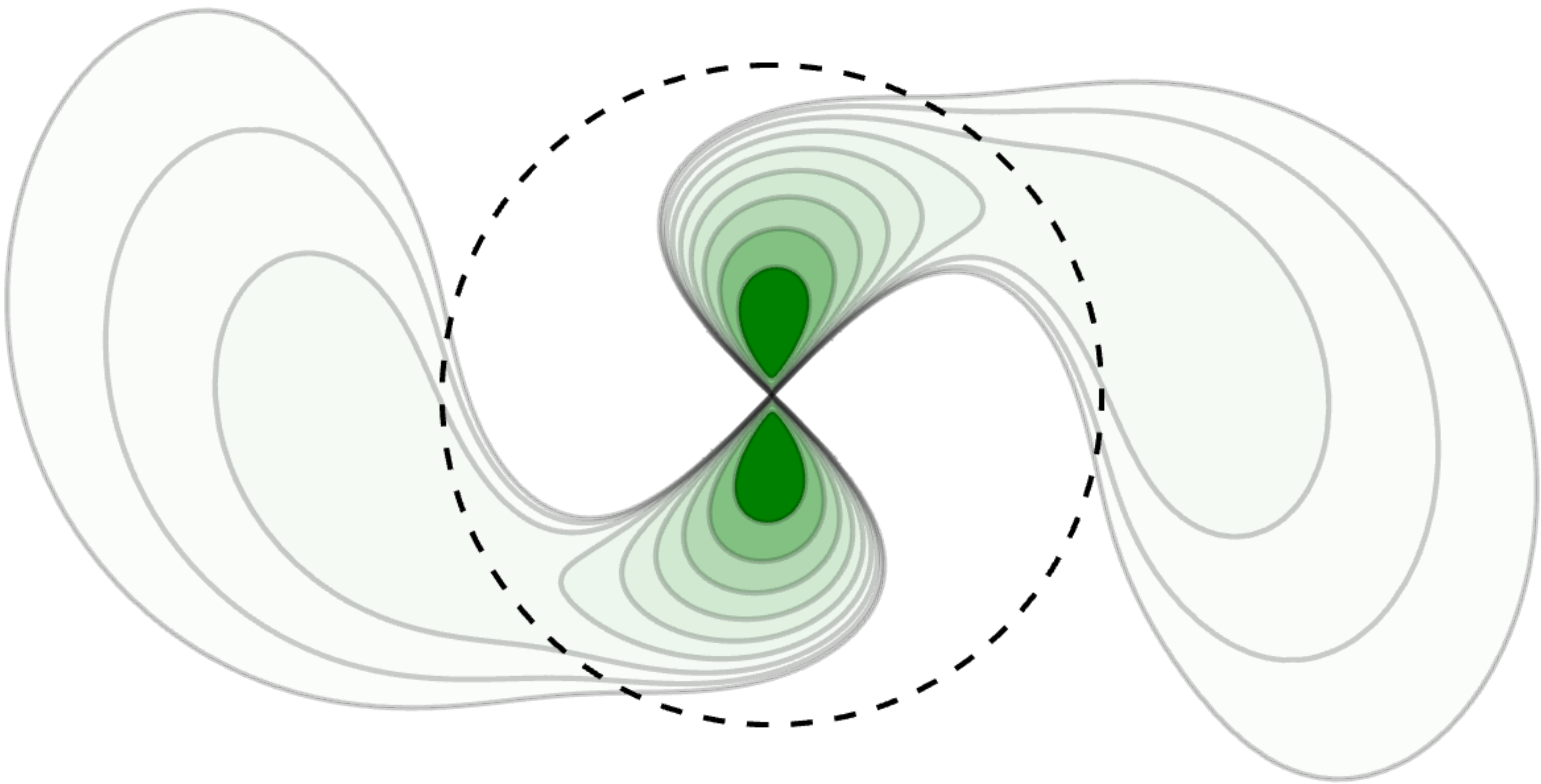
$$\frac{\partial F(\mathbf{J},t)}{\partial t} = \pi(2\pi)^3 \frac{M_{\mathrm{tot}}}{N} \frac{\partial}{\partial \mathbf{J}} \cdot \left[\sum_{\mathbf{n},\mathbf{n}'} \mathbf{n} \int \mathrm{d}\mathbf{J}' \frac{\delta(\mathbf{n}\cdot\mathbf{\Omega} - \mathbf{n}'\cdot\mathbf{\Omega}')}{\left|\mathscr{D}_{\mathbf{n}\mathbf{n}'}(\mathbf{J},\mathbf{J}',\mathbf{n}\cdot\mathbf{\Omega})\right|^2} \left(\mathbf{n}\cdot\frac{\partial}{\partial \mathbf{J}} - \mathbf{n}'\cdot\frac{\partial}{\partial \mathbf{J}'} \right) F(\mathbf{J},t) F(\mathbf{J}',t) \right]$$

$$\frac{1}{\mathscr{D}_{\mathbf{n}\mathbf{n}'}(\mathbf{J},\mathbf{J}',\omega)} = \sum_{p,q} \psi_{\mathbf{n}}^{(p)}(\mathbf{J}) \left[\mathbf{I} - \mathbf{M}(\omega)\right]^{-1}_{pq} \psi_{\mathbf{n}'}^{(q)*}(\mathbf{J}')$$



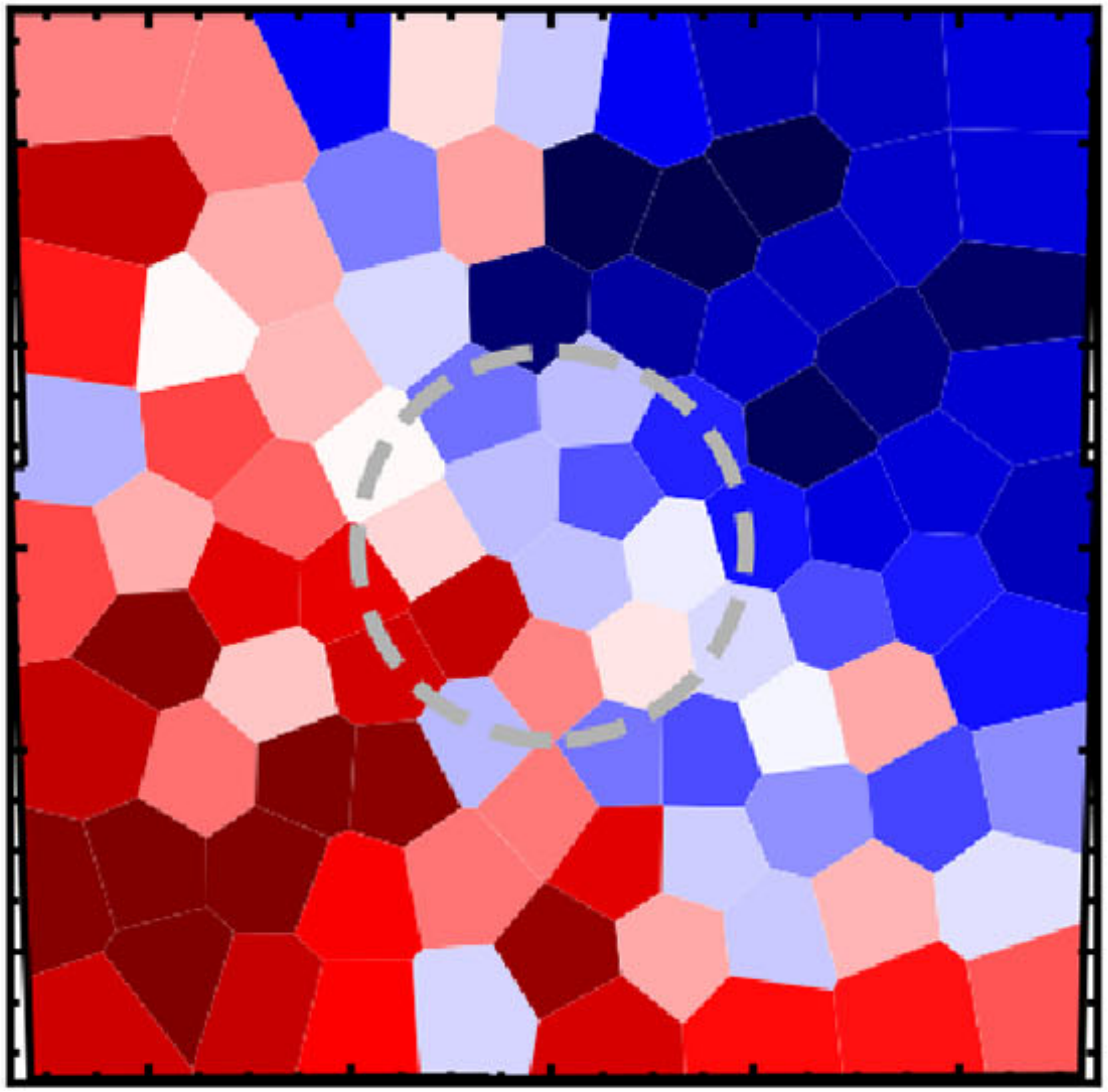
New Horizon

Thank you for your attention



Astrometry

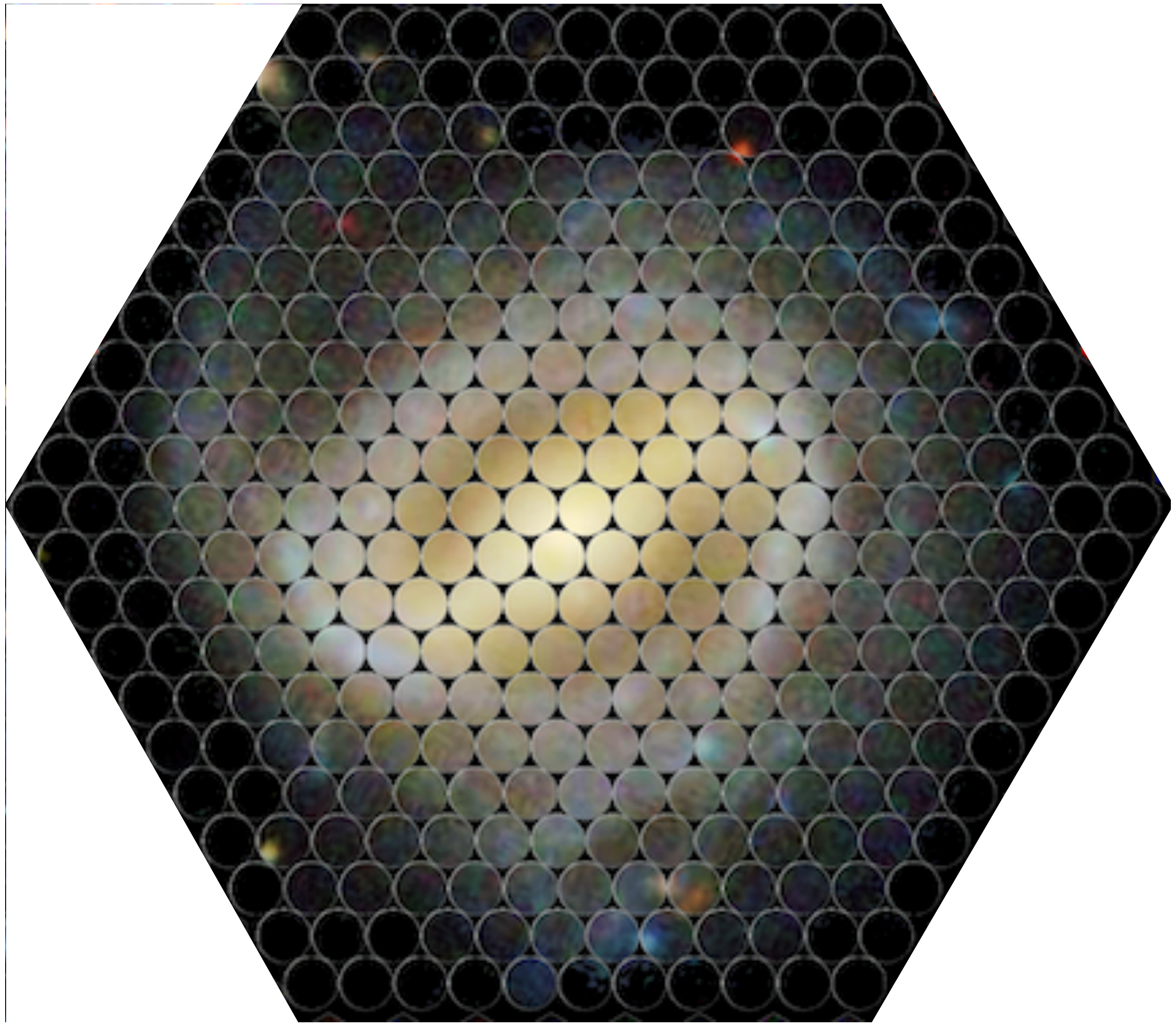
Light spectrum of each star ⟶ velocity of each star along the line-of-sight



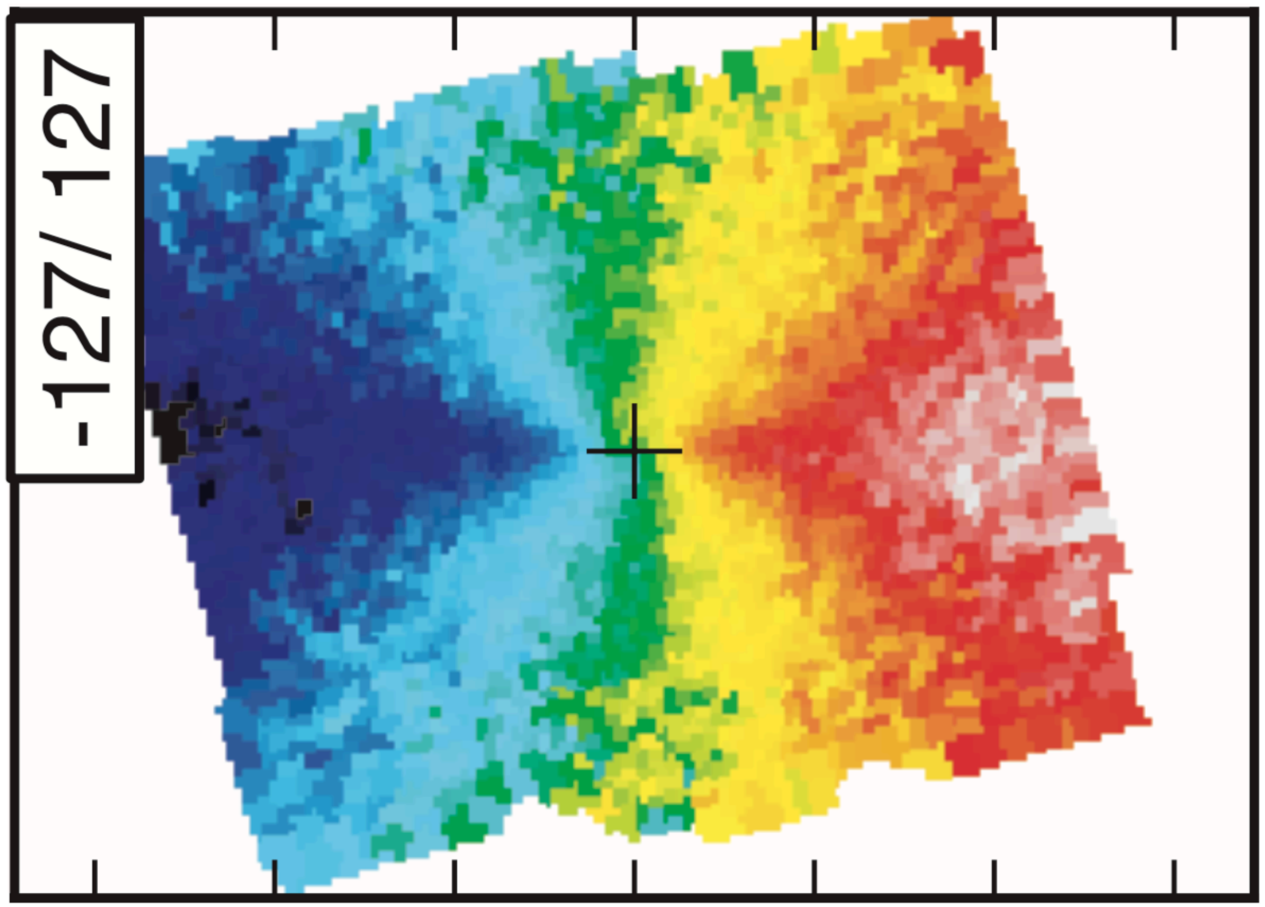
NGC 7089 (Kamann et al. 2017)

Integral field spectroscopy

Each optic fibre ⟶ mean velocity + velocity dispersion of a region of the galaxy



CALIFA (Husemann et al. 2013)



NGC 4621 (Emsellem et al. 2011)

Dynamical description of a stellar cluster

Hamiltonian

$$\boxed{H(\mathbf{x},\mathbf{v}) = \frac{v^2}{2} + \psi_0(\mathbf{x})}$$

Hamilton's equations

$$\boxed{\dot{\mathbf{v}} = -\frac{\partial H}{\partial \mathbf{x}} \quad ; \quad \dot{\mathbf{x}} = \frac{\partial H}{\partial \mathbf{v}}}$$

In angle-action variables

$$\boxed{\dot{\mathbf{J}} = -\frac{\partial H}{\partial \boldsymbol{\theta}} = 0 \quad ; \quad \dot{\boldsymbol{\theta}} = \frac{\partial H}{\partial \mathbf{J}} = \mathbf{\Omega}(\mathbf{J})}$$

Many orbits $\longrightarrow$ distribution function $F(\mathbf{J},\boldsymbol{\theta},t)$

Collisionless Boltzmann equation

$$\boxed{\frac{\partial F}{\partial t} + \dot{\mathbf{w}} \cdot \frac{\partial F}{\partial \mathbf{w}} = 0}$$

$\mathbf{w} = (\mathbf{x},\mathbf{v})$ or $(\mathbf{J},\boldsymbol{\theta})$

$\dot{\mathbf{w}}$ = dynamics of a particle

Actions are constants $\longrightarrow$ angles evolve linearly in time, at rate $\mathbf{\Omega}$

CBE in angle-action variables

$$\boxed{\frac{\partial F}{\partial t} + \mathbf{\Omega}(\mathbf{J}) \cdot \frac{\partial F}{\partial \boldsymbol{\theta}} = 0}$$

Equilibrium: $\dfrac{\partial F}{\partial t} = 0$

$$\rightarrow F = F(\mathbf{J})$$

Perturbed dynamics

Change in the potential $\longrightarrow$ change in the orbits $\longrightarrow$ change in the DF.

$\psi_0 + \psi$ $\quad$ $\mathbf{J} + \delta\mathbf{J}$ $\quad$ $\boldsymbol{\theta} + \delta\boldsymbol{\theta}$ $\quad$ $F + f$

$\psi = \psi^{\mathrm{e}} + \psi^{\mathrm{s}}$

ψ^{e} External perturbation

ψ^{s} Wake induced in the system

$$\psi^{\mathrm{s}} = \int \mathrm{d}\mathbf{v}\, \mathrm{d}\mathbf{x}' \frac{f(\mathbf{x}', \mathbf{v})}{|\mathbf{x}' - \mathbf{x}|}$$

Linearised CBE

$$\boxed{\frac{\partial f}{\partial t} + \boldsymbol{\Omega}(\mathbf{J}) \cdot \frac{\partial f}{\partial \boldsymbol{\theta}} - \frac{\partial F}{\partial \mathbf{J}} \cdot \frac{\partial(\psi^{\mathrm{e}} + \psi^{\mathrm{s}})}{\partial \boldsymbol{\theta}} = 0}$$

Perturbation described by its time frequencies (Fourier components)
-> possible resonances: the torque builds up -> changes on the orbits are larger at resonances

2 kinds of behaviours: transient waves (winding - phase mixing); self-sustained modes

Projection on a bi-orthogonal basis

Kalnajs 1976

Bi-orthogonal potential-density basis solving the Poisson equation

$$\Delta\psi^{(p)} = 4\pi G\rho^{(p)}$$

$$\int \mathrm{d}\mathbf{x}\,\psi^{(p)*}(\mathbf{x})\rho^{(q)}(\mathbf{x}) = -\,\delta_p^q$$

Compute once and for all the expression of $\psi^{(p)}(\mathbf{J},\boldsymbol{\theta})$

Same projection of ψ and ρ

$$\psi(\mathbf{x}) = \sum c_p \psi^{(p)}(\mathbf{x})$$

$$\rho(\mathbf{x}) = \sum c_p \rho^{(p)}(\mathbf{x})$$

The perturbation is fully represented by the vector c_p $\longrightarrow$ transposed into linear algebra

$\longrightarrow$ Response matrix

$$\underset{\psi^{\mathrm{s}}}{\mathbf{a}_p} = \mathbf{M}\,\underset{\psi}{\mathbf{c}_p} = \mathbf{M}\,(\mathbf{a}_p + \underset{\psi^{\mathrm{e}}}{\mathbf{b}_p})$$

Scaling relation

Gravitational field from a cluster: $g = \dfrac{GM}{r^2}$

Acceleration of a circular orbit: $a = \dfrac{v^2}{r}$

Newton's second law: $g = a$

$$\rho \propto \frac{M}{r^3}$$

$$v \propto \frac{r}{t_{\mathrm{dyn}}}$$

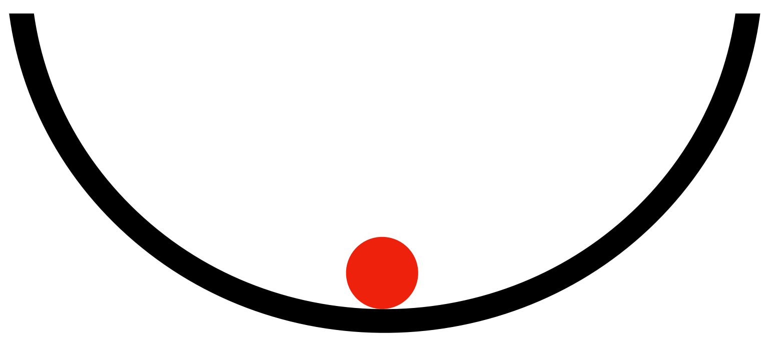
$$\longrightarrow \quad t_{\mathrm{dyn}} \propto \frac{1}{\sqrt{\rho}}$$

Changing scales amounts to changing the clock

Smaller system $\longrightarrow$ denser $\longrightarrow$ faster evolution

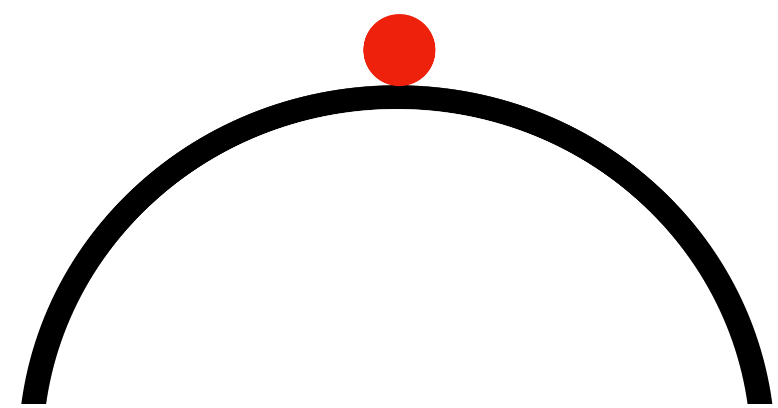
Stable VS unstable equilibria

Stable equilibrium



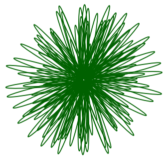
Dynamics **opposes** a perturbation

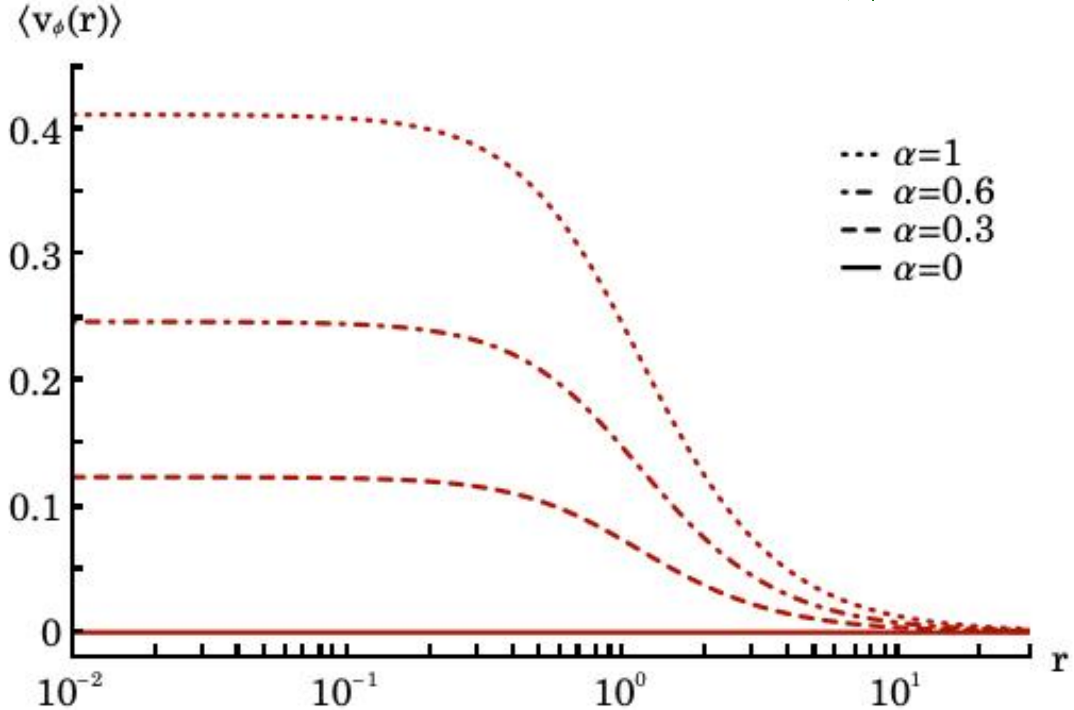
Unstable equilibrium



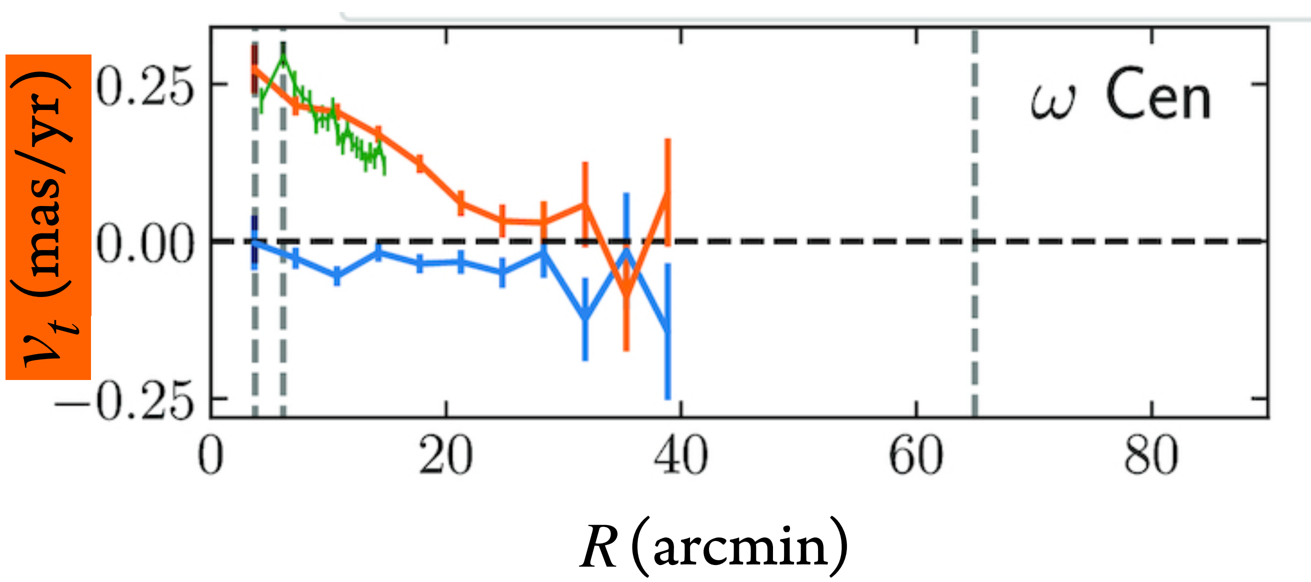
Dynamics **amplifies** a perturbation

Rotating spheres

This study (q=2) 

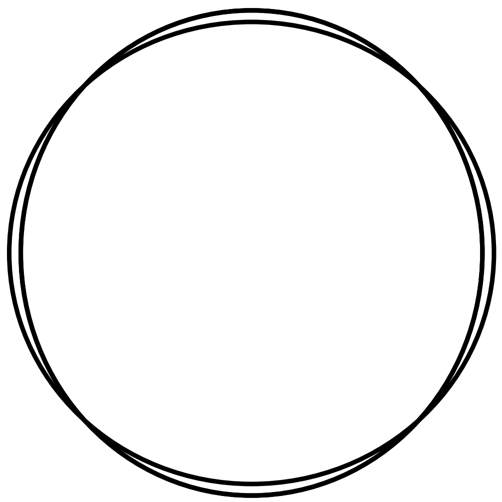


Jindal et al. 2019 (GAIA DR2)



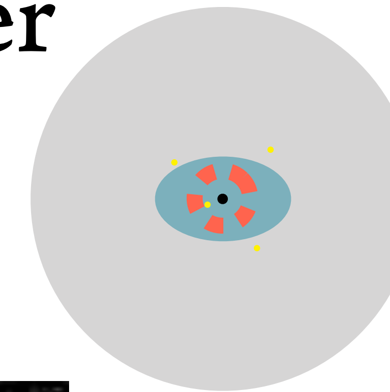
Prospects

- Build an intuitive explanation for the COI

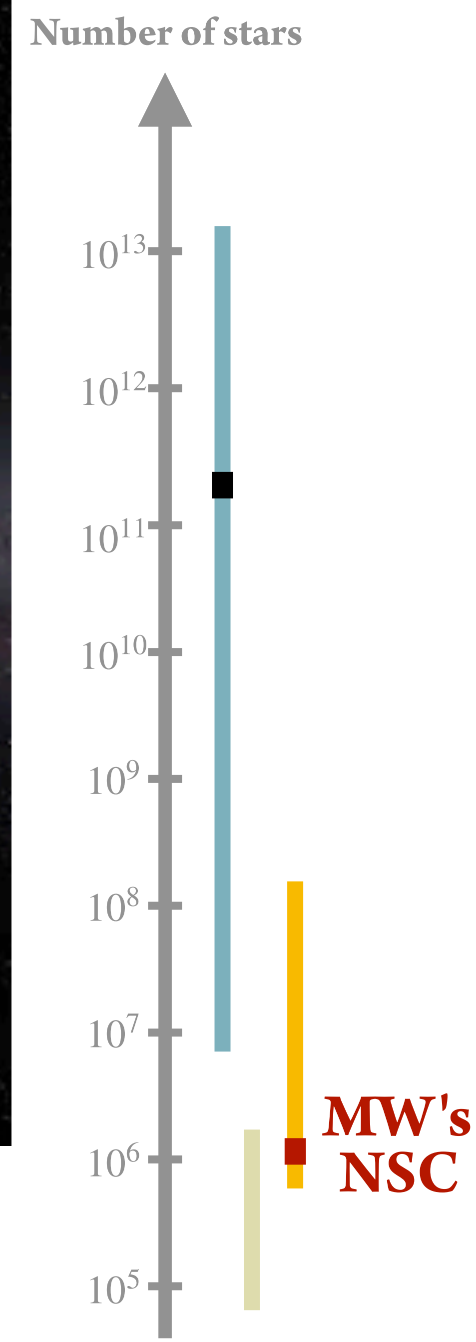


- Process setting the pattern speed?

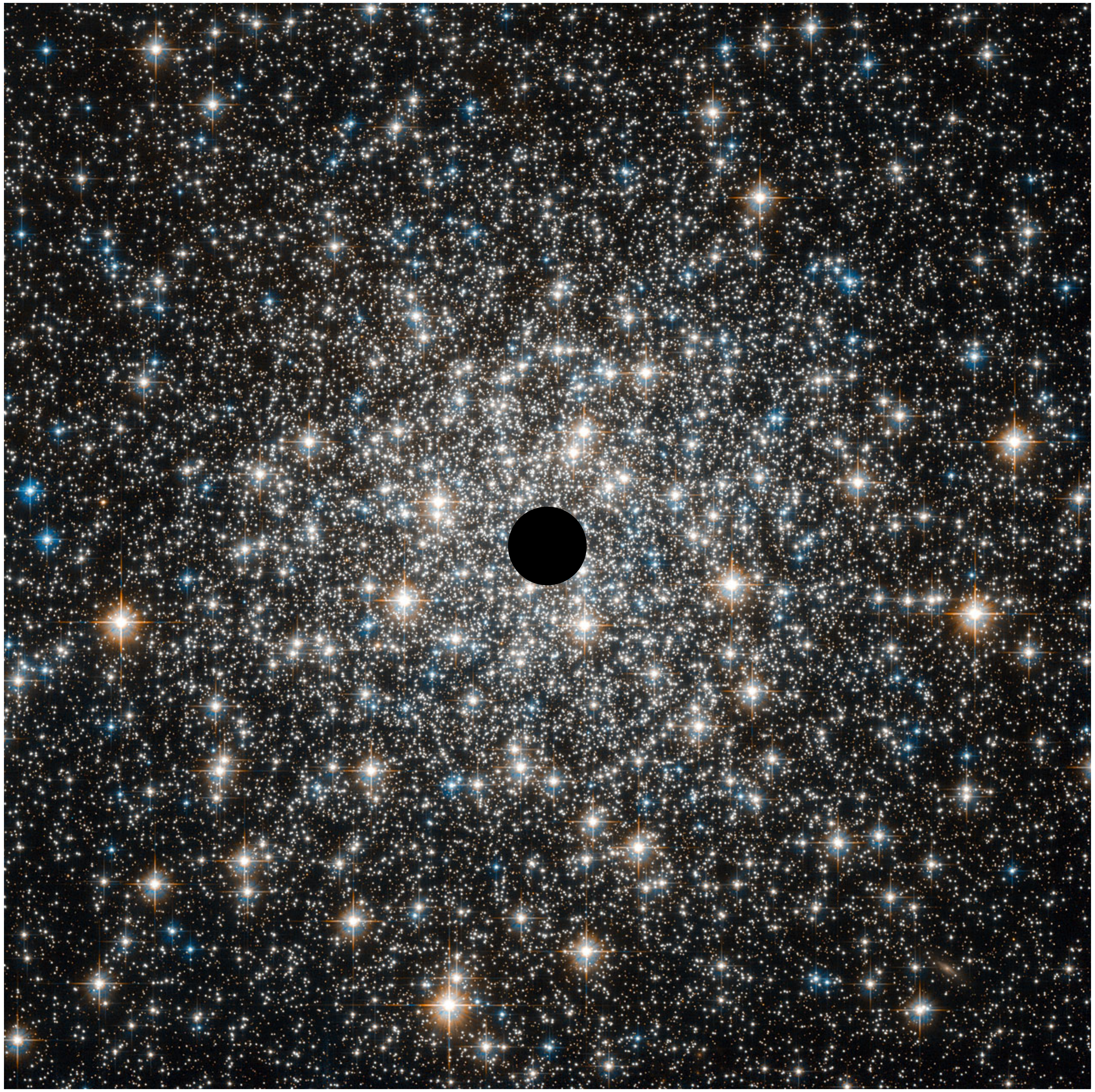
The Milky Way's Nuclear Star Cluster



Milky Way's NSC



Nuclear Star Clusters



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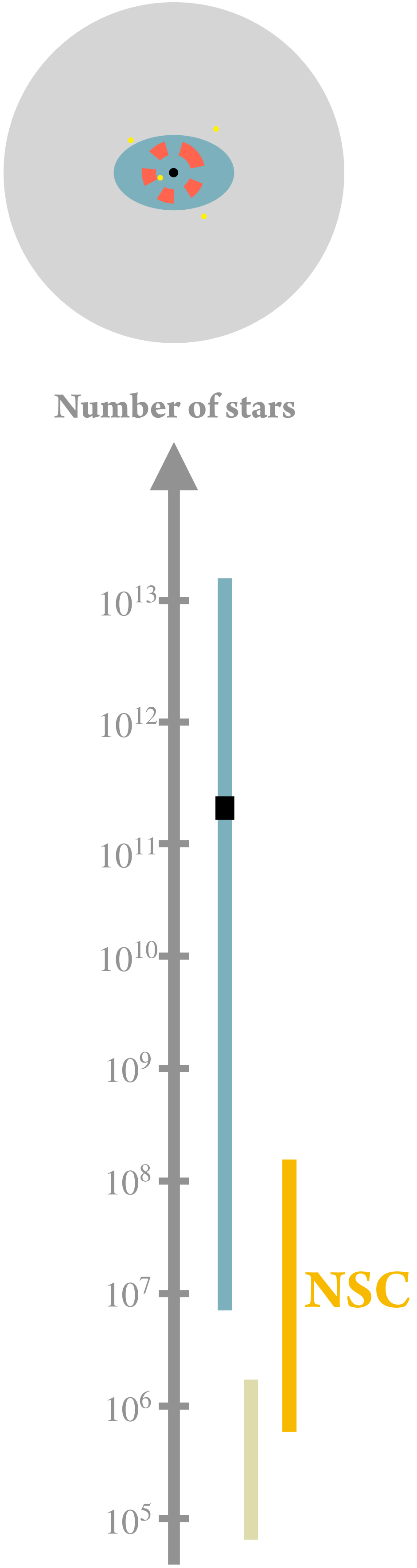


Projection on a basis

Kalnajs 1976

The basis **solves the Poisson equation**

$\psi^{\mathrm{e}}(\mathbf{x},\omega)$ ⟶ CBE + Poisson $\xrightarrow{\text{Integro-differential operator}}$ $\rho^{\mathrm{s}}(\mathbf{x},\omega) = \int \mathrm{d}\mathbf{x}' \mathcal{O}(\mathbf{x},\mathbf{x}',\omega)[\psi^{\mathrm{s}}(\mathbf{x}',\omega)+\psi^{\mathrm{e}}(\mathbf{x}',\omega)]$

Projected

Long-range force ⟶ Non-local basis

$\mathbf{b}(\omega)$ ⟶ CBE $\xrightarrow{\textbf{Response matrix}}$ $\boxed{\mathbf{a}(\omega) = \mathbf{M}(\omega)\cdot(\mathbf{a}(\omega)+\mathbf{b}(\omega))}$

Uses of the response matrix: Linear Response

Linear Instabilities

Restricted matrix method

ROI

Relies on the ILR

$(n_1, n_2, n_3) = (-1{,}2{,}2)$

Occurs in **radial** systems

COI

Relies on the ILR

$(n_1, n_2, n_3) = (-1{,}2{,}2)$

Occurs in **tangential** systems

TI

Relies on the TR

$(n_1, n_2, n_3) = (0{,}0{,}2)$

Occurs in **rotating** systems

Method to identify the **resonant processes** sourcing **instabilities**

$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \boxed{\sum_{(n_1,n_2,n_3)}} \int \mathrm{d}\mathbf{J}\, \frac{\mathbf{n}\cdot\partial F/\partial \mathbf{J}}{\omega - \mathbf{n}\cdot\boldsymbol{\Omega}}\, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J})\, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

Isolate the contribution from **each resonance**

ILR

$$\int \mathrm{d}\mathbf{J}\, \frac{-\partial F/\partial J_r + 2\partial F/\partial L}{\omega + \Omega_1 - 2\Omega_2}\, \psi_{(-1,2)}^{(p)*}(\mathbf{J})\, \psi_{(-1,2)}^{(q)}(\mathbf{J})$$

$(n_1, n_2, n_3) = (-1{,}2{,}2)$

TR

$$\int \mathrm{d}\mathbf{J}\, \frac{F}{\omega}\, \psi_{(0,0)}^{(p)*}(\mathbf{J})\, \psi_{(0,0)}^{(q)}(\mathbf{J})$$

$(n_1, n_2, n_3) = (0{,}0{,}2)$

Other resonances

$\longrightarrow$ Perform the **full mode search**, but selecting only **a fraction of the resonant terms**

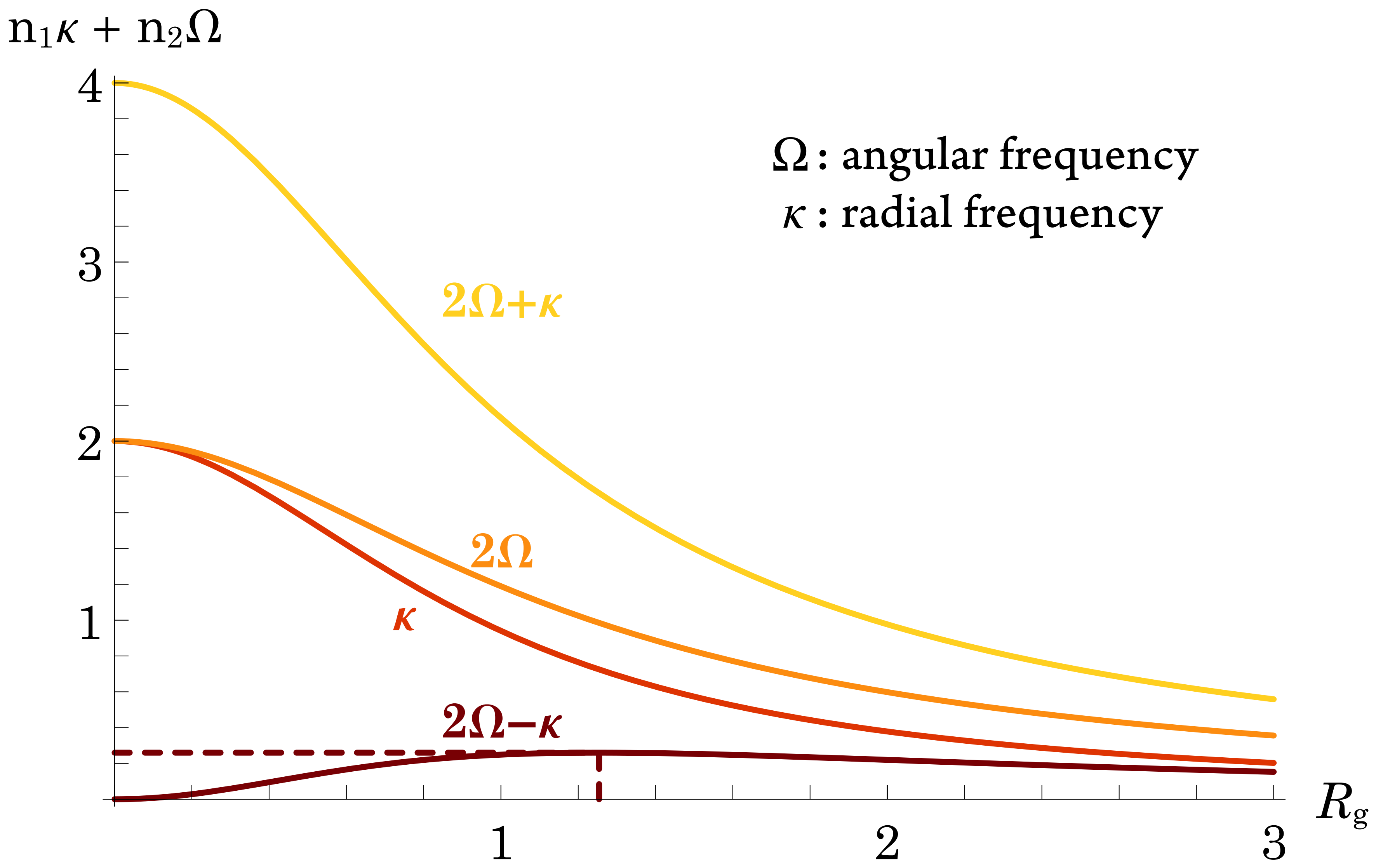
Circular orbit instability

Palmer et al. 1989

Merging of 2 resonances at the maximum of $\omega_{\mathrm{ILR}} = 2\Omega - \kappa$

Formation of a neutral mode with negative energy

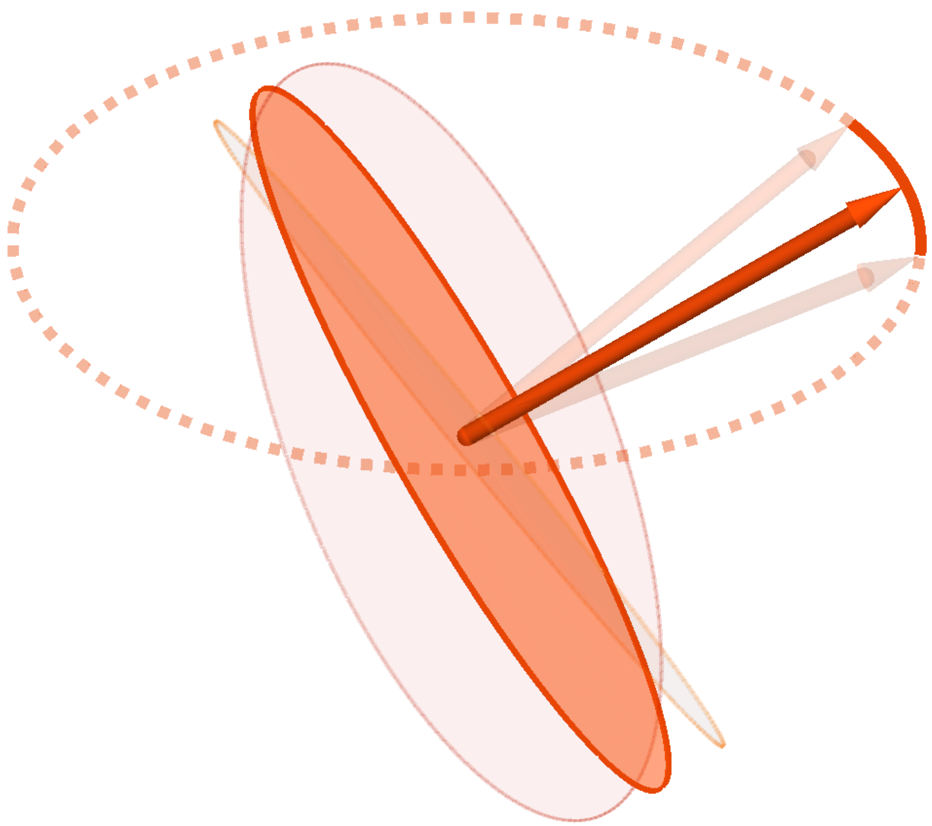
Energy dissipation by coupling with the underlying stars $\longrightarrow$ instability



Tumbling instability

Allen et al. 1992

Torque between a bar and an **inclined orbit**

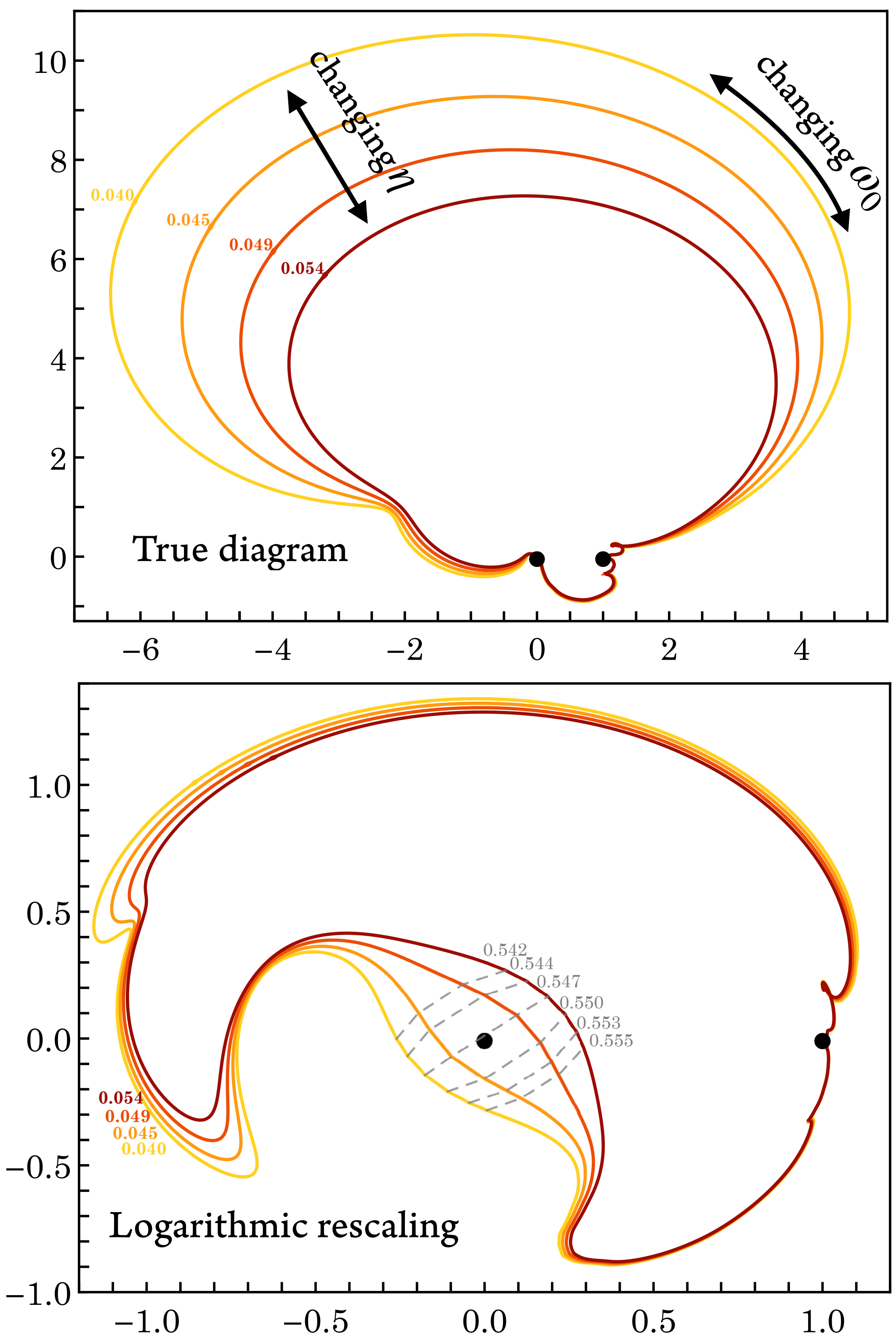


⟶ Tends to **lower the orbits' inclinations**

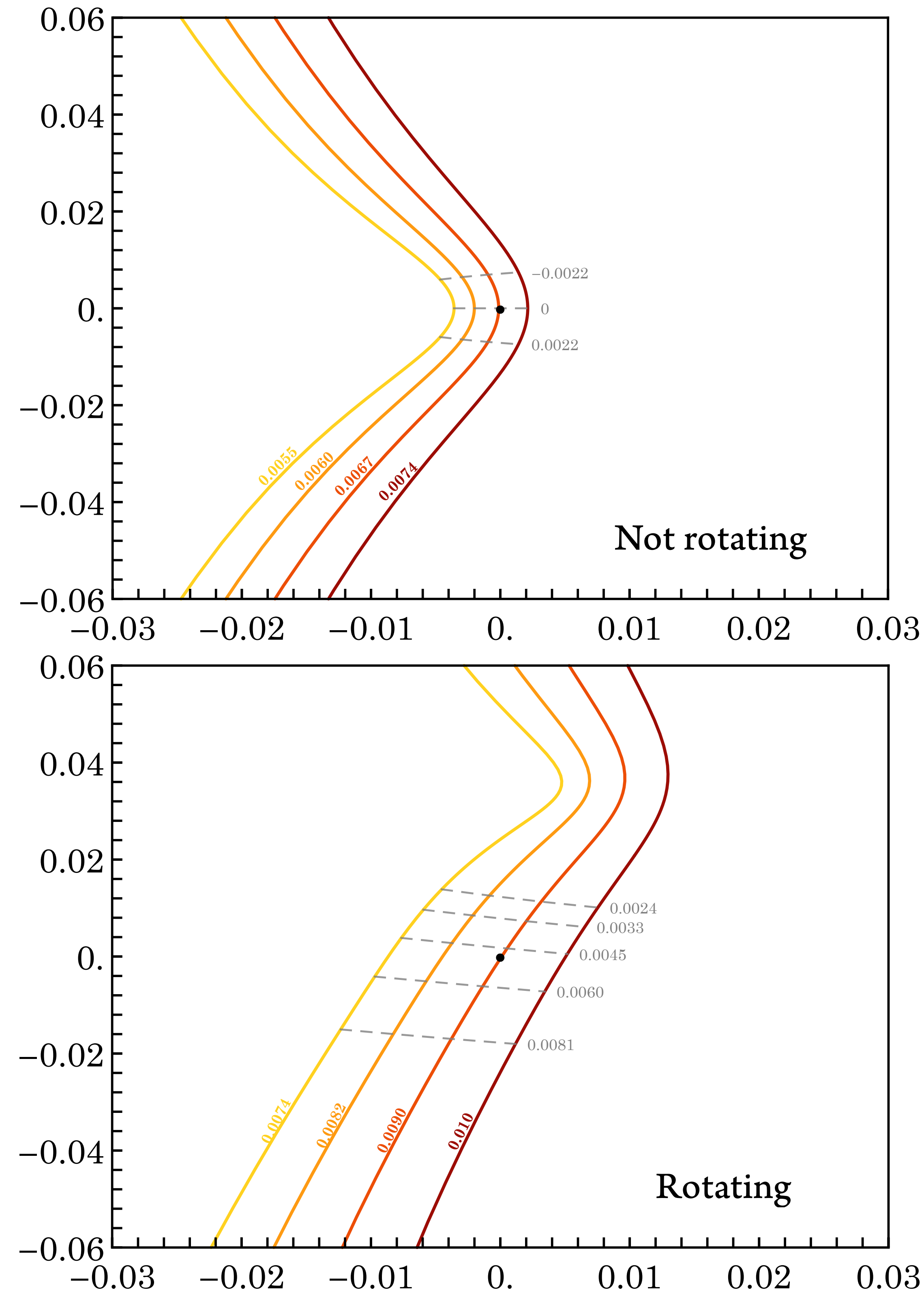
⟶ Traps the **orbital plane** into libration

Nyquist contours

Nyquist diagrams: $\omega_0 \mapsto \det[\mathbf{I} - \mathbf{M}(\omega_0 + \mathrm{i}\,\eta)]$



Influence of rotation



Questions

- How do stellar clusters **respond** to perturbations?

  Linear response

- What is the influence of **diverse kinematics**?

  Rotation - anisotropy

- What are the **important processes** at play?

  Long range force - resonances

- Why do we care?

  Nature VS nurture

Linear stability analysis

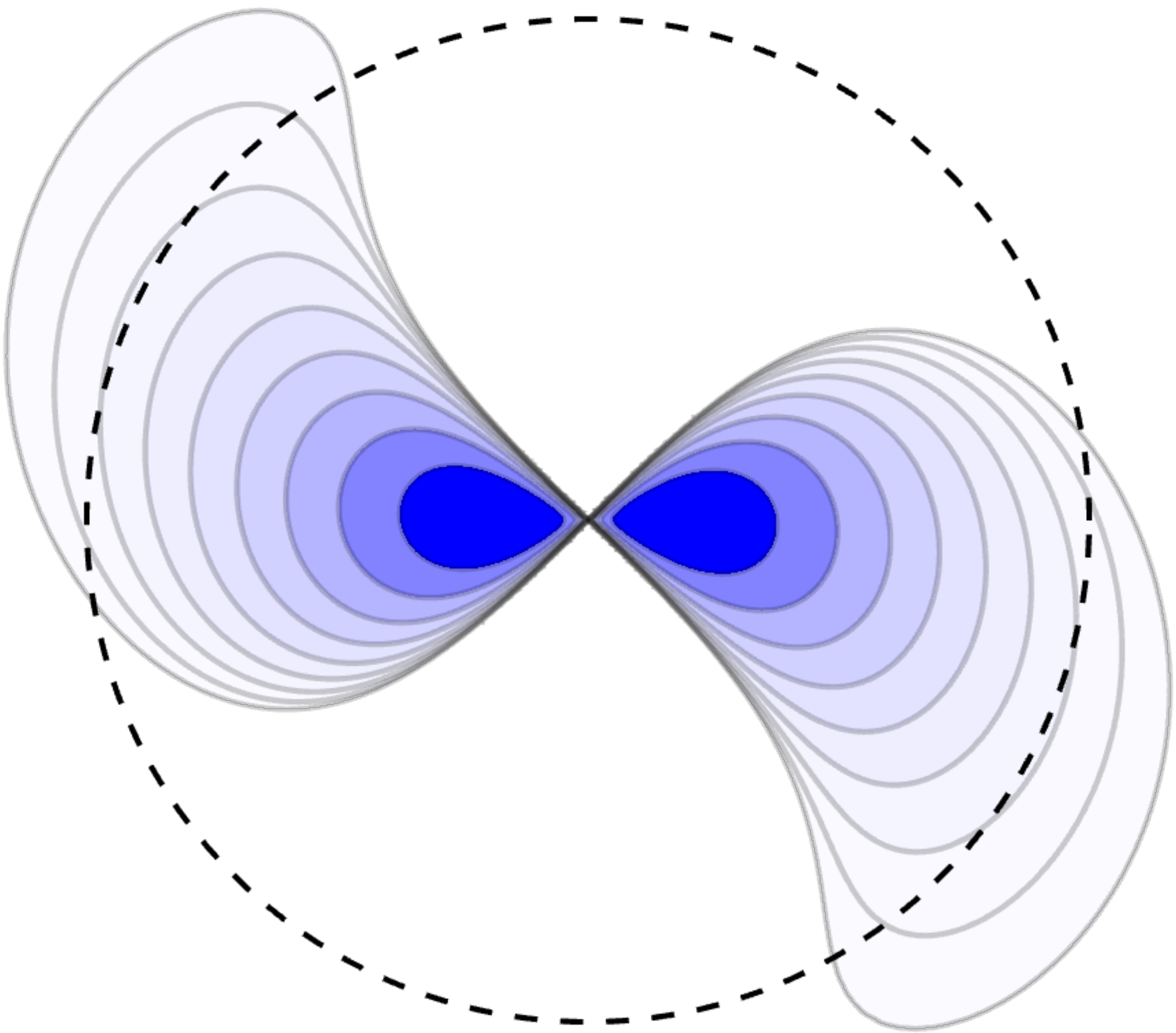
- What are the **allowed states?**

  Stable states
- What are the **favoured** states?

  Stability boundaries
- Which **processes** matter?

  Resonances
- How do systems **amplify perturbations**?

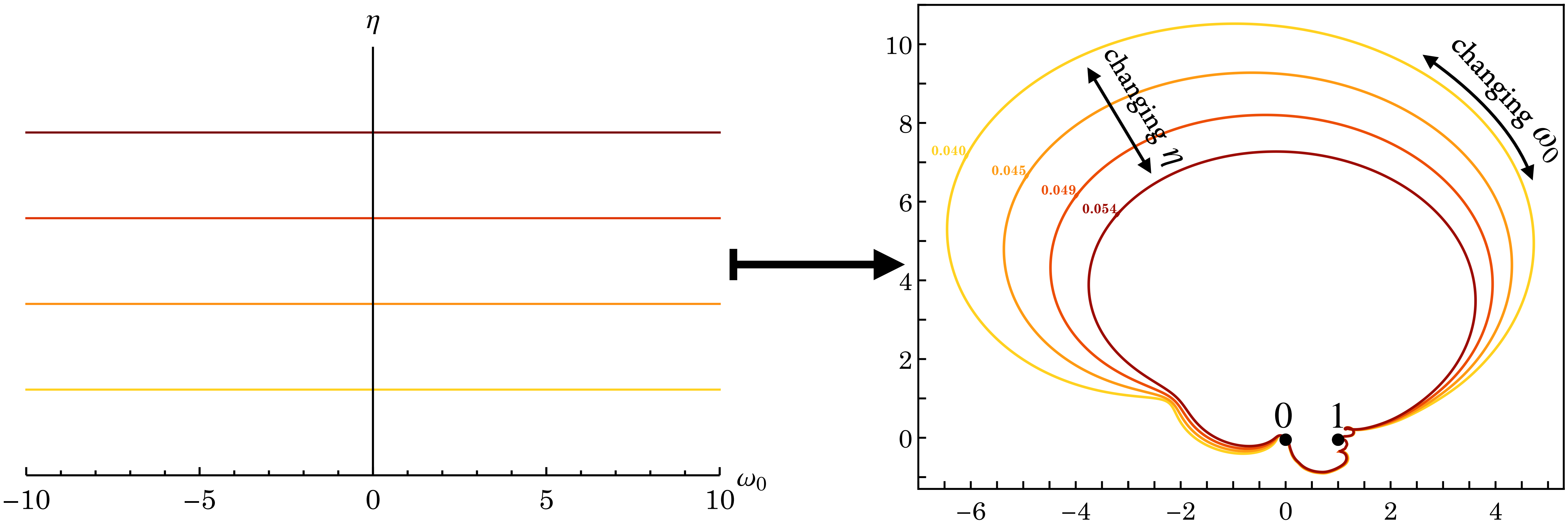
  Linear response



Nyquist contours

$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \sum_{\mathbf{n}} \int \mathrm{d}\mathbf{J}\, \frac{\mathbf{n}\cdot\partial F/\partial \mathbf{J}}{\omega - \mathbf{n}\cdot\mathbf{\Omega}}\, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J})\, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

Nyquist diagrams $\qquad \omega_0 \mapsto \det[\mathbf{I} - \mathbf{M}(\omega_0 + \mathrm{i}\,\eta)]$



Nyquist contours

$$\mathbf{M}_{pq}(\omega) = (2\pi)^3 \sum_{\mathbf{n}} \int \mathrm{d}\mathbf{J}\, \frac{\mathbf{n}\cdot\partial F/\partial \mathbf{J}}{\omega - \mathbf{n}\cdot\mathbf{\Omega}}\, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J})\, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})$$

Nyquist diagrams $\qquad \omega_0 \mapsto \det[\mathbf{I} - \mathbf{M}(\omega_0 + \mathrm{i}\,\eta)]$