Probing the Galactic center’s cluster with scalar resonant relaxation
Context

- Galactic center and S-Cluster
  - Fast Keplerian orbits
  - High density of stars
  - Supermassive black hole (SMBH) Sgr A*

- Formation
  - Intermediate mass black holes (IMBH) ?
  - Mass segregation

- Accessible data
  - GAIA
  - VLT (through the instrument Gravity)

Orbits of S-cluster’s stars near Sgr A*
Dynamics and time scales

→ Constrain the SMBH diet
→ Invisible matter experiment
→ Five processes influence stars’ orbits: from orbital time to secular time
Dynamics and time scales

1) Orbital time
2) Precession time
3) VRR time
4) SRR time
5) NR time

Dynamical processes in the Galactic center
Scalar resonant relaxation

\[
\frac{\partial P}{\partial t} = \frac{1}{2} \frac{\partial}{\partial j} \left[ j D_{jj} \frac{\partial}{\partial j} \left( \frac{P}{j} \right) \right]
\]

- Poisson noise
  → Finite-N effects
- Non-local resonances
  → Commensurable frequencies
  → Strong correlation
- Sources orbital diffusion

Resonant interactions in orbital space
\[ D_{jj}(\mathbf{J}) \propto \frac{1}{N_*} \sum_{n,n'} n^2 \int da'dj' F_{\text{tot}}(a',j') |A_{nn'}(a,j,a',j')|^2 \times \delta_D(n\nu_P(a,j) - n'\nu_P(a',j')) \]

**\[ D_{jj}(\mathbf{J}) \]** Anisotropic diffusion

**\[ |A_{nn'}(a,j,a',j')|^2 \]** Coupling coefficient

**\[ \delta_D(n\nu_P(a,j) - n'\nu_P(a',j')) \]** Resonance condition

**\[ 1/N_* \]** Finite-N effects

**\[ n^2 \]** Resonance numbers

**\[ F_{\text{tot}}(a',j') \]** Invisible cluster distribution function
Resonance line

- Precession: 2 contributions
  - Relativistic (BH) effects
  - Invisible cluster’s mass
- Asymptotic behaviours:
  - GR effects
    (very elliptic orbits close to BH)
  - Cluster mass effects
    (circular orbits away from BH)
- Loss-cone
  - Stars fall into the BH

![Iso-contours of the precession frequency graph](image)

Legend:
- Mass effects
- GR effects
- Fast frequencies
- Slow frequencies

Black hole

Semi-major axis

Eccentricity
Diffusion coefficients

SRR Diffusion coefficients

S-Cluster

Diffusion coefficient

Eccentricity

Total contribution

Contribution of each resonances

Black hole

Cut along 10 mpc

Fast diffusion

Slow diffusion

Semi-major axis

Loss cone

(1,1)

(4, -1)

(2, -2)

(2, 2)
Comparing two cluster models

- Model-dependent
  → IMBHs increase efficiency
  → Different resonances
- Efficient computation
  → Parallelized code
  → Linear (sampling) time
  → Exploration of parameter space (add more curves)
Observations

\[
\frac{\partial P}{\partial t} = \frac{1}{2} \frac{\partial}{\partial j} \left[ jD_{jj} \frac{\partial}{\partial j} \left( \frac{P}{j} \right) \right]
\]

- Diffusion equation of the PDF
  - Equilibrium: \( P(j) = 2 \ j \)
- Thermalization of stars
  - Diffusion must be efficient

Cluster distribution

Eccentricity

Thermal
Diffusion equation integration

The differential equation is given by

\[ \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \]

where

- \( c(x,t) \) is the concentration at position \( x \) and time \( t \).
- \( D \) is the diffusion coefficient.

The initial condition is

\[ c(x,0) = f(x) \]

and the boundary conditions are

\[ c(0,t) = c(L,t) = 0 \]

The solution to this equation can be found using separation of variables or Fourier series expansion. The steady-state solution is

\[ c(x) = \frac{1}{2} \left( f(x) + \frac{f(L)}{2} \right) + \frac{1}{2} \int_0^L \left[ f(y) - \frac{f(L)}{2} \right] \sin \left( \frac{n \pi x}{L} \right) \frac{\sin \left( \frac{n \pi x}{L} \right)}{\left( \frac{n \pi}{L} \right)^2} \, dy \]

where \( n \) is an integer. The time evolution is approximated using a finite difference method or a spectral method.
Stochastic approach

\[ \Delta j_t = D_j \Delta t + \sqrt{D_{jj}} \Delta t \xi_t \]

→ Monte-Carlo process
Constraining the models

→ We can constrain the models
Conclusions

Secular evolution with and without IMBH

a) birth

Sgr A

S2 cluster

w/o IMBH

b) angle averaging

c) phase mixing

d) scalar resonant relaxation

with IMBH

Secular evolution with and without IMBH
Perspectives
Last slide: key infos+images